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THE
ELEMENTS
OF
Astronomy,

PHYSICAL and GEOMETRICAL.

By DAVID GREGORY M. D. *Savilian*
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ford, and Fellow of the Royal-
Society.

Done into English, with Additions and Corrections.

To which is annex'd,
Dr. HALLEY's Synopsis of the
Astronomy of Comets.

In Two Volumes.

Vol. I.

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ADAMS

Nov. 13



THE
AUTHOR'S
PREFACE.

MY design in publishing this Book, was, that the Celestial Physics, which the most sagacious *Kepler* had got the scent of, but the Prince of Geometers Sir *Isaac Newton*, brought to such a pitch as surprises all the World, might, by my care and pains in illustrating, become easier to such as are desirous of being acquainted with Philosophy and Astronomy. The Title informs you sufficiently that the Arithmetical or Calculatory part of Astronomy is here omitted, tho' that, perhaps, may be published hereafter in its proper place. As for the Physics, it is all taken out of the abovemention'd Authors; but is here intermix'd with Astronomy, in such places as seem'd proper and convenient; the Geometry to be met with in it, I have either borrow'd elsewhere, and quoted the place where 'tis to be found,

or deliver'd it Lemmatically. Whatever is done in each Section, you have it express'd either in the Title or Preface thereof, in such a manner, as that those who are less vers'd in the more abstruse parts of Geometry, or less concern'd about the Physical parts, may pass over, and only read the Astronomy separately and distinct from them.

The Celestial Physics, or Physical Astronomy, is not only the first in dignity of all inquiries into Nature whatever, but the first in order, because it is the easiest. For the Sun and Planets are separated from one another by so immense a distance, as renders them incapable of exerting most of those forces whereby all Bodies act upon one another; so that they have no other force left them whereby they can affect one another, but the single force of universal Gravity: Whereas in the production of several Phænomena, that are observ'd upon our Earth, innumerable other forces are exerted, such as are very hard to be distinguish'd from one another; which notwithstanding, if not accurately done, in vain do we attempt Nature, and make any inquiry into it. Upon this account it is, that every Problem in the Terrestrial Physics is very operose and perplex'd, on the contrary, in the Celestial Physics, much more easy and simple; tho' even the latter has its difficulties, arising from the different distances and magnitudes of the Celestial Bodies. For the Fix'd Stars are so vastly distant asunder, that they have no mutual action upon each

each other, observable by us who are the Inhabitants of the Earth. The Primary Planets are remov'd so far from each other, that, tho' they have some small power and effect upon one another, yet we cannot be sensible of it, till after many Years observation. The Secondary Planets are not at so great a distance from their Primaries, or from the Sun, but that they may be considerably affected by the powers of both, (if regard be had to the Quantity of Matter that is in these latter,) and this is the spring of those manifold inequalities found in them, such as, for instance, manifestly shews itself in our Moon; which yet is nothing at all, if compar'd with the inequalities found among Terrestrial Bodies, which are acted upon by an innumerable variety of other forces, pressing every way upon them. So that those persons seem to apply their thoughts but to a very indifferent purpose in the study of Nature, that overlook this part of Astronomy, from whence the principal and most simple Laws of Nature are to be learn'd.

That none may think the Physics deliver'd in the following Work intirely new and unknown in Astronomy, I shall take the liberty to shew that it was both known and diligently cultivated by the most ancient Philosophers. And I shall dwell a little longer upon this argument, because there is no need of spending a Preface, either upon the order of the parts of this Work, which may be seen in the Index, or upon the usefulness, dignity, history and progress

gress of Astronomy, or even of the true System of the World, approv'd of by *Pythagoras*, and others among the Ancients; these things being all of them treated of at large by the common Writers of Astronomy. What I shall now therefore make out is, that we do still tread in the steps of the Ancients in this Physical Astronomy; inasmuch as they knew that the Celestial Bodies gravitated towards each other, and were retain'd in their Orbits by the force of Gravity; and were also apprized of the Law of this Gravity.

For if we look back to the first Rise of Astronomy, and take a view of it in its Infancy, as it were, we shall find nothing better approv'd of, nothing more universally entertain'd among the several Sects of Philosophers, than this notion of the Gravity of the Celestial Bodies. That saying is well known, so often used by ^a *Anaxagoras*, and his Scholars, ^b *Achelaus* and ^c *Euripides*. Namely, “That
“ the Sun and Stars were fiery or red-hot Stones
“ and Golden Clods.” Of the same mind also were ^d *Democritus*, *Metrodorus*, and ^e *Diogenes*,
By

^a He affirm'd the Sun to be a Mass of Red-hot Iron. Diog. Laert. in Anaxag. That the substance of the Sun was Stone, that of the Moon, Earth. Plat. in Apol. Socr.

^b That the Stars were Plates of Red hot Iron. Stob. Ecl. Phys. cap. 25.

^c They say that *Euripides* used to call the Sun a Clod of Gold, or Golden Mass. Diog. Laer. in Anaxag.

^d *Anaxagoras*, *Democritus* and *Metrodorus* affirm'd, that the Sun was a Mass of Iron or Stone red-hot. Plut. de Placit. Phil. lib. 2. c. 20.

^e *Diogenes* thought that the substance of the Stars was not unlike that of a Pumice Stone. Ibid, c. 13.

By these expressions they meant no more, than that they were heavy, dense and fix'd Bodies, (such as Stones are) so as to bear a considerable degree of heat: And that this was really their meaning will evidently appear, if we do but enquire more narrowly into the first Authors of this Opinion. For, as we are told by ^f *Democritus*, these notions about the Sun and Moon are not to be ascrib'd to *Anaxagoras* as their original, for he had really borrow'd them of the Ancients. Nor is it a difficult matter to find out, who they were that he borrow'd them of, or from whom they were handed down to him. He had them from his Master ^g *Anaximenes*, whose Opinion we know was, that the Stars were of a fiery nature and substance, that there were also mingled with them certain Earthly Bodies, which were carried round about them, tho' not visible to us: By which words he plainly means, Planets of a terrestrial nature, performing their revolutions in the System of every Fix'd Star. These notions *Anaximenes* received from *Anaximander*, *Anaximander* from ^h *Thales* himself, who was the Head and Founder of the *Jonic* Philosophy;

A 4

sophy;

^f Favorinus, in his various History, relates, that Democritus used to say of Anaxagoras, that the Opinions which he taught concerning the Sun and Moon, were not his own, but far more ancient than Anaxagoras's time; and that he had stolen them. Laert. in Democrit.

^g Anaximenes said, the nature of the Stars were fiery, and that there were certain terrestrial Bodies that are invisible, carried together about them. Stob. Ecl. Phys. c. 25.

^h Thales was of Opinion that the Stars consisted of an Earthly substance, which was continually red-hot. Stob. Ecl. Phys. c. 25, And so Plutarch. De Plac. Philos. lib. 2. c. 13.

sophy; and spread this opinion of the Gravity of the Fix'd Stars among his Sect. Nor did this Doctrine concerning the Stars stop here, but afterwards it diffus'd it self thro' the *Italic* Philosophy, the ⁱ followers of which taught, that each Star was a World in the infinite Æthereal Space, containing Earth, Air and Æther; and that the ^k Moon, not only was like our Earth, but inhabited by Animals of a larger size, and furnish'd with Plants of a more beautiful appearance.

Nor were they so absurd in their conceptions about Gravity, as to think that it was done by the virtue of any point within the Earth, or of a Center, to which all heavy Bodies placed any where tended; but they thought it was done by the ^l power of the whole Matter in the Terrestrial Globe attracting all things to it self: And as the power of the Loadstone is compos'd of the powers of the several

ⁱ The Pythagoreans affirm'd, that every Star is a World in the infinite Æthereal Space, wherein are contain'd Earth, Air, and Æther, Plut. de Plac. Philos. Lib. 2. c. 13.

^k The Pythagoreans asserted, that the Moon seem'd to be of a similar nature with the Earth, it is inhabited as our Earth is, by Animals, tho' of a larger size than ours, and fill'd with the same Plants, tho' much more beautiful than ours. Plut. de Plac. Phil. Lib. 2. c. 30.

^l And yet if every heavy Body inclines towards the same place, and does with every one of its parts tend to its middle or center, the Earth certainly will not appropriate to itself these heavy Bodies, which are its parts, because it is the Center of the Universe, but rather because it is the whole, of which they are the parts. Plut. de facie in Orbe Lunæ.

As for that which is incorporeal, 'tis not probable, nor will they themselves allow it to be possess'd with so great a power as to draw all things towards it, and retain them about it. Ibid.

several parts combin'd together, so they believ'd that the Gravity towards the whole Earth, resulted from the Gravity towards each single part of it. Besides, they believ'd there was a ^m Gravity towards the Moon and Sun, acting in the same manner as it does towards the Earth; and that each ⁿ Planet, like a Stone, whirl'd in a sling, was kept in its Orbit by the same principle, and for the same reason revolving always about us. From some things mention'd by ^o *Diogenes Laertius*, concerning *Plato*, which also are obscurely hinted at in his ^p *Timæus*, I am apt to believe with ^q *Galileo*, that the divine Philosopher suppos'd the Mundane Bodies, when they were first formed, were moved with a Rectilinear motion (by the means of Gravity,) but after that they

^m And this meeting together of Bodies here, and their coalition likewise with the Earth's Body, shew us the manner how it is probable that the parts, which are assembled at the Moon's Body, continue also there.

ⁿ But the Moon is help'd, and preserv'd from falling down, by her very Motion and that impetuosity of her Revolution; as Stones and other weighty Bodies put in Slings and swung round, are kept from dropping out by the swiftness of their Motion, and their being mov'd circularly. — Wherefore the Moon does not move downwards, as her own weight wou'd naturally carry her, her tendency that way being stop't by the violence of her circular revolution. Ibid.

^o These at first were mov'd in a confus'd and irregular manner, but when they were duely adjusted and rightly settled, then the World was establish'd by God in just order and proportion. *Diog. Laert. in Plat.*

^p He gave it a Motion altogether agreeable to its nature as a Body (that is a direct one,) and a little further. Therefore he afterwards made it continue its course in a Circle. *Plat. in Timæus.*

^q In his Cosinical System.

they had arrived to some determined places, they began to revolve by degrees in a Curve, the Rectilinear Motion being chang'd into a Curvilinear one. 'Tis from this Doctrine of Gravity, that all Bodies gravitate mutually to one another, 'tis by this that ^r *Lucretius*, taught by *Epicurus* and *Democritus*, labours to prove, that the Universe has no Center or lowest Place, but that there is an infinity of Worlds like ours in the immense Space. His Argument runs thus; If the nature of things were bound-ed any where, then the outmost Bodies, since they have no other beyond them, towards which they may be made to tend by the force of Gravity, wou'd not stand in an Equilibrio, but make towards the inner and lower Bodies, being necessarily inclin'd that way by their Gravity, and therefore having made towards one another, during an infinite space of time, would have long ago met, and lye in the middle of the whole, as in the lowest place. 'Tis evident therefore from hence that *Lucretius*, and those whom he followed, believ'd that all Bodies did Gravitate towards the Matter placed around them, and that every single Body was carried by the more prevailing Gravity, towards that region where there was most Matter.

As

^r Suppose they all had Bounds, suppose an End;
 Then Bodies which by Nature must descend,
 And from Eternity pursu'd the Race,
 Had long e'er this time reach'd the Lowest Place. *Lucr.* l. i. v. 986.

As it is manifest that the Ancients were apprized of, and had discover'd the Gravity of all Bodies towards one another, so also they were not unacquainted with the Law and Proportion which the action of Gravity observ'd according to the different Masses and Distances. For that Gravity is proportional to the Quantity of Matter in the heavy Body, ¹ *Lucretius* does sufficiently declare, ² as also that what we call light Bodies, don't ascend of their own accord, but by the action of a force underneath them, impelling them upwards, just as a piece of Wood is in Water; ³ and further, that all Bodies, as well the heavy as the light, do descend *in vacuo*, with an equal celerity. It will be plain likewise, from what I shall presently observe, that the famous Theorem about the proportion whereby Gravity decreases in receding from the Sun, was not unknown at least to *Pythagoras*. This indeed seems to be that which he and his followers would signify to us by the Harmony of the Spheres: That is, they feign'd *Apollo* playing upon an Harp of seven Strings, by which Symbol, as it is abundantly evident from ^{*} *Pliny*, *Macrobius* and *Censorinus*, they meant the Sun in Conjunction with the seven Planets,

for

¹ Besides; why have not Bodies equal Weight
 With those, whose Figure is but just as great? *Lucr.* l. 1. v. 415.
² And this I think a proper place to prove,
 That nothing of it self can upwards move. *Lucr.* l. 2. v. 178.
³ Therefore thro' Void, unequal Weights must be,
 Like Swift in Motion, all of like degree. *Lucr.* l. 2. v. 228.
^{*} *Plin.* lib. 2. c. 22. *Macrobius*, lib. 1. c. 19. *Censorinus*, c. 11.

for they made him the leader of that Septenary Chorus, and Moderator of Nature; and thought that by his Attractive force he acted upon the Planets (and called it Jupiter's Prison, because it is by this Force that he retains and keeps them in their Orbits, from flying off in Right Lines) in the Harmonical ratio of their Distances. For the forces, whereby equal Tensions act upon Strings of different lengths (being equal in other respects) are reciprocally as the Squares of the lengths of the Strings.

y For *Pythagoras* as he was passing by a Smith's Shop, took occasion to observe, that the Sounds the Hammers made, were more acute or grave in proportion to the weights of the Hammers; afterwards stretching Sheeps Guts, and fastning various Weights to them, he learn'd that here likewise the Sounds were proportional to the Weights. Having satisfy'd himself of this, he investigatèd the Numbers, according to which Consonant Sounds were generated. Whether the whole of this Story be true, or but a Fable, 'tis certain *Pythagoras* found out the true ratio between the sound of Strings and the Weights fasten'd to them. The same Tension acts upon a String as short again, four times more powerfully: For it produces an Octave, and an Octave is founded by a force that is four times greater; for if a String, put upon the stretch by a given Weight,

Weight, generates a given Tone, the same String stretch'd by a Weight four times greater, will found an Octave. Thus likewise the same Tension upon a subfesquialteran Chord, acts in a double fefquiquartan ratio: For it generates a Fifth or Diapente, and a String that founds a given Note, with a given Weight ought to be stretch'd by a Weight that is a double fefquiquartan to found a Fifth. And univerfally, the Weights which generate all Tones in Strings, are reciprocally as the Squares of the lengths of Strings of equal Tension, producing the same found in any Musical Instrument. *Pythagoras* afterwards applied the proportion he had thus found by experiments, to the Heavens, and from thence learn'd the Harmony of the Spheres. And, by comparing these Weights with the Weights of the Planets, and the intervals of the Tones, produced by the Weights, with the interval of the Spheres; and laftly, the lengths of Strings with the Distances of the Planets from the Center of the Orbs; he understood, as it were by the Harmony of the Heavens, that the Gravity of the Planets towards the Sun (according to whose measures the Planets move) were reciprocally as the Squares of their Distances from the Sun.

We have thus far been shewing what was the Opinion of the Ancients concerning Gravity; and it is evident they were perswaded that Gravity was not an affection of Terrestrial Bodies only, but of the Celestial also,
that

that all Bodies gravitate towards one another, and that the Planets are retain'd in their Orbits by the force of Gravity, and lastly, that the Gravity of the Planets towards the Sun are reciprocally as the Squares of their Distances from it. What the industry and skill of the Moderns have added to these inventions of the Ancients, the following Pages do declare at large.

T H E

THE
PUBLISHER
TO THE
READER.

THE Science of ASTRONOMY, which is as much esteem'd and admir'd for its great and manifold uses for the Service of Mankind, as it is delightful and entertaining to the more curious and contemplative, has in all ages been cultivated and improv'd, by Men the most eminent for their parts and learning; and is now brought, as it were, to the utmost degree of perfection, and that chiefly by the superior Genius and Industry of those of our own Nation. But since nothing considerable therein, has been as yet writ in our own Language, I thought I could not oblige my Country-Men more than in publishing an English Edition of the most valuable and finish'd piece of ASTRONOMY now extant. It is generally reckon'd

The Publisher to the Reader.

reckon'd to be a Book that contains not only all the Discoveries and Philosophical Sentiments of the great Kepler, and the various Hypotheses of the most noted Astronomers before and since his Time; but is chiefly valued by the best Judges, for the large and instructive Comments deliver'd in it, on the Writings of the illustrious Sir Isaac Newton, as well as on the several Astronomical Dissertations of the sagacious Dr. Halley, which the Reader will find here every where interspers'd.

And in order to render this work as compleat as possible, I shall, in a very little time, present you with another Volume, containing correct Astronomical Tables, for the ready computing of the Planets Places, Eclipses, &c. all done by a Person of known ability, from the true Theory of Gravity, deliver'd in this Book: For it was by no means judg'd proper that I should annex to so intire a piece as this is, any imperfect Tables, drawn from a different Principle from what is here established, such it seems all those as yet published are.

T H E

THE
ELEMENTS
OF
Astronomy,
Physical and Geometrical.

The FIRST BOOK.
Of the System of the World.

SECTION I.

Concerning the Order, Distances, and Periods of the Primary Planets revolving about the Sun, and the principal Phænomena thence arising.

PROPOSITION I.

TO give a general account of the Order, and Periods of the Primary Planets revolving about the Sun, and their Distances from it; as also what we are to think of the Comets and Fixt Stars.

The Sun is to be look'd upon as immovable, and placed in the midst of that immense Space,
B in

in which the Planets perform their Revolutions. And there are six opake Spherical Bodies that revolve about it, as their Center, from West to East, [*Fig. 1.*] from *A*, along *B*, *C*, and *D*, in the following Order: Mercury nearest the Sun, completing its Revolution in about three Months; next to Mercury, Venus in about seven Months and an half; then the Earth in a Year; Mars in about two Years; Jupiter in twelve; and, last of all, Saturn, which is outermost, in thirty. Their Distances from the Sun are nearly the same as they are represented in the Scheme: namely, supposing the distance of the Earth from the Sun to be divided into ten equal Parts, of these the distance of Mercury will be about four, of Venus seven, of Mars fifteen, of Jupiter fifty two, and that of Saturn ninety five.

'Tis to be observ'd, that all their Orbits are not in the same Plane, but variously inclin'd to one another; so that supposing the Plane of the Earth's Orbit to coincide with the Plane of this Scheme, one half of the Plane of any other Planet's Orbit will be above, and the other half below it; so that the Planes intersect in a Line that passes thro' the Sun.

Besides the abovemention'd Planets moving round the Sun, there are likewise Comets; and these, if they are lasting in their Nature and Motions, move in very excentric Orbits, and consequently shew themselves only, when they are in those Parts of their Orbits that are nearest to us and the Sun: Some of them move, like the Planets, from West to East, others from East to West; some again from North to South, and lastly, others from South to North. Their Orbits are various and different in Magnitude, Situation, and Inclination both to one another, and to
the

Fig: 1.
p: 2

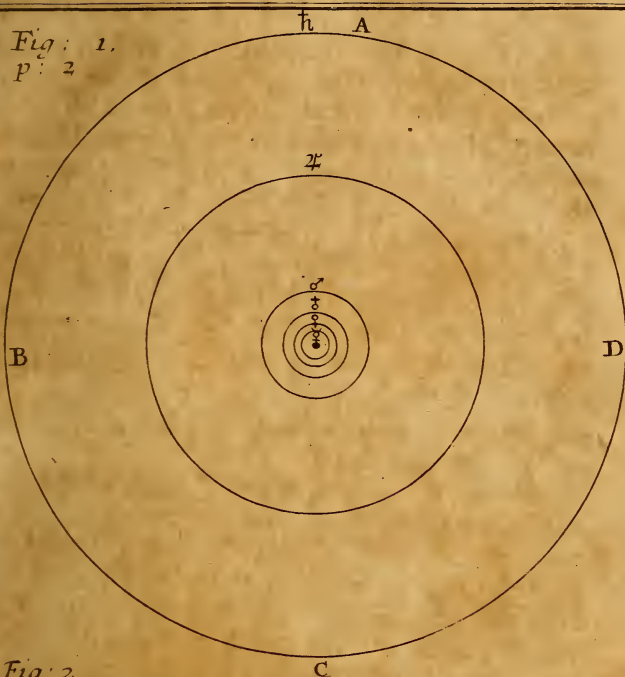


Fig: 2
p: 4

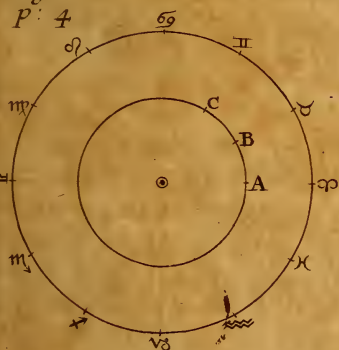
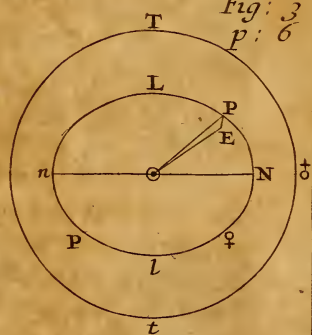


Fig: 3
p: 6



the Orbits of the Planets. Their Periods are not as yet known from Observation, nor indeed is it fully certain, that they move in Lines that return into themselves. But all these things, perhaps, will be determined by proper Observations made in future Ages.

The rest of the Mundane Space is to be conceived as divided into Spaces just like that we have been describing, each having one of those Stars in its Center, which are called *Fixt Stars*, performing the office of a Sun to it, and having, it may be, Planets and Comets of its own revolving about it.

It is enough at first to have a general conception of these things as they are here described: For tho' the Periods of the Planets, and their Distances from the Sun, as here laid down, are not exactly true, yet they are nearest the Truth in round Numbers. Again, tho' the Paths they describe are not perfect Circles, concentric to the Sun, and the Motion of the same Planet not perfectly equable; yet the difference is so small, that they need not, at present, be taken otherwise; till by Observations and Methods of using them, hereafter to be shewn in their proper places, all these things shall be precisely and exactly settled, and these niceties examined; or, at least, till the same are purposely handled, in order to inquire into their Physical Causes.

PROPOSITION II.

TO describe the *Phænomena* that arise from the situation of the Sun and motion of the Earth, as related above.

First, If the Observer be suppos'd to be plac'd in the Sun, 'tis evident the Earth will seem to him to move from West to East perpetually, as

really it does. Then since, besides the Earth, our Observer sees the Fixt Stars plac'd round him, as it were (the nature of the Eye requiring it) in a concave Sphere, that has the Eye for its Center; 'tis likewise evident, that he will observe the Earth moving, as it were, among the Fixt Stars, and approaching nearer and nearer the more Eastern ones, till, in a Year's space, having compleated its Revolution, it returns to the same place among them again. And because the Earth always goes the same track over again, the Observer will take especial notice of the Stars the Earth passes over; also the Plane of the Earth's Orbit, and the Circle in the Sphere of the Fixt Stars, called the *Ecliptic*, made by that Plane; and this will be a great Circle, because it passes thro' the Sun, or Eye, which is the Center of that Concave Sphere that terminates the Sight. But if the Observer, for the advantage of making Observations, imagines this *Ecliptic* divided into twelve equal Parts, or *Signs*, calling them by the name of any neighbouring Constellation, or Figure those Stars seem to make; in this case, I say, [Fig. 2.] the Earth will seem to move from γ to δ , and from thence to π , and so on; from West to East, thro' all the Signs, till it return to γ again.

Secondly, If you imagine the Observer to be removed from the Sun to our Earth, if the Earth be at *A*, where it is seen from the Sun among the Fixt Stars, at γ ; the Sun will appear, when seen from the Earth, in the opposite Sign π , among the Fixt Stars; the Earth being then the Center of the Sphere of the Fixt Stars: For the place of the Eye is the Centre of a Sphere, on whose surface the Stars are conceiv'd

ceiv'd to be plac'd. If the Earth be moved from *A* thro' *B*, to *C*, in its Orbit, or if look'd upon from the Sun, from γ thro' δ , to π , *in consequentia*, or according to the order of the Signs; the Sun, to an Observer on our Earth, that thinks the place he stands upon immovable, will appear to move among the Fixt Stars, according to the order of the Signs also, from α , thro' μ , to τ , &c. in the same Plane, during the same time, and towards the same Region of the Heavens, as the Earth seen from the Sun does, but in the opposite Points of the Ecliptic.

SCHOLIUM.

The like Phænomena happen in respect of the Sun and any other Planet; nay indeed, the very same, excepting that the time of that Planets revolution about the Sun, or the Sun's apparent revolution about the Planet, when view'd from that Planet, is various, according to the different Period of each, mention'd in the foregoing Proposition; and that the Plane of the Orbit of that Planet produced, will cut other Stars than those which the Plane of the Earth's Orbit does, when produced; and consequently, that the Path of the Sun among the Fixt Stars, seen from any other Planet, is different from its Path, when seen from the Earth, that is, from the Ecliptic.

PROPOSITION III.

TO describe the Phænomena of the Planets seen from the Sun, arising from their Motion in Orbits, whose Planes are inclin'd to the Planes of the Ecliptic.

Since the Orbits of the Earth and Planets are so situated, as that their Planes are inclin'd to each other, and intersect each other, (as was

shown in general in *Prop. 1.*) in right Lines passing thro' the Sun; the Inclination of the Plane of the Orbit of each, to the Plane of the Ecliptic, or Earth's Orbit, is to be taken into consideration, in explaining the Phænomena of the Planets viewed from the Sun. For the Plane of the Ecliptic is taken by Astronomers as the Standard to which the Planes of the other Orbits are judged to incline; and that with very good reason, since it is that in which the Earth (the habitation of the Astronomer) moves round the Sun, or in which the Sun seems to move round the Earth: And an Observer plac'd in any other Planet, would make the Plane of that Planet's Orbit, the Standard of all the rest, and consider them as inclin'd to it.

The right Line which passes thro' the Sun, and is the common Section of the Plane of the Orbit, with the Plane of the Ecliptic, is call'd the *Line of the Nodes* of that Planet, and the Points themselves, wherein the Orbit of the Planet cuts the Ecliptic, are call'd the *Nodes*. Thus, [*Fig. 3.*] let $\odot T \pm t$ be the Plane of the Orbit of the Earth produced indefinitely, NPn the Orbit of any Planet, intersecting the Plane of the former Orbit or Ecliptic in N and n , which are the Nodes of that Planet; so as that one part NPn of that Orbit be suppos'd above the Plane of this Scheme, and the other, npN , below it, (which makes it look like an Ellipse:) The right Line Nn joining the Nodes, being the common Section of the Plane of the Orbit of the Planet with the Plane of the Ecliptic, is the *Line of the Nodes*.

'Tis evident then, that if a Planet be seen from the Sun, when it is in one of the Nodes, as N , it will appear to be in the Plane of the Ecliptic. But when gone forwards as far as p , it will

seem

seem to deviate from the Ecliptic: and (by *Def. 5. Elem. XI.*) the Inclination of the right Line $\odot P$, to the Ecliptic, and consequently of the Planet at P seen from the Sun, call'd the Planets *Helio-centric Latitude*, is measur'd by the Angle $P \odot E$, where PE is suppos'd to be a perpendicular let fall from P to the Plane of the Ecliptic. This *Helio-centric Latitude* is continually upon the increase, till the Planet is got as far as L , its *Limit*; where it is equal to the Inclination of the Plane of the Planet's Orbit, to the Plane of the Ecliptic. But during the Planet's passage from thence to the other Node n , it is decreasing, till at last it vanishes at the Node. Having pass'd the Node n , it begins again, changing its name, because 'tis towards the contrary parts of the Ecliptic, and grows bigger and bigger, till the Planet has arriv'd at the other Limit l , from whence again it grows less and less, till it vanishes at the other Node N .

The *Orbs* of the Planets (that is, the Planes of their Orbits) are inclin'd to the Ecliptic in the following manner: The Orb of Saturn makes an Angle of $2 \frac{1}{2}$ Degrees; Jupiter, $1 \frac{1}{3}$ Degree; Mars, a little less than 2 Degrees; Venus, something above $3 \frac{1}{3}$ Deg. Mercury almost 7 Degrees. An account of the Position of the Line of the Nodes of each Planet shall be given in a more proper place. We have, in these two last Propositions, consider'd the Planets as they appear to one plac'd in the Sun: because it was necessary to the understanding their Motion when view'd from the Earth.

PROPOSITION IV.

TO describe the Phenomena arising from the Motion of the Earth, and inferior Planets, Venus and Mercury, when view'd from the Earth.

Since Venus and Mercury revolve about the Sun, in lesser Orbits than the Earth, as you see in the Scheme, [Fig. 4.] where *T* represents the Earth, carried in its Orbit *T* γ from West to East; and *ACEG*, the Orbit describ'd by Venus in a less space of Time the same way; 'tis evident, that when Venus is in *D E F*, that part of its Orbit that is farthest off the Earth, it will appear to us on the Earth to move *in consequentia*, or according to the order of the Signs, and is then said to be *Direct*. When it is in *G*, moving from thence to *H*, it will appear to move as swift as the Sun, because then its Motion tends directly towards the Earth, and it does not seem to move at all, but as its Orbit is carried along by the Sun, whose Motion is towards the East: Venus moves now therefore slower than before, but is still *Direct*. When it is got beyond *H* in its Motion, thro' *A* to *B*, it passes between the Earth and Sun, because it is nearer to us than the Sun, and moves swifter than the Earth, (the cause of which we shall hereafter assign,) consequently will seem to us to change its Place among the Fixt Stars, and move *in antecedentia*, or contrary to the order of the Signs, and then it is said to be *Retrograde*, tho' really *Direct* still, if view'd from the Sun. Between *Direct* and *Retrograde*, for instance, about *H*, it will appear *Stationary*, the right Lines that join the Earth and Venus, continuing for some sensible time parallel. Thus likewise, after its Retrogradation, before it becomes *Direct* again, it will appear *Stationary*.

Fig: 4.
p: 8⁴



Fig: 5
p: 10

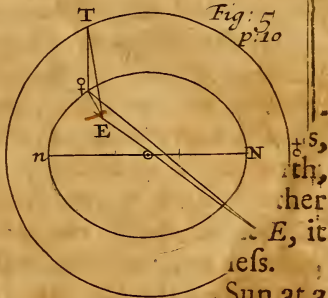


Fig: 6
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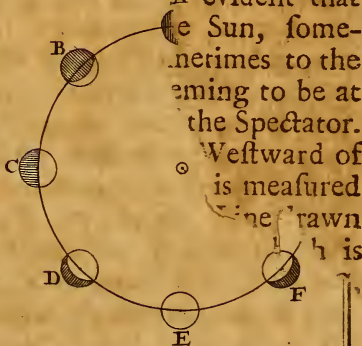
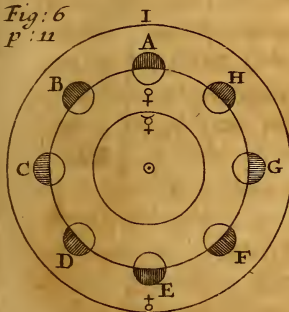


Fig: 8
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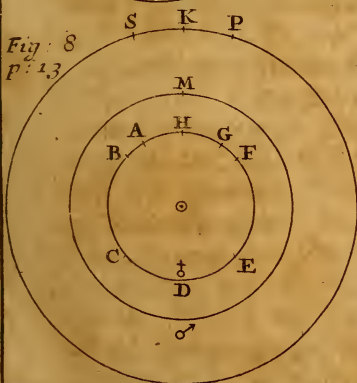
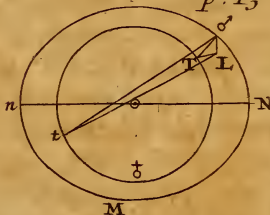


Fig: 9
p: 15



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onary a second time, and is Stationary in order to its being Direct, as it was before in order to its being Retrograde at *H*. In all this affair, regard is to be had to the Motion of the Earth, for Venus is Direct, Stationary or Retrograde, according as it is posited in such parts of its Orbit as have the same relation to the Earth in its Motion as the Points aforementioned have. From what has been said, 'tis evident that Venus, when Retrograde, as at *A*, is nearer to the Earth, and consequently appears bigger than at other times; or the contrary when Direct, as at *E*, it is farther off, and consequently appears less.

And because Venus moves round the Sun at a less distance than the Earth; 'tis evident that it will seem always to attend the Sun, sometimes to go to the West of it, sometimes to the East: all the Heavenly Bodies seeming to be at an equal distance from the Eye of the Spectator. This digression to the Eastward or Westward of the Sun is call'd the *Elongation*, and is measured by the Angle contain'd under a right Line drawn from the Eye to the Sun and Venus, which is never greater than the Angle $\odot TC$ or $\odot TG$, if the Lines *TC* or *TG*, when drawn, are Tangents to the Orbit *ADF*. Consequently the Elongation of Venus will never be above half a Quadrant from the Sun, as is evident from the Semidiameters laid down in *Prop. i*. And when it has arrived to its farthest Elongation it will return to the Sun, and pass as far beyond on the other side as if its Motion were Oscillatory.

Mercury has all the same Phænomena; but its Directions, Stations, and Retrogradations happen oftner, because it finishes its Course in a shorter time, and consequently overtakes the Earth,

Earth oftner than Venus. And since Mercury's Orbit is less than that of Venus, its greatest Elongations must be also less, and it must be a nearer and more constant attendant of the Sun, being, by *Prop. i.* never a whole Sign distant from it, and consequently seldom to be seen by us.

PROPOSITION V.

TO describe the Phænomena of the Latitude of the inferior Planets seen from the Earth.

Let $T\ \&\ t$ be the Earth's Orbit, whose Plane is the same with that of the Ecliptic; [*Fig. 5.*] and let $N\ \&\ n$ be the Orbit of an inferior Planet, for instance Venus, whose Plane is inclin'd to that of the Ecliptic, and therefore will look like an Ellipse, whose greater Axe is the intersection of the Planes, or Line of the Nodes Nn . And while Venus is in $\&$, let the Earth be in T ; in which supposition Venus will be nearest the Earth, and Retrograde, by *Prop. iv.*

'Tis evident from *Def. 5. Elem. xi.* that the Inclination of the right Line $\&\ T$ to the Plane of the Ecliptic, or the Latitude of Venus in $\&$, seen from the Earth (which is hereupon called the *Geocentric Latitude*) is measured by the Angle $\&\ T\ E$, the right Line $\&\ E$, being made perpendicular to the Plane of the Ecliptic. If Venus be suppos'd to continue in $\&$, and the Earth to be at t , in which supposition Venus is Direct, and furthest off the Earth; the Geocentric Latitude of Venus will be the Angle $\&\ t\ E$, less than $\&\ T\ E$, almost in the ratio of $T\ \&$ to $t\ \&$, tho' the Helio-centric Latitude of Venus be in both cases the same.

And all that has been now said is true of Mercury as well as of Venus; consequently all other

other circumstances being alike, the Latitude of the inferior Planets is greater, when they are Retrograde and nearest the Earth; less, when Direct and farthest off.

Moreover if any inferior Planet be most Retrograde and nearest the Earth, and at the same time in or near a Node, it will be found directly between the Observer and the Sun: If it be at a considerable distance from a Node, it will pass the Sun to the Northward or Southward of him. In like manner, if it be most Direct and farthest off the Earth, and be also near, or in a Node, it will be cover'd by the Sun; if it be otherwise situated, it will pass on one side of the Sun.

PROPOSITION VI.

TO explain the Phænomena of the Inferior Planets, arising from their being opake Spherical Bodies enlighten'd by the Sun.

Since all the Planets, as well as the Earth, are opake Spherical Bodies, reflecting the Rays of Light that fall upon them every way, 'tis evident that that half of each Planet which is exposed or turn'd to the Sun, is enlighten'd by it, while the other half, which is turn'd from the Sun, is in darkness. And since that half only which is towards the Earth is seen by the Observer, if we consider which Face of Venus is enlighten'd in this or that situation in respect of the Sun, together with the Face that is visible at the Earth, [Fig. 6.] in the part *T* of its Orbit *Tg*, it will be manifest, that when Venus is in *A*, that is, most Retrograde and nearest to us, it is not visible at all, having its dark Face turn'd to us: And if Venus were at the same time in the Plane of the Ecliptic, that is, in one of its Nodes, it would appear like a Spot in the Sun,

be

being situated directly between the Earth and Sun, by the foregoing Proposition.

When Venus is got into the situation *B*, that is, begins to be Retrograde, if view'd from the Earth, then some part of the enlighten'd half is turn'd towards the Earth, tho' the greater remaining part belongs to the obscure half. And because Venus is of a spherical Figure, and appears like a Plane, its enlighten'd part will appear Horned, and those Horns turned towards the West, or from the Sun.

When Venus is at *C*, half the enlighten'd part will be seen by us, and then it is said to be half Full; at *D* it is said to be Gibbous, above half the enlighten'd part being visible; and at *E*, where it is farthest off the Earth, and most Direct in its Motion, it appears Full, all the enlighten'd Hemisphere being turned towards us.

Venus will have the same variety of Phases in its passage thro' *F*, *G*, and *H*, that is, it will be Gibbous at *F*, half Full at *G*, and Horned at *H*, with its Horns turned from the Sun, that is, in the present case, looking towards the East, exactly contrary to what they were when it was at *B*.

For the better understanding of this matter, we have delineated the several Phases of Venus, as they appear to us, in the Scheme [Fig. 7.]; where the same Letters are used that were in the Diagram [Fig. 6.] representing its Phasis when it was in that part of its Orbit, marked by the same Letter. Thus, in this Scheme, [Fig. 7.] at the Point *A*, it is drawn all obscure, because that is its condition in respect of the Earth at *T*, when Venus is at *A*, in the former Diagram. At *B* it is represented Horned,
be-

because it appears so, when it is at *B*, in the former Scheme, and look'd upon from the Earth at *T*; and so on in the other cases of *C*, *D*, &c.

The like Phænomena will appear in Mercury, regard being had to its Orbit and Revolution.

SCHOLIUM.

As the Phænomena described in the three last Propositions, manifestly follow upon the Situation and Motion of the Earth, Venus and Mercury, laid down in *Prop. 1.* So it follows conversely, that the Observation of these Phænomena in them, establishes and confirms that Order and Situation, namely, that Venus and Mercury revolve about the Sun, in Orbits, that are included within the Earth's Orbit.

PROPOSITION VII.

TO describe the Phænomena arising from the Motion of the Earth, and of the superior Planets, Mars, Jupiter, and Saturn. [Fig. 8.]

Let *M* ☿ be the Orbit of any one of the superior Planets; for instance Mars, *AC* ☿ *G* the Orbit of the Earth, nearer to the Sun. 'Tis evident, first, that this Planet will not always attend the Sun, but sometimes be diametrically opposite to it: For the Earth finishing its Revolution sooner than any of the superior Planets, will sometimes be exactly between the Sun and that Planet; thus, when Mars is in *M*, the Earth may be at *H*; and, to speak universally, the Angle at the Earth, made by lines drawn thence to the Sun and that Planet, may be equal to any given one.

Let us suppose Mars to be in *M*, and the Earth at the same time in *A*; Mars in this case will appear Stationary, in order to its being Direct, because the right Lines that join the Earth and Planet

Planet at that moment, will continue parallel for some sensible time; during the whole of which, notwithstanding, Mars will seem to go forwards, as usual, if viewed from the Sun.

While the Earth moves along thro' *B, C, D, E, F* and *G*, Mars likewise will seem to go forwards among the Fixed Stars, upon a double account; first, because it really does move about the Sun, *in consequentia*; and then again, because the Earth in the opposite Semicircle is carried the same way, and about the same Center: And, consequently, Mars in this case being most remote from the Earth, and in conjunction with the Sun, view'd from the Earth, will seem to move faster than ordinary, *in consequentia*, and become Direct. But when the Earth is arriv'd to the Point *G*, in respect of Mars at *M* (which some time or other will happen, tho' Mars be carried in the mean while about the Sun, namely, when the Earth has almost overtaken Mars) Mars will again become Stationary, in order to its being Retrograde, as it will be soon after. For when the Earth in its Motion from *G*, thro' *H* to *A*, has pass'd Mars, and that Planet is seen in opposition to the Sun, and biggest, because nearest the Earth, which is lower and swifter, will make Mars appear to move *in antecedentia* from *S* thro' *K* to *P*; whereas in the mean while view'd from the Sun, it seem'd to move as always before, *in consequentia*.

The like Phænomena will happen to Jupiter and Saturn, excepting that Saturn's Retrogradations are more frequent than Jupiter's, and Jupiter's than Mars's: because the Earth oftner overtakes Saturn than Jupiter, and Jupiter oftner than Mars, and passes between them and the Sun.

PROPOSITION VIII.

TO describe the Phenomena of the Latitude of the Superior Planets seen from the Earth.

Let the Earth's Orbit be $Tt\delta$; [Fig. 9.] that of any superior Planet as Mars δM , whose Plane is inclined to that of the Ecliptic, and cuts it in the Line of the Nodes $N \odot n$: Let the situation of Mars and the Earth, to the Sun, be such, as that Mars being in δ , the Earth may be in T , almost between Mars and the Sun; in which case Mars is both nearest the Earth, and consequently biggest and most Retrograde, as was shown in the foregoing *Propos.* Its Geocentric Latitude will be measured by the Angle δTE , δE being suppos'd perpendicular to the Plane of the Ecliptic. But if Mars continuing in the same situation in its own Orbit, and consequently having the same Heliocentric Latitude, the Earth being supposed be in t , the Sun being between it and Mars; in which Case, by the foregoing, Mars will be farthest off, and consequently least, and most Direct in its Motion; Its Geocentric Latitude measured by the Angle δtE , is always less than the Angle δTE , in the ratio of the Distances of the Earth from Mars, that is, of the right Lines $T\delta, t\delta$. Thus in whatever situation Mars and the Earth be placed, in respect of the Sun, its Geocentric Latitude will be changed, so as, *cæteris paribus*, it will be less as Mars is nearer to a Conjunction with the Sun, and its swiftest direct Motion; and greater, as it is nearer its Retrogradation and Opposition to the Sun.

'Tis evident from what has been said, that none of the Superior Planets can ever be seen from the Earth to cover the Sun; tho' any of them may be covered by the Sun, when it is Direct and pretty near a Node.

PROPOSITION IX.

TO describe the Phænomena of the Superior Planets, arising from their being Opaque Bodies, and enlightned by the Sun.

Saturn and Jupiter being Opaque Bodies, and illuminated by the Sun, that half of each Planet which is turn'd towards the Sun, (that is, the illuminated half) is likewise turn'd towards the Earth, which is never far off from the Sun, or Center of Saturn and Jupiter's Orbit: For, by *Prop. i.* Jupiter's distance from the Sun is above five times, and Saturn's almost ten times greater than the Earth's distance from it.

In Mars indeed it is something different: For the distance of Mars from the Sun being but half as much more as the Earth's Distance from it; its inlighten'd Hemisphere, towards the Sun, is not always, as to Sense, turn'd towards the Earth. [*Fig. 10.*] Let T be the Earth's place in its Orbit $T\odot$, 'tis evident that Mars being at A or B , in Conjunction or Opposition to the Sun, has the same Face towards the Earth, as it has towards the Sun, that is, its enlightened one, and consequently appears Full; but in the situation of the Points D or C (when the Angle $\odot CT$, or $\odot DT$ is greatest, or when $\odot TC$, or $\odot TD$ is almost a right Angle) neither is the whole enlighten'd Face seen, nor is that Face that is seen entirely illuminated, but it appears Gibbous, the light being deficient a little towards those parts that are turn'd from the Sun.

SCHOLIUM.

As the Phænomena described in these three last Propositions follow from the Order and Motion of the Earth, Mars, Jupiter, and Saturn,
laid

laid down in *Prop. 1.* So by the converse of them, the Observation of these Phænomena settles and establishes that Order.

PROPOSITION X.

TO describe the Phænomena of the Motion of Comets, seen from the Earth.

Because different Comets have different Orbits, which can't be determin'd till some Observations have been made about them: 'Tis evident that during their descent, near their Perihelium, or while they are in that part of their Orbit, which is within the Region of the Planets, they have the same Phænomena as the Planet next them has, regard being had to the Velocity and Inclination of the Orbit of the Comet.

Consequently a Comet's apparent Path among the Fixt Stars, will be nearly a Great Circle; excepting that its deviation from it, on this or that side of it, will be like that of a Planet, according as the Motion of the Earth is, which carries along with it the Spectator. For were the Comet view'd from the Sun, it would describe a Great Circle among the Fixt Stars accurately. But its deviation from one, on the account of the Motion of the Earth round the Sun, will not be very sensible in a small space of time.

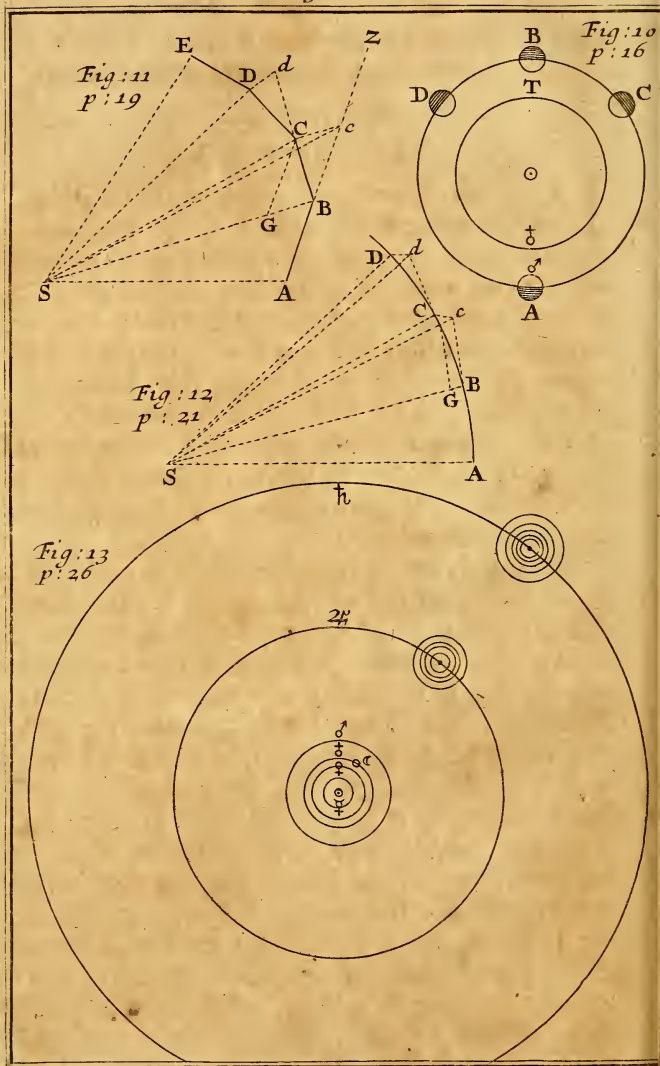
If the Earth happen to be between the Sun and a Comet, that moves *in consequentia*, the Comet will be Retrograde; for the Earth moves so much the swifter, as the right lines that connect the Earth and Comet converge towards the Region beyond the Comet: But if the Earth moves slower, then the Comet's Motion, by taking away the Earth's Motion, likewise becomes slower: But if the Sun be between the

C
Earth

Earth and the Comet, the Comet's Motion will appear swifter than really it is. On the contrary, the Comets that move *in antecedentia* are swifter than they should be, when the Earth is between them and the Sun; and slower, or perhaps moving according to the natural Order of the Signs in appearance, when the Earth is situated on the contrary side. All this happens from the Motion of the Earth, and its various Position, as it does in the Planets; that are, according as the Motion of the Earth falls in with, or is contrary to their Motion, sometimes Retrograde, sometimes Slower, sometimes Swifter than they should be, as has been shewn in *Prop. iv* and *vii*. Those Phænomena will be most sensible, and easily taken notice of, a little before the disappearance of the Comet; its apparent Motion being at that time slower, and the Earth's Motion proportionably greater, with which the Spectator is carried.

The Latitude of the Comet likewise, *cæteris paribus*, is varied by the different Situation of the Earth, it being greater in Opposition; but in Conjunction greater or less, according as the Comet is between the Earth and Sun, or the Sun between the Comet and the Earth, as was demonstrated of the Planets in *Prop. v* and *viii*.





SECTION II.

Of the Direction of the Forces, which retain the Primary Planets in their Orbits.

PROPOSITION XI.

IF a Body be moved according to the Direction of any given right line as AZ , [Fig. 11.] and at the same time be urged by a Centripetal Force tending towards a certain given immoveable Point as S , situated without the aforesaid right line; the Line describ'd by the Body will be a Curve, and Concave towards S , lying all of it in the same immoveable Plane passing thro' the right line AZ and the Point S : And the Areas contain'd under any portions of the Curve and Right lines drawn to the Center S , are to one another, as the Times wherein those portions of the Curve were described.

Let the Time be imagin'd to be divided into equal parts; in the first of which let the Body describe the line AB , a part of AZ , by its *Vis insita* alone, by which it tends to move according to the Direction of the Line AZ ; in the second part of Time it will describe the Line Bc , equal to the former AB , if nothing hinder it: For by the first and chief Law of Motion, all Bodies once put into Motion, and not meeting with any Impediment, will continue to move uniformly on, in the same right line they were first moved in. But when the Body is arrived at the Point B , let us suppose a Centripetal Force tending towards the Point S , to act upon it by a single impulse, so as that, had it been impell'd only by that Impulse, it would in the second part of Time describe the line BG . 'Tis evident

that drawing the Line cC parallel to BG , thro' the Point c , and GC to Bc , thro' G , the Body acted upon by both Impulses at once, will arrive, in the second part of time, at the Point C , describing the right line BC . For as it is well known in Mechanics, a Body describes the Diagonal of a Parallelogram, both the Forces combin'd, in the same time as it would do the Sides, with the Forces separated. 'Tis certain likewise that the right line BC is in the Plane of the Parallelogram $BGCc$, both of whose Sides BG and Bc is in the Plane of the Triangle ASB , that passes thro' the Center S , of the Centripetal Forces, and the immoveable right line AZ . Besides, the Triangles SCB , ScB are equal, because they are upon the same Base BS , and between the Parallels SB , Cc ; but ScB , $SB A$ are equal, because their Bases are equal, and height the same: Consequently SBA , SCB also are equal. By the same method of reasoning, if in the third particle of Time, a Body describes any other right line as CD ; it may be proved that the Triangle SCD is equal to the Triangle SBC , and that the right line CD is in the same Plane with the right lines SB , BC ; that is, in the same with that which is drawn through the right line AB , and the Point S . And so we may go on, as long as the Motion is continued, and in equal times the Area described by Radii drawn to the immoveable Center of the Forces, will be equally increased; and by compounding Ratios, any sums of Areas, are to one another, as the Times wherein they are describ'd. The Line describ'd by the Body will be in the immoveable Plane, and it will be concave towards S , because every part of it that is a right line, as BC , declines from the foregoing

AB ,

AB , towards the Center. If you suppose the number of the Triangles, SAB , SBC , SCD to be augmented, and their breadth to be diminished in *infinitum*; their Bases, AB , BC , CD , will form a Curve Line, concave towards the same parts, and lying in the same Plane; and the Centripetal Force, which before acted, as it were by starts, and at equal intervals of Time, whereby the Body is continually drawn off from the Tangent of that Curve (which the bases of the Triangles measure) now acts constantly: And any Areas thus describ'd, $SABCS$, $SABCDES$, will be, as before, proportional to the Times of description. Q. E. D.

PROPOSITION XII.

A Body moved in a Curve Line $ABCD$, [Fig: 12.] describ'd upon a Plane, and concave towards the same parts, and by a Radius drawn to S , an immoveable Point situated in the same Plane towards the Concavity of the Curve, describing Areas proportional to the Times, is urged by a Centripetal Force tending to the Point S .

Let such a Curve be imagined to be divided into the parts AB , BC , CD , &c. differing as little as can be from right lines, so as to be described in equal Particles of Time: Let the Centripetal Force likewise be conceiv'd to act only in the points B , C , D , &c. by starts, as in the foregoing Prop.. Let AB be produc'd to c , so as that Bc be equal to AB ; in like manner BC to d , till Cd be equal to BC ; and so on. The Triangle SAB , will be equal to SBC , because, by Hypothesis, the Areas described are proportional to the Times; and SAB is equal to SBC , because AB is equal to Bc : Wherefore SBC is equal to SBC , and consequently (by 39

Elem. 1.) Cc is parallel to SB . Moreover, the Body moved along AB , in the first particle of Time, by its *Vis insita* alone would describe Bc , in the second; but it really does describe BC , in that second Particle of Time: Wherefore the Force acting in the Point B , in conjunction with the *Vis insita* to carry the Body along BC , is directed according to a Right line parallel to Cc ; that is, according to the Right line BS . After the same manner the Force acting in the point C , and in conjunction with the *Vis insita*, (by which alone the Body would describe the Line Cd in the third Particle of Time) carrying it in the same time along CD , is directed according to a Right line parallel to dD , that is, according to CS : But the Right lines BS , CS , &c. tend towards the Point S . Therefore the Centripetal Force, that draws the Body off from the Tangent of the Curve, acts along right lines tending to the immoveable Point S . Q. E. D.

PROPOSITION XIII.

THE Forces whereby the Primary Planets, Mercury, Venus, Mars, Jupiter, and Saturn, are drawn off from their Rectilineal Motions, and kept in their Orbits, do not tend towards the Earth, but towards the Sun.

For every Body that is moved in a Curve Line, is turn'd off from its Rectilineal course, that it naturally affects, by some Force or other. And it is evident that the Planets move in Curve Lines, because their Orbits return into themselves. Now this Force exercised upon them, does not tend towards the Earth, because the Orbits of two of them, namely, Mercury and Venus, do not surround the Earth, as is evident

dent from *Prop. vi*; and consequently they are not concave in all their parts towards the Earth: Wherefore (by *Prop. xi.*) the Forces by which they are kept in their Orbits, do not tend to it. Besides, the Primary Planets, *viz.* Mercury, Venus, Mars, Jupiter and Saturn, in respect of the Earth, do sometimes go forwards from the West towards the East, sometimes backwards from the East towards the West, and sometimes stand still: But the time in which these Motions are perform'd, always flows uniformly; and therefore the Areas describ'd by a Radius drawn to the Earth from any Planet, are not proportional to the Times of description. Consequently, by the foregoing Proposition, each Planet is urged and retained in its own Orbit, by a Force that does not tend towards the Earth.

But (from the *Scholium* to *Prop. vi.*) 'tis certain, that the Orbits of Mercury and Venus surround the Sun; which is evident likewise concerning the Orbits of Mars, Jupiter and Saturn, from the *Scholium* to *Prop. ix.* And all the Planets, in respect of the Sun, are always Direct in their Motion, as is evident from *Prop. i*; and that almost uniformly, just as the Time flows. 'Tis true indeed, the Planets move a little swifter in some Points of their Orbits, even in respect of the Sun, than in others, as shall be shewn hereafter; but the difference is so small, as it may here very well be neglected. But even at that time the Planets are nearer to the Sun (for their Orbits are not Circles concentric to the Sun) and their Motion is so temper'd, as that the Areas described by Radii drawn to the Sun, are equally augmented, as will appear in the sequel of this Subject. And consequently the Forces, whereby the Planets are drawn off from a Recti-

lineal Motion, and retained in their Orbits, tend towards the Sun.

PROPOSITION XIV.

THE Forces, by which the Comets are kept in their Trajectories (if they be Curvilinear) do not tend towards the Earth, but towards the Sun.

If the Trajectories of the Comets were Right lines, they would not be urged by any Forces at all tending to a point situated without those Right lines; for if they were, they would be drawn off from a Rectilineal Motion, and made to describe Curvilinear Orbits, by *Prop. xi.* And if the Trajectories of Comets were Curve lines, yet the Force by which any such Comet is retained in that Curve, is not directed towards the Earth, because the Earth is generally found to be without the Plane of that Trajectory; besides, in respect of the Earth, the Comet is sometimes Direct and sometimes Retrograde, and consequently does not describe Areas proportional to the Times: Yet in respect to the Sun, which is placed in the Planes of all their Trajectories, a Comet always moves the same way, and the nearer the Sun, the swifter; so as to encrease its Area, which is equally described by a Radius drawn from the Sun: Wherefore what was asserted, is evident from *Prop. xii.*

SECTION

SECTION III.

Of the Order, Distances, and Periods of the Secondary Planets revolving about their Primary ones, and their Phænomena; together with the Direction of the Forces, whereby they are kept in their Orbits.

PROPOSITION XV.

TO describe the Order, and Periods of the Secondary Planets, or Satellites, about their Primary Ones, and Distances from them.

Of the Six Primary Planets, that revolve about the Sun, there are but three, as we are sure of by Observation, that have Satellites, or others revolving about them, hence called, Secondary Planets. The Earth has one, *viz.* the Moon, compleating its Revolution in $27\frac{1}{3}$ Days, and distant about 60 Semidiameters of the Earth from it.

Jupiter has four; the inmost of which revolves $1\frac{3}{4}$ of a Day, at the distance of $5\frac{2}{3}$ Semidiameters of Jupiter from his Center; the second revolves in $3\frac{3}{5}$ Days, at the distance of 9 Semidiameters; the third in $7\frac{1}{2}$ Days, at the distance of $14\frac{1}{3}$ Semidiameters; the fourth and outermost revolves about Jupiter in the space of $16\frac{3}{4}$ Days, being distant from his Center $25\frac{1}{3}$ Semidiameters of Jupiter.

Saturn has five; of which the innermost revolves in $1\frac{1}{2}$ of a Day, at the distance of $4\frac{3}{8}$ Semidiameters of Saturn, from the Center of Saturn; the second, in the space of $2\frac{3}{4}$ Days, at
the

the distance of $5\frac{1}{2}$ Semidiameters of Saturn; the third, in $4\frac{1}{2}$ Days almost, at the distance of 8 Semidiameters; the fourth, in almost 16 Days, at the distance of 18 Semidiameters; the fifth and outermost of all, that have been discovered, revolves about Saturn in $79\frac{1}{2}$ Days, at the distance of 54 Semidiameters from the Center of Saturn. And the Solar System, as far as it has been hitherto discover'd is of a Figure much like that in the Scheme [Fig 13.] But between the two last Satellites of Saturn, *Hugenius* suspects there revolves a Sixth, the space being larger than it should be, in proportion to the distances of the others, or that there are others beyond the fifth revolving about Saturn, but not seen as yet, by reason of their Obscurity. The Planes of the Orbits of the Satellites of the same primary Planet, do not coincide, but are variously inclin'd to one another, and to the Plane of the Orbit of the Primary one.

Saturn likewise is encompass'd with a thin plane *Ring*, that does not touch the Body of Saturn at all, but is like an Orbicular Arch, built round about it. The Plane of this Ring is at this time nearly parallel to the Plane of Earth's Equator. And from the various Position of it, in respect of the Sun illuminating it, it being Opake like a Planet, and in respect of the Eye of the Spectator, the various Phases of the *Anſæ* of Saturn arise, which have so long baffled the attempts of Astronomers, with their various Shapes. This was first discover'd by *Hugens*, in his *Systema Saturnium*, Printed 1659. The Diameter of the Ring is to the Diameter of Saturn, as 9 to 4; and the breadth of the space between the Ring and the Body of Saturn is equal to the breadth of the Ring it self.

Fig: 14
p: 27

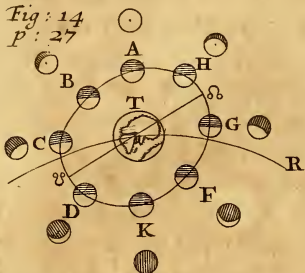


Fig: 15
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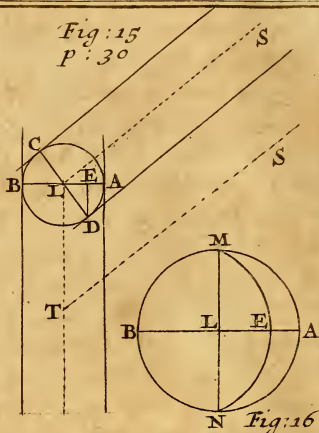


Fig: 16
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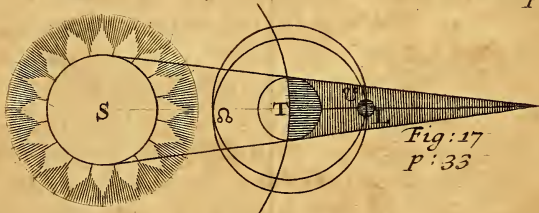


Fig: 17
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Fig: 18
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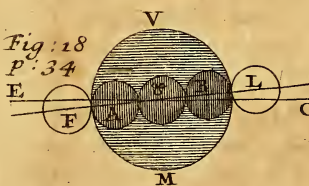


Fig: 19
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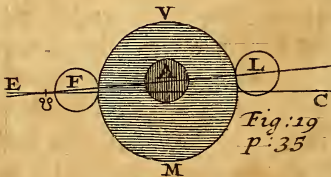
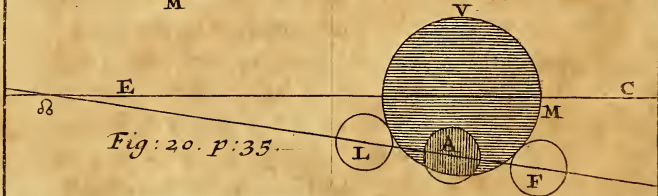


Fig: 20. p: 35.



PROPOSITION XVI.

TO describe the *Phænomena of the Moon view'd from the Earth, as it is a Spherical Body illuminated by the Sun.*

The Moon, being a secondary Planet, accompanies the Earth *T*, [Fig. 14.] during its Revolution about the Sun, in its Orbit *RT*, and moves in its Orbit the same way as the Earth does, from *A* thro' *B*, *C*, &c. The Plane of the Moon's Orbit does not coincide with the Plane of the Ecliptic, but is inclin'd to it by an Angle of about five Degrees; and the common intersection of these Planes is the Right Line $\Omega \vartheta$; the Points Ω and ϑ , where the Orbit of the Moon intersects the Plane of the Ecliptic are call'd the Nodes of the Moon, as was said before, when we treated of the primary Planets. So that the Orbit of the Moon is to be conceiv'd as being half above and half below the Plane of the Ecliptic, the former $\Omega A \vartheta$ to the North, the latter $\Omega K \vartheta$ towards the South, and on that account is represented in the Diagram, in the Figure of an Ellipse. One of the Nodes, *viz.* that where the Moon, having pass'd the Ecliptic, is ascending into the Northern Part, is call'd the *Dragon's Head*, and is mark'd thus Ω ; the other, the *Dragon's Tail*, mark'd thus ϑ . The Line of the Nodes does not always keep the same position, but is moved with an angular Motion, so as that the Nodes go backwards, *viz.* Ω thro' *G*, *F*, &c. and ϑ thro' *C*, *B*, &c. contrary to the Order of the Signs, compleating a Revolution in about 19 Years. From whence 'tis certain, that the Moon can never be found in the Ecliptic above twice in its Monthly Course, namely, when it is in the
Node

Node Ω or ϑ ; all the rest of the time it is wide of it ; and the Latitude of the Moon's deviation is measur'd by the Angle in which the Right line connecting the center of the Moon and Eye is inclin'd to the Plane of the Ecliptic, (by *Def. 5. Elem. xi.*) agreeable to what was said above concerning the primary Planets, *Prop. v. and viii.*

Besides, the Moon, like the other primary Planets, being an Opaque, Rough, Spherical Body, reflecting the Sun's Rays falling upon it ; 'tis evident, that half of it being turn'd towards the Sun is illuminated and bright, while the other half, that is turn'd from the Sun, continues obscure and dark. Now only that Hemisphere of the Moon which is towards the Earth, can be seen by an Inhabitant of the Earth viewing it: Consequently the Phases of the Moon will be various, according to the various habitude of the enlighten'd Hemisphere to that which is turn'd towards the Earth ; as was shewn in the like case concerning the inferior Planets, in *Prop. vi.*

If the Moon be at *A* in the Scheme [*Fig. 14.*] the point just opposite to the Sun, which we will suppose to be at $\odot$, and the Earth in *T* ; 'tis evident that the whole enlighten'd face of the Moon is turn'd towards the Earth ; this Phase or appearance of the Moon is call'd the *Full Moon*. If the Moon be remov'd to *B*, the Sun and Moon continuing as they were, part of the bright or illuminated half will be turn'd from the Earth, and part of the obscure or darken'd half turn'd towards it ; hereupon the Moon will not appear full, but Gibbous, or a little deficient or obscure in opposite to the Sun. The Moon arriving to the situation *C*, where the Angle *CTS* is Right, and the Moon in one
of

of its *Quadratures*, half the Hemisphere which is turn'd to the Earth is illuminated, and half dark; on which account it will appear a half Moon, and in the decrease. The Moon having got farther, for instance, to *D*, but a little of its Face that is turn'd to the Earth is inlighten'd, much the greater part remaining dark: and consequently the inlighten'd part will appear Horned, by reason of the Moon's really spherical, but apparently plane Figure; and the Horns will appear turn'd away from the Sun, as shall be demonstrated in the next Proposition, or in the present case, turned towards the West. At length when the Moon is come to the point *K*, or into conjunction with the Sun, all the inlighten'd half is turned from the Sun; and consequently the Moon will be intirely dark, and become a *New Moon*; because it will soon appear at *F*, and shew its Horns still turn'd from the Sun, and consequently now towards the East. Afterwards being upon the increase, it will appear a half Moon at *G*, at the end of the first Quarter, as it did in *C* at the beginning of the last Quarter; then Gibbous at *H*, till it become full again at *A*. For greater evidence and clearness of the whole, we have drawn the Appearance of the Moon seen by us, corresponding to every distinct situation, and plac'd it just behind it.

Tho' the time of the Moon's Revolution about the Earth be but 27 Days, 7 $\frac{1}{4}$ Hours, which is call'd a *Periodic Month*, yet by a Month, or intire *Lunation* is commonly understood all that space of time that is spent from one New Moon till the next following, which is greater than the abovemention'd Periodic Month; because during the Periodic Month, in which the Moon

Moon departing from the point K , where the New Moon now happens, returns to it again; the Earth itself, together with the Moon its Attendant, is carried an intire Sign *in consequentia*; so that the said point of the Orbit of the Moon K , is more towards the west of the Sun; and consequently the Moon is not yet arrived to a conjunction with the Sun; but still lacks 2 days and 5 hours to it, or to be become New; that is, to finish an intire Lunation, and all the several Changes of its Appearances; which space of time is call'd a *Synodic Month*, and consists of 29 days, $12\frac{1}{4}$ hours.

A little before and after a New Moon, *viz.* when the Moon is in D or F , the Rays of the Sun reflected from the Earth, meeting with the Moon, are the cause of that faint Light by which the rest of Moons Disc besides its Horns, is then render'd visible. But when the Moon is got out of the way of the reflexion of the Earth's light, that faint light vanishes, being falsely suppos'd native.

PROPOSITION XVII.

TO draw the Phasis or Appearance of the Moon at any given Time.

Let a Plane, the same with that of the Scheme, for instance, passing thro' the Centers of the Sun, Earth and Moon, cut the Globe of the Moon, and let its Section be the Circle $ADBC$, [Fig. 15.] Let CD be a Diameter of the Circle, and SL , a right line connecting the Centers of the Sun and Moon, perpendicular to it; and AB another Diameter to which the Right line TL , connecting the Centers of the Earth and Moon, is perpendicular. From the Point D , let DE fall a perpendicular to AB , meeting it in E .
Since

Since a Plane perpendicular to that of this Scheme erected upon the Diameter CD , separates the enlighten'd Hemisphere from the obscure; and a Plane erected after the same manner upon AB , separates the visible from the invisible Hemisphere: That part of the Globe of the Moon which is common to both Hemispheres, *viz.* that which lies between the Planes erected upon AL , DL , will be the enlightened part of the visible Hemisphere; and that which lies between the Planes erected upon BL , DL , will be its obscure or dark Portion. Both these Portions of the visible Hemisphere, as well the enlightened as the obscure, are terminated in the opposite Points of the Globe of the Moon; namely, those in which a Right line raised from L , perpendicular to the Plane of the Circle $ACBD$, and produced both ways, meets the Moon's Surface; that line being the common Section of the two Planes erected upon AB , and CD : The greatest breadth of these parts is in the Circumference ADB ; the breadth of obscure being BD , of the enlightened AD . The Arc BD is the measure of the Angle BLD , which is equal to the Angle SLT , contained under the Right lines drawn from the Center of the Moon to the Centers of the Sun and Earth. For if to the equal Angles SLD , TLB , (because right ones) you add the common Angle DLT , the Angles mention'd will be formed, which are consequently equal. But at any given time, the Angle of the Moon's Distance from the Sun, *viz.* LTS is known; consequently SLT , its Complement to two right: For the Distance of the Moon from the Earth is so small, in comparison with the distance of the Earth from the Sun, that the Right lines TS , LS , drawn from
the

the Earth and Sun, may be taken for parallels. Make therefore, that as twice the Radius is to the versed line of the Angle TLS , so is BA to BE . And since the Hemisphere of the Moon seen from the Earth, appears like a Circle, as it happens to any Globe seen at a great Distance, according to the Principles of Optic: Let $AMBN$ [Fig. 16.] be a Circle representing the Disc of the Moon, the same with the former, made by the Section of the Moon with a Plane erected upon AB , and consequently described upon the same Center L , and with the same Diameter. Because the Moon is at a considerable distance, each point of its Disc will be seen by Right lines parallel to the Right line LT . And therefore BE , and AE , will be the greatest Latitudes of the inlightened and obscure parts in the Disc of the Moon. Draw the Diameter MN perpendicular to AB ; and describe the Semi-Ellipse MEN , whose greater Axe is MN , and lesser equal to twice LE , $MANEM$ will be the inlighten'd Part, and $MBNEM$ the obscure Part of the Moon's Disc $AMBN$.

For the boundary of illumination on the Spherical Surface of the Moon, by the Sun, which is likewise Spherical, is a Circle; and this Circle seen at a distance obliquely, appears like an Ellipse, whose Semiaxes are LM , LE , as is evident from Optics: 'Tis seen obliquely by TL , because the Right line SL perpendicular to its Planes, is inclined to LT ; excepting when the Moon is Full, where they coincide.

In the Construction of this Problem, we suppos'd half the Globe of the Moon to be inlighten'd, and half likewise to be seen by a Spectator, tho' neither of the Suppositions are rigorously true: For the Sun being bigger than the Moon

Moon, more than half the Moon is inlighten'd; and the Spectator not being at an infinite Distance from the Moon, its intire Hemisphere will not be visible. But the difference in both these cases is so small, that it may very well be neglected: And the Phasis or Appearance of the Moon delineated as above, sufficiently agrees with Observations. That the Scheme may agree with the Heavens, the Line BA , to which MN connecting the Horns is perpendicular, must be plac'd in such a Situation, as that it may tend directly to the Sun: For the Right line LA , seen from T , coincides with the line LS tending to the Sun.

The Phases of the inferior Planets and Mars, described in *Prop.vi.* and *ix.* are to be delineated after the same manner; But then if L represent a Primary Planet, the Right lines TS , LS are sensibly inclin'd to one another, and form a Triangle.

PROPOSITION XVIII.

TO explain the *Phænomena* of the Moon arising from the Opacity of the Earth casting a Shadow, that is, an Eclipse of the Moon.

The Earth is an Opake Body, consequently when it is inlighten'd by the Sun, it casts a Shadow towards those parts which are turn'd from the Sun: and this Shadow must always be in the Plane of the Ecliptic; since both Sun and Earth are always there. [*Fig. 17.*] If at any time, the Moon when Full, should happen to be in the Plane of the Ecliptic, or pretty near it, (which it will be, when the Full Moon happens in or near a Node,) 'tis evident the Moon will be immers'd in the Shadow; and consequently deprived of the Sun's Light, by which

D

it

it shines; that is, it will suffer an *Eclipse* which will be *Total* or *Partial*, according as the whole or only a part of the Moon's Body enters the Shadow.

The Earth's Shadow terminates at last in a Point, and does not reach so far as Mars: For Mars, tho' in the Plane of the Ecliptic, and opposite to the Sun, is not Eclipsed; which it necessarily would be, if it were immers'd in the Earth's Shadow: For Mars is an opaque Body, as is shewn in *Prop. ix.* 'Tis evident at first sight, from this Figure of the Earth, that the Sun is bigger than the Earth: For if the lucid Body be bigger than the opaque one, the Shadow will be equally thick and cylindrical; But if the lucid Body be less than the opaque, the Shadow indeed will be conical, but growing bigger and bigger, and the farther off it is from the opaque Body, the thicker it is; and in both cases will be extended infinitely. As the Sun is bigger than the Earth, so the Earth is bigger than the Moon; because the Moon sometimes is totally Eclipsed by entering into the Earth's Shadow; but the Earth's Shadow is much smaller where the Moon enters it, than nearer to the Earth, as is evident from what has been already shewn.

Let the Circle VM [*Fig. 18.*] represent the transverse Section of the Earth's Shadow, where it crosses the Moon's Orbit. Let $L \& F$ be the Orbit of the Moon, CE the Ecliptic. The duration of some Eclipses is found to be so long (for instance, four hours,) as to let the Moon go the length of three of its Diameters in the Shadow totally Eclipsed. This happens when the Center of the Moon passes thro' the Center of the Earth's Shadow or Circle VM : And such an Eclipse is call'd a *Total* and *Central* Eclipse.

Some

Sometimes the Moon is not arrived at, or has already passed the Node, when it is entering the Earth's Shadow: Notwithstanding, if at that time it be near the Node, it will be totally immers'd, and consequently there will happen a Total, tho' not a Central Eclipse, nor of so long a continuance [Fig. 19.] For the way of the Moon thro' the Earth's Shadow, is less than the Diameter of the Shadow, because it does not pass thro' its Center; and consequently, *cæteris paribus*, will be pass'd thro' sooner.

But if the Node be so far off from the Shadow, [Fig. 20.] as that only part of the Moon is immers'd, as it passes in its Orbit, near the Shadow, the Eclipse will be a partial one, and is said to be of so many *Digits* as there are twelfth parts of the Moon's Diameter darken'd, when the Eclipse is greatest. For the Diameter of the Moon (like any other Integer) is imagin'd to be divided into twelve Parts or Digits: Which is to be understood likewise of any other Phasis of an Eclipse. In all these Cases, in the beginning of an Eclipse, the Moon enters the Western part of the Shadow, with the Eastern part of her Limb; and in the end of it, she leaves the Eastern part of the Shadow with the Western part of her Limb; and all the intermediate time is reckon'd into the Eclipse; but only so much into the total immersion, as pass'd while the Moon was totally in darkness. The Eastern limb of the Moon in its access to the Shadow, does not enter presently into the thickest Darkness; but grows darker and darker as it approaches nigher to the Shadow; and this arises from the Penumbra, which always accompanies a Shadow which is made by a lucid Body, that is seen under a sensible Angle. And this Pe-

numbra is diffused all about the Shadow. Let $\odot$ be the Sun, [Fig. 21.] E the Earth, and let Right lines be drawn just as in the Scheme: All about the Shadow $TVMR$, where no part of the Sun's light can come, a Penumbra $VTPMRN$ is spread, some part of the Sun's light not been stop'd therein: And this Penumbra is darker towards TV , RM which are the extremities of the perfect Shade; because fewer Rays can arrive thither, the portion of the Sun, from which they are emitted, being smaller; but less obscure towards TP , RN , where more Rays can reach: And beyond which limit all the Rays of the Sun can reach without any hindrance at all, and inlighten according to the degree of their vigour.

The Red colour, that the Moon in the middle of the compleat Shadow is affected withal, and makes her look like a Brick (for in some Eclipses the Moon intirely disappears,) seems to arise from the Sun's Rays, either refracted in their passage thro' the Earth's Atmosphere, or reflected by some particles of Matter flying about the Earth's Shadow, to the Moon; or from the light of the Stars, or all these taken together.

Some kind of Eclipse or other of the Moon, happens generally twice a Year at least: For there being two Nodes in which the Moon's Orbit crosses the Ecliptic, and they moving *in antecedentia*, (by Prop. xvi.) and the Sun appearing to go thro' the Ecliptic *in consequentia*, (by Prop. ii.) the Sun must meet one or other of these Nodes twice every Year; and consequently the Earth's Shadow must perforate, as it were, the Moon's Orb in the other Node. If therefore a Full Moon happens just at that time, the Moon must necessarily be totally and centrally Eclips'd,

as

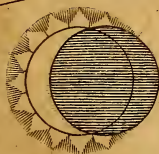
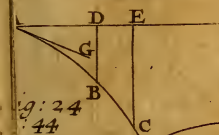
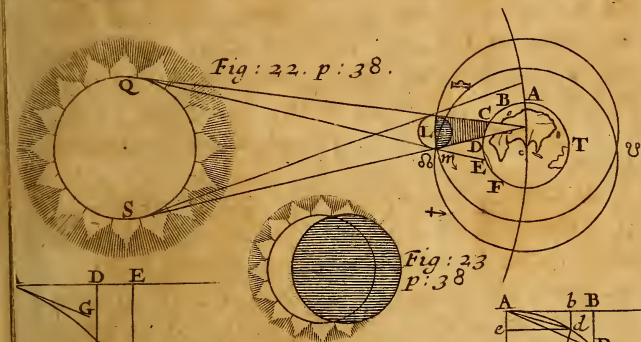
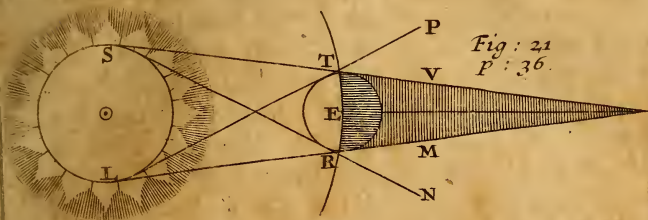


Fig: 23
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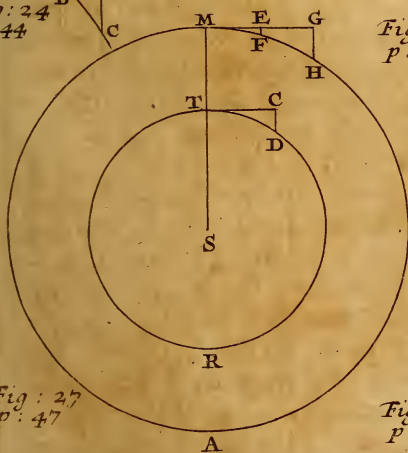
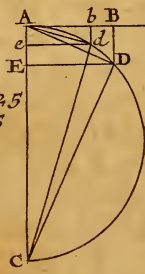


Fig: 25
p: 45.





as was demonstrated above. And tho' a Full Moon does not happen just as the Sun and Node meet; yet the inclination of the Orbit of the Moon to the Ecliptic, and the depth of the Earth's Shadow is so great, that tho' the Full Moon is distant above ten Days from the aforesaid time, before or after it, and it can be distant but fifteen Days, yet the Moon will scarce get clear of the Shadow. But if the abovemention'd meeting of the Sun, and Lunar Node, happen on the very Day of the New Moon, or a Day or two before or after, which can happen but seldom, the Moon will be far enough off from the Earth's Shadow in the next Full Moon, whether preceding or following, and so will escape an Eclipse; consequently there will be no Eclipse that half Year.

PROPOSITION XIX.

TO explain the Phenomena of the Sun seen from the Earth, and arising from the Opacity of the Moon, or Eclipses of the Sun accounted for.

As the Moon upon the interposition of the Earth is deprived of the Sun's light, and said to be Eclipsed; so, in like manner, if the Earth should be robbed of the Sun's light, by the interposition of the Moon, this Phenomena ought to be call'd an Eclipse of the Earth. But the Observer of it being on the Earth, and allowing of no loss of Light or Eclipse, nor Motion, nor any other thing that seems to argue an imperfection in his place of abode, calls this Phenomenon an Eclipse of the Sun; for the same reason as an Inhabitant of the Moon, would say the Sun was eclipsed, when the Moon is really entering into the Earth's Shadow.

'Tis evident an Eclipse of the Sun will happen in every such New Moon, as has the Moon at or near any of the Nodes. For then the shadow of the Moon lying directly betwixt the Sun and Earth, reaches to the Earth, and causes a total Eclipse to the Inhabitants of the tract CD [Fig. 22.] that are immers'd in the thickest shade. But the Moon's Shadow not being large enough to cover the whole Earth, the circular tract BC, ED , which surrounds the former CD , is cover'd with the Penumbra, and its Inhabitants see only a partial Eclipse of the Sun; which is greater towards C and D , because a greater portion of the Sun will be cover'd by the Moon; but less towards B and E , the extremities of the Penumbra, and the defect of light is scarce sensible. At the same time, in other places, as EF , the bigness of the Earth is the reason why there is no Eclipse at all; the Sun inlightning it without any hindrance or impediment.

All the preceding account happens in Nature, just as it has been related. But if we look upon the Sun and Moon from our Earth, the Moon will seem to cover the Sun more or less according as the Spectator is nigher or farther off from the total Shadow [Fig. 23.] And because the Moon which covers the Sun can't be seen, the Sun will appear to be darken'd in that part which the Moon covers, the rest only remaining lucid.

Sometimes a central Eclipse happens not to be total; but a bright Ring seems to surround the Moon: The reason is, at that time the Moon's shadow is so short that it cannot reach the Earth; either because the Moon is then so far off from the Earth; or because the Sun's Rays that graze along the extreme parts of the

Moon

Moon are more than ordinarily inflected, and so shorten the Moon's shadow. The bigness of a Solar Eclipse is to be estimated just as in the Moon, by the digits of the Sun's diameter, that are darken'd by the Moon at the time given.

'Tis evident likewise that the Moon moving towards the East, or from $\sphericalangle$, thro' m to z , [Fig. 22.] the western part A of the Earth will be in the shadow first, which will pass along the Earth's disc, like a Spot, thro' B, C, D, E , to F , where it leaves the Earth. But if the Moon be look'd upon from the Earth, the eastern limb of the Moon will first cover the western of the Sun, and the western of the Moon will last uncover the eastern limb of the Sun: And the greatest darkness that happens in a total Eclipse is soon at an end; some part of the Sun's disk being presently uncover'd, almost so soon as the whole was cover'd: and that part, tho' never so little, will mightily enlighten the Air.

Tho' the Moon must be in a Node the very moment of the New Moon, to cause the biggest Eclipse of the Sun that is possible, and that the shadow of the Moon may go along the middle of the Earth: Yet if she be not far off from thence, the shadow of the Moon, or at least part of the Penumbra, will fall upon some tract of the Earth, being so big, and there cause a total, or at least a partial Eclipse: And in this sense there are more Eclipses of the Moon than of the Sun. But Eclipses of the Sun, in any given place, are much fewer than the Eclipses of the Moon; because the Moon's shadow is less than the Earth's, and consequently it does not involve any given place upon the Earth, so often as the Earth's shadow does some part of the Moon.

S C H O L I U M.

The same Phænomena will appear in Jupiter and Saturn : For their Satellites or Moons, will be eclipsed, by being immers'd in the shadow of their primary Planet ; and those Eclipses of them are observed by us, just as the Eclipses of our Moon may be observed from them. In like manner, every Satelles coming between the Sun and its Primary, casts a shadow upon the Primary, which seems to move along the Disk of the Primary from East to West like a Spot. But the Duration, Phases, Periods, &c. of these Phænomena are various, and differing from the like seen by us, and arising from our Moon, according to the diversity of the Shadows, Motions and Magnitudes, both of the Primary and Secondary Planet.

PROPOSITION XX.

Each of the Secondary Planets mention'd in Prop. xv. is urged by a Force compounded of a Centripetal Force, tending to the center of the Primary, about which it revolves, and of all the Accelerating Force with which the Primary is urged. And therefore the Forces whereby the Satellites are retain'd in their Orbits about the Primary ones, tend towards the centers of their Primary ones respectively.

Because the Satellites of Jupiter and Saturn revolve equably in circular Orbits, concentric with Jupiter and Saturn, they describe Areas about these Centers respectively, proportional to the Times. In like manner, if the Moon's Orbit differ from a Circle concentric with the Earth, the less the Moon appears, (that is, the farther it is from the Earth) the slower it moves round the Earth, so that still the Area, that the Radius drawn to the center of the Earth describes,

scribes, is equally augmented. So that universally, each of the abovemention'd Satellites describes Areas, by a Radius drawn to the center of its Primary Planet, about that Center, proportional to the Times. If therefore the System of any Primary Planet, and its revolving Satellites, be suppos'd to be urged along parallel Lines, by a Force equal and contrary to that, whereby the Primary Planet in that Point of its Orbit, where it then is, is urged towards the Sun, the Primary Planet will no longer descend towards the Sun; and the Secondary will continue to describe the same Areas about the Primary as before; that is, proportional to the Times: (For if the Bodies move any how in respect of one another, and are urged by equal accelerating Forces along parallel lines; or, which is all one, if the Space in which they perform their Motions be moved uniformly in a right line, they will move all after the same manner, as they would do if those Forces were absent, or the Space were at rest, in which they are included). So that each Satelles, only urged by the difference of the Forces, will go on to describe, about the center of its Primary, Areas proportional to the Times. Therefore, by *Prop. xii.* the difference of those Forces tends to the Primary Planet as a Center. But before that the whole System was urged along parallel lines, by a Force equal and contrary to that, whereby the Primary Planet is urged towards the Sun, that is, in its natural State, each Satelles is urged by a Force compounded of the Centripetal Force tending to the center of its Primary, and of all the accelerative Force that the Primary is urged by. Consequently the 'foresaid difference of the Forces, is that whereby the
Satelles

Satelles is retained in its Orbit about the Primary, the remaining accelerative Force, whereby the Primary Planet is urged in any Point of its Orbit, conferring nothing to this: But this difference has been shewn to tend to the Center; therefore the Forces whereby the Secondary Planets are retained in their Orbits about the Primary, tend towards the Centers of the said Primary respectively. Which was to be demonstrated.

SECTION IV.

Of the Periods of the Primary Planets about the Sun, and Secondary about their respective Primary, compar'd together, and of their Distances likewise compared together, with their mutual relation to one another, the reason and causes of it.

PROPOSITION XXI.

THE Motion of the Secondary Planets revolving about a Primary, is such, as that the Squares of the Periodic Times, are in the same ratio as the Cubes of their Distances from the Center of the Primary.

This Theorem is abundantly evident from the Observations of Astronomers: And by the Squares of the Periodic Times, you must understand, the Squares of Right lines, or of Numbers, having the same ratio as the Periodic Times; in this Sense also are the Squares of the Velocity, and the like.

Thus in the Satellites of Jupiter, where the Periodic Times of the inmost, second, third, and

and outmost, are respectively as $1\frac{2}{40}$, $3\frac{3}{4}$, $7\frac{1}{2}$, and $16\frac{3}{4}$ almost; and their Distances as $5\frac{2}{3}$, 9, $14\frac{1}{2}$ and 25; the Square of the Periodic Time of the inmost, namely 3, is to 13 the Square of the Periodic Time of the second Satelles, as 170, the Cube of the distance of the inmost from the Center of Jupiter, to 736, the Cube of the distance of the second from thence. In like manner 3 is to 51, the Square of the Periodic Time of the third Satelles, as 170 to 2890, the Cube of the distance of the third from the Center of Jupiter. And 3 is to 280, the Square of the Periodic Time of the outmost Satelles, as 170 to 15800, the Cube of the distance of the outmost from Jupiter's Center. And therefore, *ex æquo*, the same ratio holds between any other two of them compar'd together; as the second with the third or the last, or the third with the last. This Ratio comes out more exact, if the Distances and Periodic Times are taken more accurately.

The same will be found to hold in the Satellites of Saturn, if you take the numbers laid down in *Prop. xv*, that their distances from Saturn, and their Periods about him may be estimated. But the Moon being a solitary Satelles, this Proposition can't be applied to her.

PROPOSITION XXII.

THE Motion of the Primary Planets about the Sun, is such, as that the Squares of their Periodic Times are in the same ratio as the Cubes of their Distances from the Sun.

Thus, for instance in round numbers, the Period of Saturn is 30 Years, Jupiter 12; their Squares are 900 and 144. The distance of Saturn from the Sun, is, by Observation, to the distance

distance of Jupiter from the same, as 9 to 5 almost; their Cubes are 729 and 125. But 'tis evident those Squares are nearly as these Cubes. Also the Period of the Earth is a little more than four times greater than the Period of Mercury; consequently their Squares are as 17 to 1 almost: Supposing the Distance of the Earth from the Sun to be ten parts, the Distance of Mercury is, by Observation, about 4, or 3, 9; and their Cubes as 1000 to 59: But 17 is to 1 almost as 1000 to 59; And so on in the other Planets. The more correct the Distances and Periods are taken, the nearer you'll come to this Proportion. What has been now said, may suffice at present: The Proposition being more exactly to be made out in the third Book.

PROPOSITION XXIII.

THE Spaces that a Body describes, acted upon by any kind of finite Force, (whether it be determinate and unchangeable, or the same in continual increase, or continual decrease) are in the very beginning of the Motion, in the duplicate ratio of the Times.

Let the Times be express'd by the Right lines AD , AE ; [Fig. 24.] and the Velocities acquired at the end of those Times, by the Ordinates DB , EC : And the Spaces described with these Velocities will be as the Areas ABD , ACE , made up, as it were, of these Ordinates; namely, the Space describ'd in every smallest particle of Time, being as the Velocity and that Particle of Time conjunctly. But, in the beginning of the Motion, the Ordinates DB , EC , are very near the Point A ; and in that case, the Trilineal Figures ADB , ACE , are similar rectilineal Triangles, that part of the Curve

ABC ,

ABC , which belongs to the infinitely small trilineal Figures, not extending it self beyond the Right line AG , which is a Tangent to the Curve in A . Now these similar rectilineal Triangles, are in the duplicate ratio of their homologous Sides AD , AE . Consequently the Spaces describ'd in the beginning of the Motion, are in the duplicate ratio of the Time. Which was to be demonstrated.

C O R O L L A R Y.

From hence we may gather, that the Errors of Bodies describing similar parts of similar Figures in proportional Times, generated by any equal disturbing Forces similarly applied to the Bodies, and measur'd by the distances of the Bodies from those places of the similar Figures, to which these Bodies would arrive, in the same proportional Times, without those Forces, are very near as the Squares of the Times wherein they are generated. For these Errors are the Spaces which the Bodies acted upon by the disturbing Force describe. But if these disturbing Forces are not equal, but in a given ratio; the Errors are as those Forces and the Squares of the Times conjunctly; supposing the Forces similarly applied.

P R O P O S I T I O N XXIV.

THe nascent or evanescent subtense of the Angle of Contact, in any Circle, is in the duplicate ratio of the conterminous Arc.

Let ADC [Fig. 25.] be a Circle, the right line AB a Tangent in A , consequently the Angle of Contact BAD . I say, any Subtense that is the nearest to the Point A , for instance, BD is as the square of the Arc AD ; that is, the subtense BD is to another subtense bd , in the same
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Circumstances, as the Square of the Arc AD to the Square of the Arc Ad .

Draw the Diameter AC , which will be perpendicular to AB , join the Right lines DA , dA , DC , dC , and draw DE , de parallel to AB .

First, let the Subtense BD be parallel to AC . Now (by 8 and 17. *Elem. vi.*) $AD^2 = CA \times AE$, and $Ad^2 = CA \times Ae$. Wherefore $AD^2 : Ad^2 :: (AE : Ae ::) BD : bd$. But since BD , bd are nearest to the Point A , or in their nascent condition, the Arcs AD , Ad , and their Subtenses do not differ from one another; that is, they are equal. Consequently, in this case, BD is to bd , as the Square of the Arc AD , to the Square of the Arc Ad .

Secondly, if the Subtense BD of the Angle of Contact be not parallel to AC , but to AG , [*Fig. 26*] the same still holds: For, drawing DF , df parallel to AC ; then, because DFB dfb are similar Triangles, $BD : bd :: DF : df$; but it has been already shewn that DF is to df , as AD^2 to Ad^2 . Wherefore BD is to bd , as AD^2 to Ad^2 .

Thirdly, if BD be suppos'd to be drawn according to any other certain Law, (for instance converging toward the Center;) since BD , bd are as near as may be to the Point A , the Angles B , b will be equal; and consequently, in that case, $Bd : bd :: (DF : df ::) AD^2 : Ad^2$. Which was to be demonstrated.

SCHOLIUM.

This is true likewise in any other Curve, to which a Circle equi-curve may be drawn; such as all the Conic Sections are. For the Points D , d (being the nearest that can be to A) must be in that other Curve, as well as in the Circle, which is equi-curve to it, by Hypothesis: Consequently

frequently the Properties of BD , bd , which agree to the Circle, agree likewise to this Curve, to which the Circle is equi-curve. And since the foregoing Proposition is true of all Circles, it must likewise be true of all Curves, that can have equi-curve Circles drawn to them.

PROPOSITION XXV.

Bodies describing different Circles with an equable Motion, are acted upon by Centripetal Forces tending to the Centers of those Circles. And the Forces are to one another as the Squares of the Arcs, described in the same time, applied to the Radii of the Circles. This likewise is true of the Centrifugal Forces of the Bodies thus moved.

Because the Bodies, by their *Vis insita* alone, would describe Tangents, 'tis evident that they are drawn off from their Rectilineal Motion, and retain'd in Circular Orbits by Forces tending to Points within the Circles. But since, by supposition, they are carried in the Circumferences by an equable Motion; the Areas described by a Radius drawn to the Centers are equally augmented; and therefore are proportional to the time which flows equably; and consequently these Forces, by *Prop. xii.* tend to the said Centers.

Let the Bodies M and T [*Fig. 27.*] revolving in the Circumferences of the Circles MA , TR , describe the infinitely small Arcs MF , TD ; From their extremities F , D , draw the Right lines FE , DC as far as the Tangents, either parallel to SM , ST , or diverging from S : (For it comes to the same, since MF , TD are only nascent Arcs.) These Right lines are the effects of the accelerating Centripetal Forces; and

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consequently proportional to them, as their adequate Causes; that is the Centripetal Force of the Body M is to the Centripetal Force of the Body T , as EF to CD . Make the Fig. MGH similar to the Fig. $TC D$; wherefore GH will be a nascent Right line: And so CD is to GH , as the Arc TD , to the Arc MH . And, by the præced. GH is to EF as MH^q to MF^q : And therefore, *ex æquo*, CD is to ET , as $(TD \times MH^q$ to $MH \times MF^q$; that is, as $TD \times MH$ to MF^q , or as) $TD \times MH \times ST$ to $MF^q \times ST$: And since $TC D$, MGH are similar Figures, the Arcs TD , MH are similar; that is, $ST:SM::TD:MH$. Wherefore $MH \times ST = TD \times SM$. Then by Substitution $CD:EF::(TD^q \times SM:MF^q \times ST::) \frac{TD^q}{ST}:\frac{MF^q}{SM}$.

But the Centripetal Forces of the Bodies T and M were shewn to be proportional to CD , EF . Therefore the Centripetal Force of the Body revolving in TR , is to the Centripetal Force of the Body revolving in MA , as $\frac{TD^q}{ST}$ to $\frac{MF^q}{SM}$. And, (because the Motion in both is equable) TD and MF , being describ'd at the same time, have the same ratio with any others describ'd at the same time. Wherefore the Centripetal Force of a Body revolving in TR , is to the Centripetal Force of a Body revolving in MA , as the Square of an Arc describ'd in any time in TR applied to the Radius ST , is to the Square of an Arc describ'd in the same time in MA applied to the Radius SM .

If the abovementioned Bodies be imagined to be tied to S , by two Strings, and kept revolving

ving in the Circles by the help of them; the same Force, whereby the String is stretched, consider'd as it is in the String pulling the Body, is call'd the Centripetal Force, but considered as it is in the Body stretching the String, may be call'd the Centrifugal Force: And consequently all that has been said concerning the Centripetal Force, is true also of the Centrifugal Force. Which was to be demonstrated.

COROLLARY I.

From hence it follows, that if a Body should be let fall towards the Center, being acted upon by the Centripetal Force, that kept it in the Circle; so as that this Force be neither augmented nor diminished in its descent, but continue the same, and to the first impulse continually super-add a new one equal to it, (which is the *Galilean Hypothesis of Gravity*;) In the time that the said Body describes an Arc in the Circle, it would, in its descent towards the Center, describe a Right-line, equal to that which is produc'd by the Square of that Arc applied to the Diameter of the Circle. For by this *Prop.* 'tis evident it is done in the very beginning of the Fall; and in the present Hypothesis of descent, the Spaces run are in the duplicate Ratio of the Times; and since the Circle is equably described, the Squares of the Arcs described are also in the duplicate Ratio of the Times of description: Now the application of these Squares, to the Diameter of the Circle, being a constant Quantity, does not alter the Ratio. Consequently it holds perpetually. So that in this Hypothesis of an uniform Centripetal Force, a Body would in half the Time that it describes an Arc of a Circle in, descend the length of a line, equal to that produced by the application

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of the Square of half that Arc to the Diameter of the Circle; namely, the subquadruple of the Right-line, it would descend in double the time, *viz.* in which it describes the intire circular Arc.

COROLLARY 2.

From hence likewise it follows, that the centripetal Forces of Bodies, describing different Circles by an equable Motion, are in a ratio compounded of the duplicate ratio of the Velocities directly, and of the simple ratio of the Radii inversely. For the Motion in any Circle being equable, the ratio of the Velocities will be the same with that of the Arcs described in the same time; that is, of the Spaces run in the same time: But (by this Proposition) the centripetal Forces are in a ratio compounded of the duplicate ratio of the Arcs described at the same time directly, and of the simple ratio of the Radii inversely: Wherefore the ratio of these Forces is compounded of the duplicate ratio of the Velocities directly, and of the simple ratio of the Radii inversely.

PROPOSITION XXVI.

THe same things being supposed, I say that the centripetal Forces are reciprocally as the Squares of the Periodic Times applied to the radii of the Circles; or that they are in a ratio compounded of the simple ratio of the Radii directly, and of the duplicate ratio of the Periodic Times inversely. The same also is true of the centrifugal Forces of these Bodies.

Let the centripetal Force of one of the Bodies, moving in one of the Circles, be call'd V , in the other v ; the Celerity in one Circle C , in the other c ; the Arcs at the same time described in the first Circle A , in the second a ; the Radius, or

or Distance of the Body from the center, in the first Circle D , in the second d ; the Periodic Time in the first Circle T , in the second t .

'Tis well known, that in comparing the motions of Bodies together, the Spaces run are in the compound ratio of their Times and Celerities directly. But the Radii of the Circles are proportional to the intire Circumferences, which are the Spaces run in the Periodic Times. And therefore $D : d :: C \times T : c \times t$, so that $C \times T \times d = c \times t \times D$: therefore $C^2 \times T^2 \times d^2 = c^2 \times t^2 \times D^2$. Therefore, dividing by $D^2 \times d^2$, 'tis $\frac{C^2 \times T^2}{D^2}$

$= \frac{c^2 \times t^2}{d^2}$. Wherefore $\frac{t^2}{d} : \frac{T^2}{D} :: \frac{C^2}{D} : \frac{c^2}{d}$. But,

(by Cor. 2. of the prec.) $V : v :: \frac{C^2}{D} : \frac{c^2}{d}$. And

therefore $V : v :: \frac{t^2}{d} : \frac{T^2}{D}$. But the ratio of $\frac{t^2}{d}$ to

$\frac{T^2}{D}$, is the same with that of $t^2 D$ to $T^2 d$; or of t^2 to T^2 , and D to d . Therefore the ratio of V to v is equal to the ratio of t^2 to T^2 , and of D to d .

What has been said of the Centripetal Forces of two Bodies revolving about a Center, is true of their centrifugal Forces; since they differ only in the manner of conception; as was shown in the foregoing Proposition. Q. E. D.

COROLLARY I.

From hence (and from Corol. 2. Prop. preced.) it follows, that if the Periodic Times are equal, then, as well the centripetal Forces, as the Velocities, will be as the Radii; and *vice versa*.

For, by this Proposition, $\frac{V}{v} = \frac{D \times t^2}{d \times T^2}$. Wherefore if $T=t$, or $T^2=t^2$, then will $V:v::D:d$. And *vice versa*, if $V:v::D:d$, since $V:v::Dt^2:dT^2$, then $D:d::Dt^2:dT^2$; and consequently $T^2=t^2$, or $T=t$. The second part of the Corollary appears from hence. In the comparison of Motions, the Velocities are as the Spaces run in the same Time. But, because the Periodic Times are equal, the Spaces run in the same time are the Circumferences of the Circles; but they are as their Radii. Consequently the Velocities are as the Radii. The converse of this Proposition is drawn after the same manner.

COROLLARY 2.

Hence also it follows, that if the Squares of the Periodic Times are as the Radii, the centripetal Forces are equal, and the Velocities are in the subduplicate ratio of the Radii; and *vice versa*. For from this Prop. $\frac{V}{v} = \frac{D \times t^2}{d \times T^2}$: But by supposition, $T^2:t^2::D:d$, or $D \times t^2 = d \times T^2$. Therefore V and v , proportional to them, or the centripetal Forces themselves, are equal. And *vice versa*, if $V=v$, the quantities proportional to them will likewise be equal, namely, $D \times t^2 = d \times T^2$: And therefore $D:d::T^2:t^2$. In like manner, because by Cor. 2. of the preceding Prop. $\frac{V}{v} = \frac{C^2 \times d}{c^2 \times D}$, and in the present case, $V=v$, then $C^2 \times D$ will be equal to $c^2 \times d$, or $C^2:c^2::D:d$; that is, $C:c::\sqrt{D}:\sqrt{d}$. The converse of this is drawn after the same manner as that of the former.

COROL-

COROLLARY 3.

If the Circles described are equal, the centripetal Forces are reciprocally as the Squares of the Periodic Times : For the ratio of the Radii, in this case, being that of equality, makes no alteration in the composition.

PROPOSITION XXVII.

THE same things being supposed, if the Bodies revolve in such a manner, as to have the Squares of the Periodic Times as the Cubes of the Radii, then the Centripetal Forces are reciprocally as the Squares of the Radii : And the Celerities of the Bodies are reciprocally as the Square Roots of the Radii ; and vice versa.

Retaining the Characters used in the former Proposition, 'tis evident from it, that $V:v :: \frac{t^2}{d} : \frac{T^2}{D}$. From whence it follows, that $\frac{VT^2}{D} = \frac{vt^2}{d}$, or $VT^2d = vt^2D$. And therefore $Dv :: dV :: T^2 : t^2$. And by supposition, $D^3 : d^3 :: T^2 : t^2$; therefore $Dv : dV :: D^3 : d^3$. Consequently $Dvd^3 = dVD^3$; or $vd^2 = VD^2$. And therefore $V:v :: d^2 : D^2$.

Again, in the demonstration of the foregoing Proposition, it was shewn that $C^2 \times T^2 \times d^2 = c^2 \times t^2 \times D^2$. Wherefore $c^2 \times D^2 : C^2 \times d^2 :: T^2 : t^2$. But by supposition, $D^3 : d^3 :: T^2 : t^2$. Therefore $c^2 \times D^2 : C^2 \times d^2 :: D^3 : d^3$. And so $c^2 : C^2 :: D : d$ (or by extracting the Square-root, and inverting the Proportion) $C:c :: d^{\frac{1}{2}} : D^{\frac{1}{2}}$; that is, $C:c :: \sqrt{d} : \sqrt{D}$. Which were to be demonstrated.

The Converses of these are evident. For as from the putting $T^2 : t^2 :: D^3 : d^3$, it follows that $V:v :: d^2 : D^2$, and that $C:c :: d^{\frac{1}{2}} : D^{\frac{1}{2}}$; so

conversely, from the supposition of any one of these, you may return to the former Proportion, *viz.* $T^2 : t^2 :: D^3 : d^3$; and from hence again to the other Conclusion. And consequently any one of these three being supposed, the other two will follow.

SCHOLIUM.

The three præceding Propositions (and their Corollaries) are true, not only of Bodies revolving in concentric Circles, but of such also as describe Circles whose Centers are intirely different, (for the identity of Centers was not supposed in the Demonstration;) and of those that describe similar parts of any Figures, having their Centers similarly posited. For what was assumed in the Demonstration of these Theorems in Circles, are equally true of all other Figures like those we but just now described; namely, that the nascent lineola GH (in *Fig.* 27.) is to the nascent lineola EF , as MH to MF (for this is demonstrated of any Figure that a Circle is equicurve to, in *Prop.* 24.) and that MH is to TD as SM to ST , is true of any similar Figures; provided the Center S be similarly posited in both Figures, and MH , TD , similar, and similarly posited parts of those Curves in respect of the Center. And instead of an equable Motion in Circles, if the Figure be different from a Circle, the Motion of the Body in the Perimeter of it, must be such, as that the Areas described, by rays drawn to their Centers similarly posited, be proportional to the Times; otherwise (by *Prop.* 12.) the Forces, whereby the Bodies are drawn aside from their Rectilineal Motion, and kept in their Curvilineal Orbits, do not so much as tend to those Centers.

PROPOSITION XXVIII.

THE Forces, whereby the Primary Planets are perpetually retracted from their Rectilineal Motions, and retained in their Orbits, are reciprocally as the Squares of their Distances from the Center of the Sun, which they respect.

That these Forces respect the Center of the Sun, has been shewn in *Prop. 13.* But that the Law of these Forces is such, as that they are reciprocally as the Squares of their Distances from the Center, or that they increase as the Squares of the distance decreases, and decrease as the Squares of the distance increases: As for instance, that the Force whereby the Earth is made to tend towards the Sun, and is retained perpetually in its Orbit, so as not to fly off in a Tangent to it, is to the like Force in Mars, as 5 to 4, to that in Jupiter as 27 to 1; because the distance of the Earth from the Sun, is to the distance of Mars from the same, as 2 to 3, and to the distance of Jupiter, as 5 to 26; and that the Centripetal Force in the Earth is but half that in Venus, because the Earth's distance from the Sun is to the distance of Venus as 10 to 7; and so on in the others, is thus demonstrated. The Motion of the Planets revolving about the Sun, and tending towards it as a Center, is such, as that the Squares of the Periodic Times are as the Cubes of the distances from the Sun, by *Prop. 22.* Now by the preceding *Prop.* the Centripetal Forces of Bodies revolving according to this Law, are reciprocally as the Squares of their distances from the Center: Consequently 'tis evident that the Forces respecting the Sun, by which the Planets are acted upon, and retain'd in their Orbits, are re-

ciprocally as the Squares of their distances from the Center of the Sun.

PROPOSITION XXIX.

THE Forces whereby the Secondary Planets are retained in their Orbits, are reciprocally as the Squares of their Distances from the Center of their respective Primary, to which these Forces tend.

For since (by Prop. 21.) the Motion of the Circum-Jovials revolving about Jupiter, and Circum-Saturnials revolving about Saturn, is such, as to have the Squares of the Periodic Times in the same ratio with the Cubes of their Distances from the Center of Jupiter and Saturn respectively; and since (by Prop. 23.) the Centripetal Forces of Bodies revolving by this Law are reciprocally as the Squares of their Distances from their Center: 'Tis evident that the Centripetal Forces, whereby the Secondary Planets are respectively urged towards Jupiter and Saturn, are reciprocally as the Squares of their Distances from the Center of their respective Primary Planet.

But this Force in any Secondary Planet is only one part of the whole Force, whereby the Satelles is urged; and the other Force is the whole accelerating Force whereby the Primary is urged; as has been shewn in Prop. 20.

As for the Moon, the Earth's attendant, because it is solitary, it has no other Secondary to be compared withal, as to Distance and Periodic Times. So that the Law of its Centripetal Force, whereby it is urged towards the Earth, can't be deduced after this manner. Yet hereafter you will find it drawn from the Figure of the Orbit that the Moon describes about the Earth; which hitherto we have considered as

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not differing sensibly from a Circle. But if it really were a Circle concentric to the Earth, the Law of the Forces whereby it is retained in its Orbit, could not be from hence discovered: For such an Orbit may be described by a Body, let the Law of the Centripetal Force be what it will.

COROLLARY.

It follows from these two Propositions, that among the Primary Planets, that nearest the Sun, and among the Secondary, that next the respective Primary, moves fastest. For it has been demonstrated, that in both of them, their Centripetal Forces are reciprocally as the Squares of their Distances from their Center: And in *Prop. 27.* it was demonstrated that the Celerities in this case are reciprocally as the Square Roots of their Distances from the Center of Motion; or that a mean proportional between the Distances of that which was nearer, and that which was more remote, is to the distance of the nearer, as the Velocity of the nearer, to the Velocity of the more remote. And since the first of these Quantities is greater than the second, the third will be greater than the fourth; that is, the Velocity of the nearer is greater than the Velocity of the more remote.

SECTION V.

Of the Motion of the Primary Planets about their proper Axes, and of the Phænomena thence arising.

PROPOSITION XXX.

TO explain the Motion of the Celestial Bodies about their proper Axes, given in Position, and the Revolutions of them.

When we say a Body is not moved, we mean by it, that every line, and every point in it, is at rest. But if we say a Body is moved along a line, without any other Motion, we intend to signify, that the Center of Gravity of the Body describes that line, while the parts of the Body, in respect of the whole, remain at rest; that is, every line in that Body retains a situation parallel to that it had before it was moved; and consequently every line in the Body thus moved, remains always parallel to it self. For after this manner, and this manner only, will it happen, that the Sum of all the Motions will be the least, (that is, the motion of the whole the least, as is supposed) and that any point of the Body thus moved, is as much moved as any other, and describes a line similar and equal, and only differing in situation: For the line describ'd by the Center of Gravity, is suppos'd to be described by the Body it self, from the known nature of a Center of Gravity.

If a Body be said to be moved about a given Axe, being in other respects not moved, that Axe is suppos'd to be unmov'd, and every point
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out of it to describe a Circle, to whose Plane the Axis is perpendicular. And for that reason, if a Body be carried along a line, and at the same time be revolved about a given Axe; the Axe, in all the time of the Body's motion, will continue parallel to it self. Nor is any thing else required to preserve this Parallelism, than that no other Motion besides these two be impressed upon the Body; for if there be no other third Motion in it, its Axe will continue always parallel to the Right-line, to which it was once parallel.

These things being premised, 'tis evident from Observation, that the greater Celestial Bodies, (if not all, yet at least of every kind) besides the Motions spoken of in general above, do revolve about their Axes. And, first, from observations, of Spots in the Sun, 'tis evident that the Sun, the only Body of its kind, in our System, revolves from West to East about an Axe, inclin'd to the Plane of the Ecliptic, in an Angle of about $87\frac{1}{2}$ Degrees, in the space of about 25 Days. And since the Sun has no progressive Motion, its Axis continues unmoved. Among the Primary Planets, the Earth revolves with an equable Motion, from West to East, about an Axe, inclin'd to the Plane of the Ecliptic, in an Angle of $66\frac{1}{2}$ Degrees, in the space of a natural Day. And since the Earth also is carried about the Sun in the space of a Year, its Axe must however continue parallel to it self, tho' it can't be intirely at rest. And this will come to pass by means of these two Motions alone, without the assistance of a third, which some have groundlessly invented and call'd by the name of a *Parallelism*. But the Axis of the Earth, tho' it continue parallel to it self, as

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to sense, during one Revolution, yet after several, it changes this Site. Of the Causes and Phænomena whereof, we shall speak in a proper place.

After the same manner Jupiter is revolved, from West to East, about an Axe in the space of about 10 Hours; Mars in $24\frac{2}{3}$ Hours, and Venus in 23 Hours, about their Axes; every one of which therefore continues always parallel to itself, if these Bodies are not moved by any other kind of Motion beside their Revolution about the Sun, and Circumrotation about their own Axes.

Among the Secondary Planets, the Moon, the Earth's attendant, besides its monthly Motion about the Earth, and annual about the Sun, (by which alone each line in it would be always parallel to its self,) it likewise revolves about itself in the same space of a Month, so as to turn always the same Face to the Earth.

We are not so certain yet of the other Planets, both Primary and Secondary, from Observations. Notwithstanding, 'tis probable that they, like the former, revolve about a certain Axe; for this reason, that every part of them may be brought under, or removed from, the Rays of the Sun, more than once in one Revolution about the Sun, and may undergo Changes and Vicissitudes suitable to their Nature.

PROPOSITION XXXI.

TO describe the Figure of the Sun and Planets, revolving about their own Axes.

If the Sun and Planets were deprived of all Motion about their Axes, they would put on a Spherical Figure; supposing their parts like the parts of the Earth, to gravitate towards their Cen-

ter,

ter, (such as really they are, as we shall afterwards demonstrate,) and either are now or were formerly fluid. This is demonstrated by *Archimedes* in *Hydrostatics*. From a Motion about their Axes, it comes to pass, that their Parts receding from the Axis as much as they can, endeavour to ascend about the Circle that lies betwixt the two Poles, by a Centrifugal Force arising from their circular Motion. And therefore if the Matter of the Sun or Planet be fluid, it will increase the Diameters of that middle Circle between the two Poles, by its ascent; and lessen the Axe towards the Poles by its descent, and consequently constitute an oblate Spheroid, generated by the rotation of an Ellipse about its lesser Axe; being depressed towards the Poles, and elevated towards the Circle that lies exactly between the Poles, more or less according to the quantity of the Centrifugal Force. This Spheroidical Figure is observed in *Jupiter* by the help of a Telescope; and is proved to be in the Earth by Experiments, (since we can take a view of her at a distance.)

SCHOLIUM.

As this Spheroidical Figure follows upon the rotation of the Body about its own Axis, the fluidity of its Parts; so, on the contrary, from this observed Figure, we may justly infer its Motion about its Axis, and the (former) fluidity of its Parts.

PROPOSITION XXXII.

TO explain the *Phænomena* arising from the Diurnal Motion of the Earth about its Axis, seen from the Earth.

In the foregoing Propositions I have shewn, what would be the *Phænomena* of the Motion
of

of the Sun and Planets seen from the Earth, if the Earth were a Point; as that the Sun would seem to move among the Fixt Stars from West to East; that the Planets would sometimes seem to go forwards, at another time backward, according to the situation of the Sun and each Planet. But since the Earth is an opaque Body, and large enough, in respect of the Observer, that small part of its Surface, tho' Spherical, which comes at the same time under his confin'd view, will seem to be extended like a Plane. And the Eye taking a view of the Heavens all around itself, defines a concave spherical Superficies, concentric to the Earth, or rather to the Eye; which the abovemention'd Plane of the Earth's Superficies, because drawn thro' the Center, will divide into two equal Parts, one of which will be visible, but the other, by reason of the Earth's Opacity, will lie conceal'd. Because the Earth moves upon its own Axis, the Spectator standing upon it, together with the said Plane he stands upon, call'd his *Horizon*, dividing the visible from the invisible Hemisphere of the Heavens, is carried round the same way, *viz.* to the East. From whence it is, that Objects situated towards the East, and invisible, become visible, the Horizon sinking as it were below them; and Objects situated towards the West, are hid and become invisible, the Horizon being elevated above them. And the former Objects therefore will appear to the Spectator, who usually reckons the Place he stands in, to be immoveable, to ascend above the Horizon or rise, the latter to descend below the Horizon or set.

Since the Earth and the Horizon of the Spectator fix'd to it, continues to move always towards

wards the same Quarter and about the same Axis equably; all Bodies, and all Appearances that don't partake of that Motion (that is, all such things as are entirely separate from the Earth) will seem to move in the like time uniformly, but towards the opposite parts, *viz.* from East to West, excepting such things as the Earth's Axis produced will meet withal, they will appear at rest. And every one of these Objects according to Sense, will describe the Circumference of a Circle, to whose Plane the Axis of the Earth is perpendicular at the Center. And because all these Circles, together with the visible Objects describing them, appear to be in the concave Spherical Superficies of the Heavens, concentric to the Earth; every visible Object will seem to describe a greater or lesser Circle, according to its greater or lesser distance from the Points above described, that seem at rest. In short, the Celestial Sphere will seem to be moved from East to West, in the Space of a Day; about the same Axe in Position, about which the Earth really moves in that time from West to East: And those Points will be the Poles of this Motion, in which the Axis of the Earth produced meets with its Superficies; and that will be the middle Circle betwixt the Poles, and consequently the greatest, (call'd the *Aequinoctial* and *Equator*, for reasons to be explain'd in the next Proposition,) where the Plane of that Circle of the Earth, which lies exactly between the Poles, being produced, meets the concave Sphere. And universally, any Circle on the Surface of the Earth, mark'd by any Point of it, in the diurnal Motion of the Earth, has a correspondent Circle in the Celestial Sphere, made by the intersection of the

Surface

Surface of the Celestial Sphere with the Superficies of a Cone, whose Base is a Circle marked on the Earth, and Vertex the Center of the Earth. The Axis of the World, and the above-mention'd Celestial Circles, have always the same situation among the Fix'd Stars; because their primitive Terrestrial ones, from whence they have their original, produced so far, fall upon the same Stars, as to Sense, in what Point soever of its Orbit the Earth be placed.

PROPOSITION XXXIII.

TO explain the Phenomena of the Sun seen from the Earth, arising from the Diurnal Motion of the Earth about its Axis, in conjunction with its Annual Motion about the Sun; that is, to explain the changes of Day and Night, and their variety in different Places, and different Seasons of the Year.

Tho' each place of the Earth be inlighten'd by every Star that is above its Horizon, yet the illumination of the Sun being so very great above all others, its being above the Horizon alone is called *Day*; and its abode below the Horizon is called *Night*: And because the Sun is intirely separate and remote from the Earth, and so does not partake of its Diurnal Motion, it will Rise and cause Day, in respect of a given place; and being carried thro' the midst of the Heavens, describing a Circle to appearance, having the Axis of the World perpendicular to its Plane, will at last Set again; making the Night to continue, till by the apparent universal Revolution of all the Celestial Bodies, it come up again above the Horizon of the said Place.

To explain the variety of the Days and Nights, and the Seasons of the Year thence arising, imagine the Circle $\gamma \text{ } \ominus \text{ } \oplus \text{ } \varpi$ [Fig. 28.] to represent the



the Orbit describ'd by the Earth, in a Year, about the Sun plac'd in the Center at *S*. The Circle in this Scheme is designedly reclined, and made to appear in the form of an Ellipse, for the more commodious delineation of the various Positions of the Earth.

In this Orbit, let the Earth $A \odot B E$ be supposed to be carried after the manner described in *Prop.* 30. from $\vee$, $\oslash$, to π , &c. revolving at the same time about its own Axis BA , towards the same quarter, or from $\odot$ upwards towards E . The extremities of the Axis of the Earth will be the Poles; A the Southern Pole, and B the Northern: $E \odot$ the Equator, whose Plane is inclin'd to the Plane of the Ecliptic, by an Angle of $23\frac{1}{2}$ Degrees; viz. the Complement of $66\frac{1}{2}$ Degrees to a right Angle, by which the Axis of the Earth (perpendicular to the Plane of the Equator) is inclin'd to it.

As $A \odot B E$ represents the Earth, so it may be imagin'd to represent the Celestial Sphere, (describ'd in the preced. *Prop.*) concentric to the Earth, and surrounding it as it were, and carried along with it; for this Sphere may be describ'd at any distance. And the Place of the Sun or any Star or Celestial Point in this Sphere, will be there, where a Right line joining the Center of the Earth and the said Point, meets its Surface, for there the Eye plac'd in the Center of the Earth refers it.

Let the Earth be suppos'd to be at $\sphericalangle$, where a Right line $S \sphericalangle$, joining the Sun and Earth's Centers, is perpendicular to the Axis of the Earth AB , that is, where the Sun appears in the Plane of the Earth's Equator produced, or where being seen from the Earth, it appears in the Æquinoctial Circle, marked in the Celestial Sphere;

F and

and consequently declines neither to the North Pole, nor the South, but seems to describe, in its diurnal Motion, the Equinoctial it self.

The Sun always seems to be in the Plane of the Ecliptic, (as was shewn in *Prop. 2.*) it will therefore appear to be in the common intersection of the Circles of the Ecliptic and *Æquator* described in the Celestial Sphere ; namely, in *V*. Now in this Situation, the illumination of the Earth, made by the Sun, reaches both the Poles *A* and *B* ; (because the Boundary or Circle of illumination is a great Circle on the Earth, to whose Plane, the Right line joining the Centers of the Sun and Earth, is perpendicular) consequently half the Terrestrial Equator *E Q*, or any other Circle parallel to it, is illuminated by the Sun, while the other half continues in darkness. And therefore every Plane of the Earth being carried round by the equable diurnal Motion, will be as long in darkness as it was in the light, that is, the Day, all over the Earth, will then be equal to the Night. From whence the Circle that the Sun then describes by its Diurnal Motion, lying between the two Poles in a Celestial Sphere, is called the *Equinoctial Circle*.

The Earth moving a little forwarder towards *m* and *2* in its Annual Motion, has the Plane of its Equator *E Q* no longer directed towards the Sun ; but subsiding below it towards the South : From whence the Sun seems by little and little to decline from the Celestial Equator, towards the North Pole. For the Earth being at rest to all appearance, its Equator must likewise be apparently at rest. Consequently the Celestial Equator corresponding to it, is moved only by the apparent Diurnal one : And therefore

fore the Sun, which changes its Situation to it, will seem to move. And the light of the Sun, which before reached to both Poles, *A* and *B*, will by degrees be spread beyond *B*, and fall short of *A*. But when the Earth arrives at *v* and the Sun hereupon appears in *☉*, where it will seem to have its greatest declination Northward, (equal to the inclination of the Ecliptic to the Equator,) and will afterwards return towards the Ecliptic. The Circle in the Celestial Sphere parallel to the Equator, and express'd by *TC*, to which the Sun in its North declination seems to have arriv'd, and describe by its Diurnal Motion at that time, is call'd the *Tropic of Cancer*; borrowing its name from the Sign of the Ecliptic the Sun is then in; with which name likewise the correspondent Circle on the Earth is call'd. The Earth being in this Situation, 'tis evident that the rays of the Sun, inlightening half of it, will reach beyond the North Pole *B*, to *L*, and stop short on this side the South Pole *A*, at *F*, so as that the Arc *BL* or *AF*, is equal to the Arc *ET*; the measure of the Sun's greatest declination, or of the Inclination of the Ecliptic to the Equator. If Circles be imagined to be drawn thro' *L* and *F*, parallel to the Equator *EQ*, (namely *KL*, *FG*,) they will be the *Polar Circles*; the one the *Arctic*, and the other the *Antarctic*. These things being suppos'd, 'tis plain that all that Tract of the Earth, which is included in the Arctic Polar Circle *VL*, notwithstanding the Diurnal revolution, is all the while inlightened, and injoys therefore a continual Day; and on the contrary, that all within the compass of the Antarctic Circle *FG*, have continual Night or Darkeness covering it. 'Tis further evident (from *Prop. 19. Book 2. of the Sphaerics*

of *Theodosius*,) that the greater part of any Circle, parallel to the Equator EQ , lying between it and the Arctic Circle KL , is inlightened; the greater part of any one lying between the Equator and the Antarctic Circle FG , is in darkness; and that so much the more (by *Prop. 20. of that Book*) as the Circle is farther distant from the Equator; and (by *Prop. 11. Book 1.*) one half of the Equator is always in darkness, and the other half always inlightened. And therefore in that Situation of the Earth, wherein the Sun appears in $\odot$, to the Inhabitants of the Northern parts of the Earth, the Days are longest, and Nights shortest; consequently it is Summer: But to the Inhabitants of the Southern part, the Days are shortest, and Nights longest, consequently 'tis Winter. And these longest Days are so much the longer, and shortest Nights so much the shorter, according as the Place is more Distance from the Equator: But to such as inhabit the Equator, the Days and Nights are still equal; and consequently are so all the Year long.

The Earth going forward thro' ϖ , $\times$, to $\vee$; during which time, the Sun seems to move thro' the Signs $\odot$, $\cap$, and III , the Sun returns towards the Equator; till at last at $\cap$, the Sun appears in the Equator of the Heavens, (because at that time, the Plane of the Earth's Equator produced does again pass thro' the Sun,) making the Days equal to the Nights all over the Earth. And now again the extremities of illumination reach to both Poles; so that the Day under the Pole B , which has been all the while inlightened, is equal to the space of half a Year, (the time the Earth has spent since its departure from $\cap$;) and the Night under the Southern Pole A , as long.

The

The Earth moving forwards thro' the Signs γ , φ , π , the Sun will seem in the mean while, to go thro' $\triangle$, $\mathfrak{m}$ and $\uparrow$, and to decline gradually from the Equator towards the South; till at last, the Earth being really in $\mathfrak{S}$, and the Sun appearing to be at $\mathfrak{W}$, all the same Appearances will happen to the Inhabitants of the Southern Hemisphere, as have happen'd to the Inhabitants of the Northern Hemisphere, when the Earth was at $\mathfrak{W}$; and what happen'd to the Inhabitants of the Southern Hemisphere before, will now happen to them of the Northern. And while the Earth is carried thro' $\mathfrak{S}$; $\mathfrak{Q}$, and $\mathfrak{W}$, the Sun will seem to move thro' $\mathfrak{W}$, $\mathfrak{Z}$, and $\mathfrak{X}$ to the Equator, and γ , compleating the Year: And the Sun will cause the same Phænomena of Day and Night, the former decreasing, the latter increasing, to the Inhabitants of the Southern Hemisphere, as it did to the Inhabitants of the Northern Hemisphere, while it went thro' the three opposite Signs.

SECTION VI.

Of the Orbits of the Planets and Comets, and their Figure; together with the Law of Centripetal Forces, requisite to make them move in such Orbits.

PROPOSITION XXXIV.

Each Primary Planet describes the Perimeter of an Ellipse, having the Sun in one of its Focus's.

Having hitherto been treating of what was general, 'twas sufficient to consider the way of

F 3

each

each Planet, as if it were the Circumference of a Circle, having the Sun for a Center, since it does not differ very much from a Circle. He that considers the Phænomena of the Planets mov'd about the Sun more closely, will find, that they are not every where equally distant from the Sun: For instance, the Earth is more distant from the Sun in Summer than in Winter; as is evident from the apparent Diameter of the Sun, which is greatest about the middle of *June*, and least about the middle of *December*. 'Tis further evident from the swifter Motion of the Earth about the Sun, in the midst of Winter, than in the midst of Summer, near a fifteenth Part. And for this reason it is, that there are about eight Days more from the Vernal Equinox to the Autumnal, the Summer Season, than from the Autumnal Equinox to the Vernal, the Winter Season; tho' in both cases the Earth or Sun seems to have moved but a Semicircle. Since therefore the Earth moves swifter about the Sun in Winter than in Summer, it must then be nearer to the Sun, by *Prop. 12.* to make the Area, described by a Right-line drawn from the Sun, augment equably as the Time, the greatest length of the Base compensating for the lesser Altitude of the Trilineal Figure. The same holds likewise in the Motion of the other Primary Planets about the Sun: Nay the Orbit of any of them (except Venus) is more different from a Circle, having the Sun for its Center, than the Orbit of the Earth, as shall be shewn hereafter.

This deviation of the Planets in their Motions about the Sun, from the Circumference of a Circle concentric with the Sun, is so great and sensible, that all Astronomers have been oblig'd to
suppose

suppose *Excentric Orbs*: And because they thought it was past dispute, that all Celestial Motion must be perfectly Circular, they imagined each Planet to be moved in a Circle, as *AIPR* [Fig. 29.] having its Center *C*, at such a distance from the Sun *S*, as might agree with its Motions. The remotest Point *A*, of this Excentric Orb from the Sun, they call'd the *Aphelium*; *P*, the *Perihelium*; both the *Apsides*; the Right-line *AP*, connecting the *Apsides*, and in which they placed the Sun, the *Line of the Apsides*; the part of it *CS*, the *Excentricity*. And by the help of such a *Theory* of the Planets as this suppose, the Places of the Planets found by Calculators, agreed exactly enough with their true ones in the Heavens; especially of them whose Orbits were very little different from Circles; for instance, the Earth. But when they came to Mars, its Place, by computation from this Theory, was so different from its Place by observation, that they concluded its Orbit was not Circular, but something depress'd in the Points *I* and *R*, which are remotest from the *Apsides*, and therefore, as it were, an Oval. This Oval the sagacious *Kepler*, (in his *Commentaries upon Mars*,) after indefatigable pains in Calculations, found to be a perfect Ellipse, having the Points *A* and *P* for its Vertices, which were the *Apsides* in the Circular Orbit, and still retain the same Names, and the Center of the Sun for one of its Foci: Because this Ellipse alone exhibits, the Distances from the Sun, (or Right-lines *SE*) agreeable to observation, as it does the true Equation for determining the Angle *ASE*, from the mean Motion of the Sun given: That is, only the before mentioned Ellipse is of that nature, as to have the Periodic Time, or intire Revolution,

tion, in the same ratio to the Time since its departure from the Aphelium in A , as the entire Area $A E P L A$, is to the Area $A S E Q A$, intercepted between the line of the Apfides $S A$, and the line $S E$ (known by observation) drawn from the Sun to the Planet: But that this is a circumstance in the Motion of every Planet about the Sun, is evident from *Prop. 13*. Besides, the Place of the Earth in an Elliptic Orbit is such, as that the Places of the other Planets about the Earth, found in their respective Orbits by Calculation, and seen from the Earth, agree very exactly with their Places found by Observation in the Heavens; which happens so exactly in no other Orbit whatever. Since therefore the Places of the Planets, found by a Calculus accomodated to Elliptic Orbs, agree so exactly with their Places by Observation; but do not at all agree with a Calculus founded upon Orbs of any other Figure, but upon the account of their being something near, or approaching to an Elliptic form; 'tis evident that the Orbits described by the Planets are Elliptic.

The Elliptic Orbits of the Primary Planets described about the Sun in the common Focus, are almost immovable; that is, when the Planets are carried to the Apfides of their Orbits, or any other given Points, they return to the same Points of the Mundane Space nearly: For such kind of Orbits only, solve the observed Phænomena of the Places of the Planets among the Fixt Stars, seen from the Earth.

PROPOSITION XXXV.

A Comet moves in a Conic Section, that has the Sun in one of its Foci.

For such Orbits, and such only, agree with the

the Places of the Comets observed in the Heavens. 'Tis true that *Kepler*, and several Philosophers after him, supposed the Trajectories of Comets to be Right lines; and have Calculated their Places upon that Hypothesis, agreeing pretty exactly with their Places by Observation. But this might have been done, tho' the Comet were moved in a Conic Section, if the Observation were made only while it is in that part of its Orbit which is not much different from a Right line. Let $APVBC$ [Fig. 30.] be a very excentric Section of a Cone; that is, the Distance of whose Foci is almost equal to its Transverse Axe: Let S , the Center of the Sun, be one of the Foci; it may be the Comet is observ'd while it is describing the part AP of its Orbit, but during the rest of the time, while it pass'd from P , thro' V , B , and C , (where it disappears, and goes off into very distant Regions,) it lies hid by the Rays of the Sun, in respect of the Observer upon the Earth, moved about the Sun S : Or it may be the Comet was hid during its Motion along $APVB$, under the Beams of the Sun, (the Situation and Motion of the Earth allowing of it, and concurring;) and then at last is observed, when it is come to B , and is about to describe the line BC ; and in both cases, the Path of a Comet differs not sensibly from a Right-line. In the former case, the Comets will seem to be swallow'd by the Sun; because they were seen before their access to the Sun, and afterwards were seen no more; and consequently are look'd upon as destroy'd: But in the latter case, they will seem to arise out of the Sun, because they were first seen in their ascent from the Sun, towards the remote Regions. Besides, when a Comet in its

descent

descent towards the Sun (as in AP , comes under our Observation, approaching afterwards to the Sun, and lying hid for some time in his Rays (namely, while it describes PVB ,) and unexpectedly emerges on the other side of the Sun, with a Tail considerably increased, (for reasons afterwards to be assigned in their proper place;) it is commonly taken for a new Comet, and different from the former; the Trajectory of the one being the Right line AP , but of the other BC . Now the difference and seeming contrariety of their Trajectories, have deceived some Astronomers, who have asserted that it was not the same Comet that described them both. Nor is this Mistake easily corrected in a Comet that never returns to the same place, as we know of, by Observation. From all which, the reason is plain, why Rectilineal Trajectories, for the most part, agree with the Motion of Comets; namely, because only one portion of it, is taken for the whole Trajectory: But if the whole be considered together, as well in its descent towards the Sun, as in its ascent from it, no other line but a Conic Section would be found to agree with it.

Of the three Sections of a Cone, an Ellipse, having the Center of the Sun in one of its Foci, is most agreeable to the Motion of a Comet, as shall be demonstrated in its proper place: And if the Comets be durable Bodies, and return, none but an Ellipse will suit their Motion. But instead of a portion near the Vertex of a very excentric Ellipse, which the Motion of a Comet requires, a portion of a Parabola, having the same Vertex and Focus, may be substituted; because not sensibly differing from it. As the Parabolic Curve is its true Trajectory, if it never

ver return; or else perhaps an Hyperbolic one, if there be any such Comet, whose Motion an Hyperbolic Trajectory will better agree to, than a Parabolic.

PROPOSITION XXXVI.

EACH Secondary Planet revolves about its Primary, in the Perimeter of an Ellipse, having the Center of the Primary in one of its Foci.

As for the Moon, the Earth's Secondary, it appears once, in its Revolution about the Earth, lesser, and at the same time slower; and in the opposite point greater, and swifter: 'Tis plain therefore, that its Orbit is excentric to the Earth. That it is an Ellipse having a Focus in the Earth's Center, may be proved by reasons like those by which (in the præced. Prop.) we demonstrated the Orbits of the Primary Planets about the Sun, to be Elliptic: namely, because assuming an Elliptic Orbit for the Moon, its distances from the Earth, come out agreeable to Observation, and the Æquations of its Motion, such as are not to be found from the Supposition of any other kind of Orbit. All this agrees likewise with the Orbits of the Satellites of Jupiter and Saturn; tho' they are nearly Circular and concentric with Jupiter and Saturn.

Tho' the Elliptic Orbits of the Primary Planets are at rest, as was said in the præceding Proposition, yet 'tis quite otherwise in the Secondary: For their Motions are so disturbed by causes to be explain'd hereafter, that the same Ellipses are not described again, abstracting from the Motion of the primary Planet. But to reduce their Orbits to Ellipses, we must imagine the Ellipse of each Secondary Planet, to be moved so, as that the Plane of the Ellipse,
and

and Focus fixed in the Center of the Primary, continuing the same, the Axis of the Ellipse is carried, *in consequentia*, by an angular Motion, round the immoveable Focus. But this Motion is so slow, and insensible, (being in the Moon an hundred times slower than the Moon it self,) that it may very well be neglected in this place; and the Ellipses described by the Secondary about their Primary, may be look'd upon as unmoved, abstracting from their Motion round the Sun in common with their Primary.

PROPOSITION XXXVII.

IF a Body be projected according to the direction of any Right line, and at the same time urged by a Centripetal Force tending to the Center S, [Fig. 31.] so as by a Motion compounded of both, it describes the Curve APP; and the Right line PR be a Tangent to it, in a point P; and from the point B, nearest to P, a line BD be drawn perpendicular to the Right line SP, and the Right line BR parallel to the same line SP; and if the like construction be made at any other Point p, of the Curve: I say the Centripetal Force in P, is to the Centripetal Force in p, as $\frac{Sp^q \times bd^q}{br}$

is to $\frac{SP^q \times BD^q}{BR}$: Or that the Centripetal Force

in P, is reciprocally as the Solid $\frac{SP^q \times BD^q}{BR}$, when the Figure PRBD is indefinitely small.

Let the Force in P, that tends to the Center S, be called *V*. Let the Time wherein the indefinitely small Arc PB, is run, or in which a Body, by its *Vis insita* alone, would run the indefinitely small Tangent PR, be called *T*. Let the Centripetal Force acting in p be called *v*:
and

and the Time that pb , or the Tangent pr , is run in call t . Take an Arc $P\beta$, run in the same time with the Arc pb ; and draw βp parallel to PS . These things being supposed, 'tis evident that the ratio of the lineola BR to the lineola br , is compounded of the ratio of BR to βp , and of the ratio of βp to br : But (by *Prop.* 24.) BR is to βp in the duplicate ratio of the Arc PB to the Arc $P\beta$; and these Arcs, since they are indefinitely small, are as the Triangles BSP , βSP , of the same Altitude; that is, (by *Prop.* 12.) as the Times in which they are described by a radius drawn to S ; or, by Construction, as the Times wherein the Arcs PB , pb , are described. Again, the lineola βp , is to the lineola br , as the Causes producing them; that is, as the Centripetal Force in P , to the Centripetal Force in p . And therefore the ratio of the lineola BR to the lineola br , is compounded of the duplicate ratio of the Times wherein PB and pb are described, and of the ratio of the Centripetal Force in P to the Centripetal Force in p :

That is, in Symbols, $\frac{BR}{br} = \frac{T^2}{t^2} + \frac{V}{v}$; or

$$\frac{BR}{br} = \frac{T^2 \times V}{t^2 \times v}. \text{ And consequently } \frac{V}{v} = \frac{BR \times t^2}{br \times T^2}$$

But (by *Prop.* 11.) T is to t , as the Area SBP to the Area Sbp ; or as its double $SP \times BD$ to $Sp \times bd$ the double of the latter: Substituting

therefore instead of the ratio $\frac{t^2}{T^2}$, its equal

$$\frac{Spq \times bdq}{SPq \times BDq}, \text{ and then } \frac{V}{v} = \frac{BR \times Spq \times bdq}{br \times SPq \times BDq}$$

or V to v , as $BR \times Spq \times bdq$ to $br \times SPq \times BDq$; that is, (dividing them by $BR \times br$)

as $\frac{Sp^q \times bd^q}{br}$ to $\frac{SP^q \times BD^q}{BR}$. The Force therefore in P tending towards S , is reciprocally as the nascent or evanascient Solid $\frac{SP^q \times BD^q}{BR}$.

Which was to be demonstrated.

COROLLARY.

From whence, if any Figure (APp) be given, and in it a Point S , to which the Centripetal Force is directed; the Law of the Centripetal Force, whereby a Body drawn from its rectilineal course, shall be made to keep in the Perimeter of that Figure, and describe it in its revolution, may be found; provided this Force be compounded of a proper one impress'd along the Tangent of the Figure. For the Law sought (by this *Prop.* 37.) is, that the Force be reciprocally as the nascent Solid $\frac{SP^q \times BD^q}{BR}$, which is to be computed from the property of the Figure being given. A remarkable instance of which, we shall give, taken from nature.

PROPOSITION XXXVIII. LEMMA.

IF a Right line ZPR [*Fig. 32.*] touch the Ellipse API , whose Foci are S and F , in any Point P , and a Diameter as IK be drawn parallel to it, thro' the Center C ; the portion EP , of the Right line SP , joining either of the Foci (as S) and the point of Contact, is equal to half the greater Axe CA .

Thro' the other Focus F , draw FH parallel to PR , meeting SP in H . Because (by *Prop.* 48. *lib.* 3. *El. Conic. Apollonius*) the Angles FPZ , HPR , are equal, and consequently their alternate ones PFH , PHF are equal; therefore PH is equal to PF . Again, (by 2. *Elem.* 6.) SE is equal to EH , because

Fig: 33
p: 79

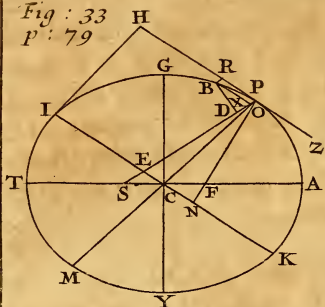


Fig: 36
p: 83

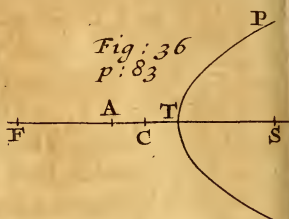


Fig: 34
p: 82

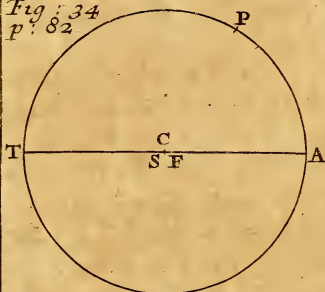


Fig: 37
p: 83

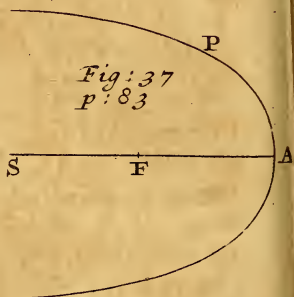


Fig: 35
p: 82

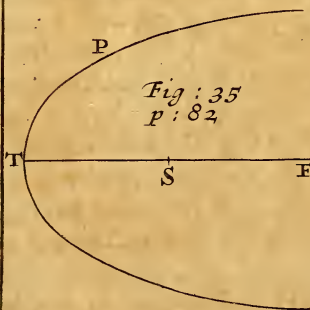
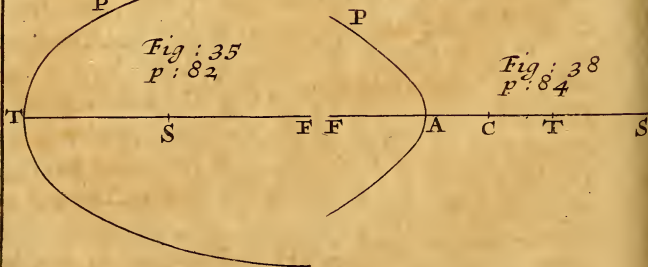


Fig: 38
p: 84



cause SC is equal to CF ; and since SH is the difference of the Right lines PS , PH , it will likewise be the difference of the lines PS , PF ; and EH , half this difference. And therefore PE , (made up of the lesser and half the difference) is equal to half the Sum of PS , and PF : But (by *Prop. 52. lib. 3. El. Conic.*) the Right lines SP , PF , taken together are equal to the greater Axe: Wherefore half their Sum (that is, EP) is equal to half the greater Axe CA . Which was to be demonstrated.

PROPOSITION XXXIX.

L *Et a Body revolve in an Ellipse; the Law of the Centripetal Force tending to the Focus of the Ellipse is required.*

Let APM [Fig. 33.] be the Ellipse, whose Perimeter is described by the revolving Body; its Focus or Umbilical Point S , to which the Force is directed, whereby the Body drawn from its rectilineal Motion is retained in the Elliptic Orbit: required the Law of this Force.

Draw the conjugate Axes of the Ellipses TA , GY , crossing one another in the Center C . And thro' any Point P , in its Perimeter, draw a Tangent PZ , and a Diameter PM , and a conjugate to it ICK parallel to PZ , to which, from P , let fall the perpendicular PN . Compleat the Parallelogram $PCIH$, and the Right line IH will touch the Ellipse: (by 17. *Book I. Conic.*) Join SP cutting IK in E . Thro' B , the nearest Point possible to P , draw BR parallel to PS , and BX to PZ , meeting the Right line SP in X , and MP in O .

The centripetal Force tending to S , is (by *Prop. 37.*) reciprocally as the solid $\frac{SP^2 \times BD^2}{BR}$.
Which

Which therefore must be computed from the nature of the Ellipse. In order to which, let the *Principal Parameter* of the Ellipse, or that which belongs to the greater Axé TA , be call'd L . The Ratio of $L \times BR$ to BD^2 is compounded of the Ratios of $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BO^2 , BO^2 to BX^2 , and BX^2 to BD^2 : But the Ratio of $L \times BR$ to $L \times PO$, or (by *Prop. 1. El. 6.*) of BR or PX to PO , is equal to the Ratio of PE to PC ; (by *Prop. 2. El. 6.*) because XO is parallel to EC ; and (by præced.) PE is equal to AC : Therefore the Ratio of $L \times BR$ to $L \times PO$ is equal to the Ratio of AC to PC . The Ratio likewise of $L \times PO$ to $MO \times PO$ is the same with the Ratio of L to MO . And the Ratio of $MO \times PO$, or the rectangle MOP to BO^2 , is the same (by *Prop. 21. B. 1. Conic.*) with the Ratio of the rectangle MCP (that is, CP^2) to IC^2 . And in the present case, the Ratio of BO^2 to BX^2 is the Ratio of equality; for the Point B coinciding with the Point P , BO becomes equal to BX . Lastly, the Ratio of BX^2 to BD^2 is equal to the Ratio of PE^2 to PN^2 : (For the Triangles BDX , PNE are equiangular, the Angles at D and N being right; and BXD , PEN , being alternate, and therefore equal, because of the two Parallels BX , EN .) And PE^2 is equal to CA^2 , because the lines themselves are equal (by the præceding Proposition.) Therefore the Ratio of BX^2 to BD^2 is the same with the Ratio of CA^2 to PN^2 . Besides, (by *Prop. 72. Book 4. of Gregory St. Vincent*,) the rectangle under GC and CA is equal to the Parallelogram $PCIH$, that is, to the rectangle under IC , PN : Wherefore (by 16. *El. 6.*) CA is to PN as IC to GC ; therefore CA^2

is to PN^q as IC^q to GC^q : The ratio therefore of BX^q to BD^q , is the same with the ratio of IC^q to GC^q . Since therefore the ratio of $L \times BR$ to BD^q is compounded of the ratios of $L \times BR$ to $L \times PO$, $L \times PO$ to $MO \times PO$, $MO \times PO$ to BO^q , BO^q to BX^q , and BX^q to BD^q ; 'tis compounded also of ratios respectively equal to these; namely, of the ratios of AC to PC , L to MO , CP^q to IC^q , and IC^q to GC^q ; and therefore will be equal to the ratio of $AC \times L \times PC^q$ to $PC \times MO \times GC^q$: But (by *Prop. 13. Book 1. Conic.*) $AC \times L = 2GC^q$. Substituting therefore the one in the room of the other, the ratio of $L \times BR$ to BD^q , is equal to the ratio of $2GC^q \times PC^q$ to $PC \times MO \times GC^q$; that is, to the ratio $2PC$ to MO : But (in the present case, since the point B is the nearest possible to P ;) the point O must be the least distance possible from the point P ; that is, MO is equal to $2PC$; and therefore, in this case, $L \times BR$ is equal to BD^q : Because therefore, in an Ellipse, $BD^q = L \times BR$, 'tis $\frac{SP^q \times BD^q}{BR} =$

$\left(\frac{SP^q \times L \times BR}{BR} \right) = SP^q \times L$: The Centripetal Force therefore is reciprocally proportional to this Solid; and L in it is a constant invariable Right line.

The Law therefore of the Centripetal Force tending to the Focus of an Ellipse, (whereby a Body, turn'd off perpetually from its Rectilineal course, is kept in the perimeter of that Figure, and describes it in its Motion,) is, that it is reciprocally proportional to the Square of the distance of the Place P from the Center S . Which was to be found.

COROLLARY 1.

If the Focus S , of the Ellipse, to which the Force tends, together with the next Vertex T , continue as before; [*Fig. 34.*] but the Focus F , approach it, and at last coincide with it; the Body P , will now be kept revolving in the Perimeter of a Circle, by the Centripetal Force tending to S , after the same manner, and by the same Law, as it was in the Perimeter of the Ellipse. And as a Circle may be described by a Body, acted upon by any Centripetal Force tending to a point situated without the Right line, according to the direction of which it would have moved by its *Vis insita* alone, (as was said in *Prop. 29.*) so a Right line will be described by a Body that is urged by no Force tending to a point situated without the said Right line: For if the two Vertices continue the same, and the Foci approach to them, the Ellipse will be turned into a Right line. But to describe any other Lines, besides a Right one and a Circular, a particular Law of the Centripetal Force is necessary.

COROLLARY 2.

If the Focus S , [*Fig. 35.*] of the Ellipse, to which, as a Center, the Force be directed, together with the next Vertex T , remain as before, and the other Focus F , go farther off *in infinitum*, the Body P will be kept in the Perimeter of the Figure TP , by the Centripeaal Force tending to S , after the same manner as it was before, and by the same Law. But in this case, the Ellipse will be turn'd into a Parabola: The Law therefore of a Centripetal Force tending to the Focus of a Parabola, retaining a Body in the Perimeter of such a Figure, is this, that it is reciprocally proportional to the Square of the Distance from the Focus,



Fig: 39
p: 85

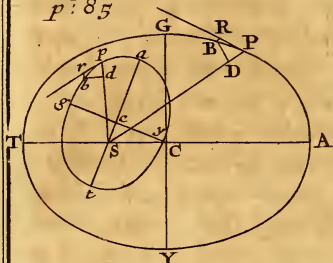


Fig: 40
p: 87

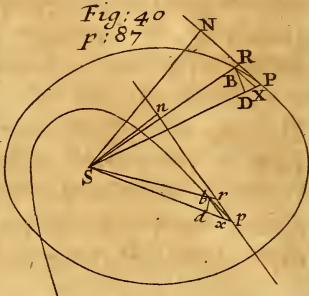


Fig: 41
p: 90

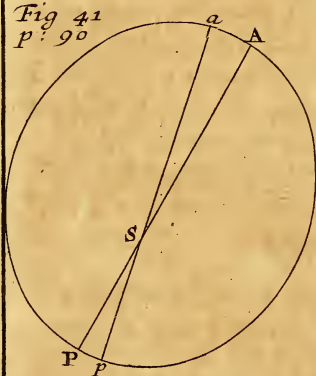


Fig: 42
p: 90

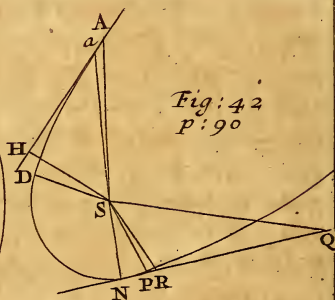


Fig: 43
p: 96

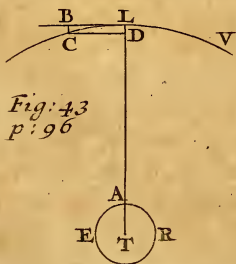
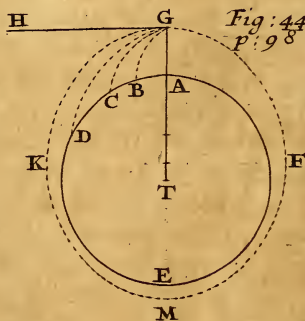


Fig: 44
p: 98



COROLLARY 3.

If, all the rest continuing as before, the Focus F , [Fig. 36.] go off to a more than infinite Distance; (that is, if it appear on the other side of T , and make the Vertices A and T lie between the Foci S and F ; whereas before, the Foci S and F were between the Vertices A and T , in which case, the Ellipse is turn'd into an Hyperbola; the Body acted upon by the same Law of the Centripetal Force as before, (that is, so as to increase, as the Square of the Distance from the Center decreases,) will then move in the Perimeter of this Hyperbola.

COROLLARY 4.

But if the Focus F , [Fig. 37.] of the former Ellipse, and the Vertex A continue fixt, and in the mean while the other Focus S , to which the Force is directed, goes off further and further, to an infinite Distance, (that is, if the Ellipse be turned into a Parabola;) the Centripetal Force directed to the Point S at an infinite Distance, or according to Right lines parallel to the Axe AF , by which the Body moves in its Perimeter, will become equable: For Distances from an infinitely distant Center, whose intervals are finite, are equal. And therefore the Force, which is reciprocally as the Square of that Distance, is equable. And conversely, a Body acted upon by an equable Force, tending to a Center infinitely distant, and which consequently is directed according to Right lines parallel to a given one, will describe in its Motion a Parabola, whose Axis is parallel to the given Right line. And this is the famous Theorem of *Galileo*, concerning the path or tract of a Projectile.

COROLLARY 5.

All things continuing as before, if the Focus S , (or Center to which the Force is directed) go off to more than an infinite Distance, as in *Corol.* 3. it happen'd to the Focus F , then the Ellipse will be turned into an Hyperbola, [*Fig.* 38.] and the Body made to move in the Perimeter of the Figure thus changed, by a Force acting according to the direction of the Right line SP , that is, reciprocally proportional to the Square of the Distance of the Place from the Center S : and that consequently is a Centrifugal Force, in regard of the Point S , because the Curve AP , is convex towards A .

As an Ellipse is described by a Body urged by a Force tending to either of the Foci (because within it,) and acting according to the Law of the preceding Proposition; and an Hyperbola, while the Centripetal Force of the same Law tends to a Focus within it, or while the said Centripetal Force is changed into a Centrifugal one, tending to a Focus, without the Curve: So a Parabola, which, as it were a mean betwixt these two Sections, partakes of the nature of both; and is described by the action of a Centripetal Force of the same Law, tending to a Focus within the Curve, (as in both the preceding,) and by the action of a Centripetal Force tending to one Focus, or a Centrifugal Force tending from the other, (after the manner of an Ellipse and Hyperbola respectively,) according as that other Focus is suppos'd to be placed towards the convex or concave parts of the Curve: For both ways it may be considered as infinitely distant.

COROLLARY 6.

From this Proposition and its Corollaries, it follows, that if any Body be projected from any
Point

Point P , according to the direction of any Right line PR , with any Velocity, and be acted upon at the same time by a Centripetal Force, which is reciprocally proportional to the Square of the distance of the Place from the Center S , situated without the abovemention'd Right line; it will be moved in one of the Conic Sections, having its Focus, as the Center to which the Force is directed. For the Ellipse of the former Proposition can be changed into no other Line but a Right line and a Conic Section; (by the motion of either Focus in the Axe produced if need be.) And the principal Parameter of this Section or L , by the demonstration of the Proposition, (where it was shewn that $L \times BR = BD^2$), is a third proportional to BR and BD considered as nascent.

PROPOSITION XL.

IF several Bodies revolve about a common Center, and the Centripetal Force be reciprocally as the Square of the Distance from the Center; I say the Squares of the Periodic Times in Ellipses are as the Cubes of the Transverse Axes.

Let there be any two Orbits APT , apt , about a common Focus S , (fig. 39.) in which the former Construction is suppos'd to continue. Their Parameters L and l , (by Corol. 6. of the preceding Proposition) are respectively equal to

$\frac{BD^2}{RB}$, $\frac{bd^2}{rb}$; the Points B and P , likewise

b and p , coinciding. But RB is to rb (being generated in the same time) as the Centripetal Force in P , to the Centripetal Force in p ; that is, by supposition, as Sp^2 to SP^2 : And therefore

$L : l :: \left(\frac{BD^2}{Sp^2} : \frac{bd^2}{SP^2} :: \right) BD^2 \times SP^2 :$

$bdq \times Spq$. And $BD \times SP$ is to $bd \times Sp$, as their halves; that is, as the Areas described at the same time by the Bodies P and p : And therefore L and l are as the Squares of the Areas, that are described in the same time, by Radii drawn to the common Center S ; that is, these Areas, described at the same time, are in the subduplicate ratio of the Parameters. And since the intire Areas of the Ellipses are as any lesser parts of them, described at the same time, drawn into the respective Periodic Times; the intire Areas will be to one another, in a ratio compounded of the subduplicate of the Parameters, and simple ratio of the Periodic Times.

Since (by *Prop. 191. Book 4. of Gregory St. Vincent*) the Ellipse $AGTY$ is to $agt y$, as $AT \times GT$ is to $at \times gy$: And therefore the rectangles under the Axes of the Ellipses, are in a ratio compounded of the subduplicate ratio of the Parameters, and of the simple ratio of the Periodic Times; that is, as $L^{\frac{1}{2}} \times$ Periodic Time in APT to $l^{\frac{1}{2}} \times$ Periodic Time in apt . And since from the nature of an Ellipse, GT is a Geometric mean between L and AT , and gy betwixt l and at ; or $GT = L^{\frac{1}{2}} \times AT^{\frac{1}{2}}$, and $gy = l^{\frac{1}{2}} \times at^{\frac{1}{2}}$; Therefore $L^{\frac{1}{2}} \times AT^{\frac{3}{2}} : l^{\frac{1}{2}} \times at^{\frac{3}{2}} :: (GT \times AT : gy \times at ::) L^{\frac{1}{2}} \times$ Periodic Time in $APT : l^{\frac{1}{2}} \times$ Periodic Time in apt . Dividing therefore the Antecedents by $L^{\frac{1}{2}}$ and the Consequents by $l^{\frac{1}{2}}$, and squaring the Quotients you will have the Cube of AT , to the Cube of at , as the Square of the Periodic Time in the Ellipse APT , to the Square of the Periodic Time in the Ellipse apt . Which was to be demonstrated.

COROLLARY I.

Because a Circle is a Species of an Ellipse; where both the Foci coincide with the Center, and the greater Axe is equal to any Diameter,

'tis manifest that the Squares of the Periodic Times of Bodies revolving in an Ellipse and Circle about the same Center, are as the Cubes of the Transverse Axes: And therefore, if the Axes and their Cubes are equal, the Squares of the Periodic Times, and consequently the Periodic Times themselves, will be equal: that is, when the Center of the Forces is the same, the Periodic Times in Ellipses, will be the same with those in Circles, whose Diameters are equal to the greater Axes of the Ellipses.

COROLLARY. 2.

'Tis evident from the Demonstration of the Proposition, that the Parameters of any Orbits, described by Bodies revolving about a common Center, and acted upon by a Centripetal Force, which is reciprocally as the Squares of the Distances from the Center, are as the Squares of the Areas described in the same time, by Radii drawn to the Center: For this was demonstrated universally of any Orbits, whether Elliptic, Parabolic, or Hyperbolic, before the reasoning was confin'd to Ellipses, in which only, the Periodic Time has any place.

PROPOSITION XLI.

THE Law of the Centripetal Force being as before; I say the Velocities of any Bodies revolving about the same Center, in any points whatever, are in a ratio compounded of the subduplicate ratio of the principal Parameters belonging to the Lines they describe, and of the reciprocal ratio of the perpendiculars let fall from the common Center upon Right lines that are Tangents at the points where the Bodies are.

Let the Bodies *P* and *p* [Fig. 40.] revolve about the common Center *S*, in any kind of Lines *PB*, *pb*. Because the Bodies, by supposition,

tion, are acted upon by a Centripetal Force, reciprocally proportional to the Square of the Distance of the Place from the Center; these Lines will (by *Corol. 6. Prop. 39.*) be Conic Sections, having S for a Focus. Let their principal Parameters be call'd L and l . From S , let fall SN , sn , perpendicular upon the Right lines PR , pr , Tangents in P and p . I say the Velocity of the Body P , is to the Velocity of the Body p , in a ratio compounded of the ratio of $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$, and of the ratio of Sn to SN ; or as $L^{\frac{1}{2}} \times Sn$ to $l^{\frac{1}{2}} \times SN$.

Take the Arcs PB , pb , described in the same infinitely small time; and thro' B draw BX parallel to RP , and BD perpendicular to SP ; let the same be done at b . The Velocities in P and p are as the nascent Arcs PB , pb ; because they are the Spaces run in the same time; that is, as the Lines PR and pr equal to them. The right angled Triangles BDX , SNP , having the Angles BXD , SPN equal, because BX , NP are parallel; so that $SN : SP :: BD : BX$; and therefore BX , or what is equal to it $PR = \frac{SP \times BD}{SN}$. And $pr = \frac{Sp \times bd}{Sn}$, because of similar ratios. But $SP \times BD$, $Sp \times bd$ are as their halves, that is, SPB , Spb ; or the Areas described at the same time by the Bodies P and p , which again (by *Cor. 2. Prop. preced.*) are as the Square Roots of the Parameters L and l ; or as $L^{\frac{1}{2}}$ to $l^{\frac{1}{2}}$. And therefore PR is to pr , or the Velocity of the Body in P , is to the Velocity of the Body in p , as $\frac{L^{\frac{1}{2}}}{SN}$ to $\frac{l^{\frac{1}{2}}}{Sn}$, or as $L^{\frac{1}{2}} \times Sn$ to $l^{\frac{1}{2}} \times SN$: That is, in a ratio compounded of the subduplicate ratio of the Parameters to the Figures PB , pb , and of the reciprocal ratio of the

the Perpendiculars, let fall from the Center of the Forces *S*, upon the Tangents. Which was to be demonstrated.

COROLLARY 1.

Hence it follows, that the Velocities of the Bodies, in their greatest and least Distances from their common Focus, about which they revolve, are in a ratio compounded of the subduplicate ratio of the Parameters, and of the reciprocal ratio of the Distances from the common Focus. For, in their greatest and least Distances from the Focus, the Perpendicular to the Tangent is the Distance it self. If these Distances from the Center are equal, the Velocities, in that moment of time, are in the subduplicate ratio of the Parameters; that other ratio of the distances becoming a ratio of equality. And if one of the Figures be a Circle, the Velocity in the Conic Section, in the greatest or least distance from the Focus, will be to the Velocity in the Circle, as the Square Root of the Parameter of the Conic Section is to the Square of the Distance.

COROLLARY 2.

In the same or different Figures, having equal Parameters, the Velocity of the Body is reciprocally as the Perpendicular let fall from Focus, or Center of the Forces, on the Tangent. For the subduplicate ratio of the Parameters, in this case, is a ratio of equality. And therefore the Velocities of a Body revolving in an Ellipse, in the greatest and least distances from the Focus, are reciprocally as the distances. For, in this case, the distances are the perpendiculars to the Tangents of the Orbits.

COROLLARY 3.

The apparent Motions of a Body in the principal Vertices A and P , [Fig. 41.] viewed from the Center of the Forces S , are reciprocally as the Squares of the Distances SA , SP . Take at the Vertices A and P , the indefinitely small Arcs Aa , Pp , described in equal indefinitely small Times, by a Body revolving about S ; and join Sa , Sp . The apparent Motions of the Body from S , are the Angles ASa , PSp . The ratio of these Angles ASa , PSp , is compounded of the Ratio of Aa to Pp (since these Arcs do not differ from Circular Arcs described upon the Center S ,) and of the ratio of SP to SA ; or the reciprocal of the Radii. But because Aa , Pp are described in equal Times, they will be as the Velocities in the points A and P , which (by *Corol.* preced.) are as SP to SA . And therefore ASa is to PSp , as SP^2 to SA^2 .

COROLLARY 4.

Hence it follows, that, if the Orbit be very nearly a Circle, the apparent Motions of the Body viewed from the Center S , are very nearly reciprocally as the Squares of the Distances from it. For, in this case, all the Right lines that fall upon the Orbit from S , are nearly perpendicular to it.

COROLLARY 5.

From *Corol.* 2. it follows, that if a Body, moved in any Conic Section ADS [Fig. 42.] as before, should leave the Curve, and go on uniformly in a Right line PQ , touching the Curve in P , with the Velocity it had in the point P ; the Area SPQ , which it would describe by a radius drawn to S , would be equal to the Area SAD , which (in the same or equal time,) it would describe, being kept still moving in

in the Conic Section. Take the lineola Aa , PR , described in the same indefinitely small time, by two Bodies; and from the Focus S of the Section, to the Tangents AH and PQ , let fall the Perpendiculars SH , SN . By *Corol.* 2. the Velocity in A is to the Velocity in P , as SN to SH . But as the Velocities in A and P are, so are the Spaces run in the same time, by the Bodies; namely, Aa and PR . And therefore Aa is to PR , as SN to SH . Therefore the Triangle SAa is equal to the Triangle SPR . And since this holds in the indefinitely small Triangles that constitute the Trilineal Figures SAD , and SPQ , and the Spaces SAD , SPQ are made up of an equal number of Triangles, the Times of running AD and PQ being taken equal; the Triangle SPQ is equal to the Area SAD .

PROPOSITION XLII.

THE Primary Planets and Comets moves about the Sun according to this Law, that the Motion of each is compounded of an equable one along the Tangent to the Orbit, and another tending towards the Center of the Sun, in which the accelerating Force is reciprocally proportional to the Square of the distance from that Center.

Each of the Primary Planets (by *Prop.* 13.) is acted upon by a Force tending toward the Center of the Sun; and (by *Prop.* 34.) describes the Perimeter of an Ellipse, having the Sun in its Focus; and all this is true also of any Comet, (by Propositions 14 and 35;) or at least it describes some other Conic Section having the same Focus, seeing it is acted upon by a Force tending towards the Sun. And therefore (by *Prop.* 39. and its *Corollaries*) every Comet is acted upon by an equable Force impress'd along a Tangent

Tangent to the Orbit, and another whereby it is drawn off from the Tangent and kept in a Curvilinear Orbit, which tends towards the Sun's Center, placed in the Focus of the Section, and is reciprocally proportional to the Square of the distance from it. Which was to be demonstrated.

The Planets and Comets therefore are to be conceiv'd as so many Projectiles, which are acted upon by two Forces. For in this manner, and this only, do they describe Orbits, such as Observation finds them to be. And the Law of the Force that draws off these Projectiles from their Rectilinear Motion, is, that it increases as the Square of the distance of the Comet or Planet projected, from that Center of the Sun, increases: Which is the Law whereby the Planets tend to the Sun, as we have demonstrated, otherwise than from the Figure of its Path, in *Prop. 28.*

PROPOSITION XLIII.

THE Nodes and Apfides of the Orbits and Planets are at rest.

For, by *Prop. 40.* every Planet revolves in an immoveable Plane, and the common Interfection of any two Planes continues immoveable: But the Interfection of the Plane of any Planet, with the Plane of the Earth's Orbit, is the Line of the Nodes of that Planet. The Nodes therefore of that, and in like manner of all the Planets, are at rest. Besides, every Planet describes, in a Plane that is at rest, always the same Ellipse, which therefore is at rest, (by *Prop. 34.*) and consequently its Apfides are at rest. Which was to be demonstrated.

COROLLARY.

Since the office of every Fixt Star is like that of the Sun, (as far as Men are capable of judging;) namely, to have several lesser Bodies like so many Planets revolving round its vast Body; its other affections likewise will be similar; as, that its Motion, like the Sun's, will be none or insensible; which is farther evident from the constant and always the same mutual Distance of the Fixt Stars. Consequently the Situation of the Nodes and of the Apfides of the Planetary Orbits, in respect to the Fixed Stars, continues the same.

SCHOLIUM.

This Proposition is true, if the Centripetal Force of the Planets towards the Sun be only considered, as has hitherto been done: But if the mutual Actions of the Planets and Comets upon one another, be taken into the consideration, things will be a little otherwise; as shall be shewn in its proper place. But the Effects of these Actions, by reason: of the smallness of the Forces that produce them, are very small, and as it were none at all, and therefore to be neglected in this place. Nay, the Motions of the Nodes and Apfides of the Primary Planets are so small, that no notice at all have been taken of them by very considerable Astronomers, in their Calculations, and the Nodes and Apfides look'd upon to be at rest.

PROPOSITION XLIV.

THE Planets and Comets move in the Heavens with the greatest Freedom; and consequently their Motion may be preserved for a very long space of time.

For, in Prop. 39. (where we determined the Law of the Centripetal Force, whereby a Body

is

is urged, that revolves in the Perimeter of an Ellipse, or any other Conic Section, having the Center to which the Force tends for a Focus,) we supposed both the Motions, of which the Motion in an Elliptic, Parabolic, or Hyperbolic Curve is compounded, to be perfectly free, and having nothing to hinder or resist it. Since therefore the Planets describe in their Motion, perfect Ellipses, whose common Focus is the Sun, and continue their revolutions in the same Ellipses; 'tis manifest from the converse, that the Motions of the Planets are intirely free from all resistance; that is, that the Planets move in the Heavens very freely. And if the Planets move freely in the Heavens, without any resistance, the Comets will do so too; because they are moved along the regions of the Planets, during their being visible to the Inhabitants of the Earth, in Orbits that are either true Ellipses or species of Ellipses; namely, Parabola's or Hyperbola's, having the Sun in the Focus; to the description of which the same freedom from all resistance is necessary. And that the resistance of the medium in the regions of the Solar System, beyond the Planets, is not greater than in the Planetary regions, is very probable. For if there were the least sensible resistance in the Heavens, the same Orbits could not be described by the Planets carried about the Sun, like so many Projectiles, as have been demonstrated to agree to them, moving freely: But, just as in our Air, the way of a Projectile is found to be very different from a Parabola, which it would describe, if there were no resistance; so the ways of the Planets at least after several revolutions, would be found to be very different from Ellipses; and the Ellipses the same, for Magnitude and Position, with such as are to be de-

described by Planets, moved in free Spaces, would not so exactly agree with the Places observed in former times, as well as in the present Age, as we find by a due Calculation they do. Consequently there is no resistance at all: But the Planets and Comets move in the Heavens very freely, and therefore their Motion may be continued, so as to endure for a very long time. Which was to be demonstrated.

PROPOSITION XLV.

THE Secondary Planets move about the Primary according to this Law, that besides the accelerating Force of the Primary, the Motion of each is compounded of an equable one, along the Tangent to the Orbit, and of another tending to the Center of the Primary, in which the accelerative Force is reciprocally proportional to the Square of the distance from the Center.

Laying aside all the accelerating Force where-
by the Primary is urged, (as was done in Prop. 20.) each Secondary Planet (by Prop. 37.) revolves about its Primary in the Perimeter of an Ellipse, having a Focus in the Center of the Primary, to which (by Prop. 20.) the Centripetal Force tends, by which it is drawn off from its rectilineal Motion: (For the deformity of the Ellipse arising from the Motion of the Apfides, or from any thing else, is so small that it may well be neglected in this place:) And therefore (by Prop. 32.) the Law of the Centripetal Force tending to the Center of the Primary, is, that it is reciprocally proportional to the Square of the distance from the Center: For by this Force only is a Body drawn off from a Rectilineal Motion in a Tangent, arising from its *Vis insita*, and kept in an Elliptic Orbit. But these two Motions (the one equable in the Tangent, the other

other tending towards the Center of the Primary, and increasing as the Square of the distance of the Satellite from it lessens,) being compounded, make the Secondary Planet to describe an Elliptic Orbit like a Projectile.

SECTION VII.

The Planets and Comets are kept in their Orbits by Gravity, which is propagated thro' the whole Solar Systeme according to the same Law.

PROPOSITION XLVI.

THE Force whereby the Moon tends to the Center of the Earth, is the same with the Force of Gravity, whereby all Bodies about the Earth tend to the same Center.

Let RAE [Fig. 43.] represent the Earth, T its Center, VL the Orbit of the Moon, LC , a part of it run by the Moon in the space of a Minute. Because (by *Prop. 15.*) the Moon finishes its Course or whole Circle in 27 days, 7 hours, 43 minutes; that is, 39343 minutes of Time; therefore LC is $\frac{1}{37\frac{1}{3}43}$ of the whole periphery, or 33 Seconds of a degree. Now the Circumference of the Earth (according to the late mensuration of Mr. Picart) is 123249600 Paris Feet; consequently its Semidiameter TA is 19615800 Feet: wherefore TL the Semidiameter of the Moon's Orbit is 1176948000 Feet; or sixty times TA , (by *Prop. 15.*) And the versed Sine LD of the Arc LC , of 33", and BC , which is equal to it, is $15\frac{1}{15}$ Feet, nearly. The Force therefore, whereby the Moon tends toward the Center of the Earth (as by which it is drawn

drawn off from its Rectilineal Motion along the Tangent LB , and retained in its Orbit,) is such, that it impels the Moon towards the Center of the Earth, about $15\frac{1}{12}$ Paris Feet in a Minute. By the preceding Prop. this Force in the Moon is increas'd, as the Square of the distance from the Center of the Earth is lessen'd; if therefore it descend to the Surface of the Earth, where the Square of the distance from the Center is diminished 60×60 times, it will be increased 60×60 times, and therefore 'twould impel the Moon placed on the Surface of the Earth, $60 \times 60 \times 15\frac{1}{12}$ Feet, towards the Center in a minutes time. But Gravity is that Force, which impels any Body placed on the Surface of the Earth, the length of $15\frac{1}{12}$ Paris Feet towards the Center, in the space of a second of time, as Mr. *Hugens* has determined by very accurate Experiments, in *Prop. 25. Part 4. of his Horolog. Oscil.* Therefore the same Force of Gravity, in the space of a minute, impells a Body $60 \times 60 \times 15\frac{1}{12}$ Feet towards the Center; because the Spaces run by a heavy Body, in its fall, are as the Squares of the Times. Since therefore the Force, whereby the Moon is kept in its own Orbit, and the Force of Gravity, perform the like and equal things, and would produce the very same Effects, in the same Circumstances, and tend toward the same point, (namely, the Center of the Earth,) they are the same Forces; that is, the Force whereby the Moon is drawn off from its Rectilineal Motion, and kept in its Orbit, is the same Force with that we call Gravity. Which was to be demonstrated.

A distance of the Moon, a little greater than sixty times the Semidiameter of the Earth, would fit the preceding Calculation better, as $60\frac{1}{3}$. Which number likewise is more agreeable

to Astronomical accounts: Though we have thought it sufficient to give the measure in round Numbers.

SCHOLIUM.

After the same manner as the Moon revolves about the Earth, any other heavy Body projected with a Force strong enough, from a given Point without the Surface of the Earth, according to the direction of an Horizontal Right line, would describe an Orbit, and compleat a revolution, like a Planet, without touching the Earth; and that continually. Let the Circle *ABCD* [Fig. 44.] represent the Earth, *T* its Center. Take any Point without the Surface of the Earth, as *G*, from which, let any heavy Body be projected according to the direction of the Line *GH*, perpendicular to the Line *TG*. 'Tis evident, that if the Body be projected with little or no Force, (that is, if it be let fall freely) it will fall down to the Point *A*, directly under *G*; but if with some Force, it will reach the Earth at the Point *B*, distant from *A*, towards *H*; if with a greater, the Body projected will meet the Earth at *C*, beyond *B*. If the projectile Force be increased, so as that the Body does not reach the Earth, till it has got beyond the Point *E*, opposite to the Point *A*, then ascending again towards *G*, it will compleat the Ellipse *GKMF*; and, if not hindered, will perpetually describe it; and so become a Planet. If the Projectile in the Point *M* of its Way, opposite to *G*, be less distant from the Earth, than at *G*; then the Center of the Earth, *T*, is the Focus of the Ellipse, more remote from *G*. If the projectile Force increased still, till the Points *M* and *G* are equally distant from *T*, the Path or Orbit will be a Circle; and, still augmenting the projectile Force, (all other things continu-

ing

ing the same,) it will become an Ellipse, in which T is the Focus that is nearest G , and the other Focus will be removed from T , proportional to the augment of the projectile Force, till at length, it turns into a Parabola, and then into an Hyperbola. The same things will happen tho' HG be not perpendicular to TG , excepting that in this case, G will not be the principal Vertex of the Orbit. The higher the Point G is above the Earth, the less Force is needful to turn the Projectile into a Planet; and the lower it is, the greater: So that if the Moon should be projected with the same Celerity as it is carried now in its Orbit, from a height above the Earth of but a mile, like a common Projectile, it would meet and strike the Earth before it had run the distance of 15 miles. For the Arc which the Moon runs in $20''$, is less than 15 miles, and a heavy Body in $20'$ near the Earth in falling, runs $20 \times 20 \times 15 \frac{1}{12}$ or 6033 Feet; that is, more than a mile; therefore the Moon would strike the Earth, before it could be projected to the distance of 15 miles. But if, the action of Gravity continuing the same, and the Earth should be annihilated, all Projectiles (like the Planets) would describe Ellipses, or some other Conic Sections, whose common Focus would be that point which now is the Center of the Earth.

PROPOSITION XLVII.

THE Sesondary Planets of Jupiter gravitate towards Jupiter, the Satellites of Saturn towards Saturn, and both the Primary and Secondary Planets gravitate towards the Sun; and all of them are drawn off from their Rectilineal Motions, and kept in their Orbits by the Force of their Gravity.

By reason of that Simplicity of Nature which

is observable in all her Works, more Causes ought not to be admitted, than are sufficient to explain the Phenomena of Natural things; and the natural Effects of the same kind are to be looked upon as having the same Causes: Since therefore the revolution of the Medicean Stars about Jupiter, and Saturn's Attendants about Saturn, and of the Primary Planets about the Sun, are Phenomena of the same kind with the revolution of the Moon about the Earth; their Causes will be the same. For the Forces whereby the Secondaries of Jupiter and Saturn, and the Primary Planets, are drawn off from their Rectilineal Motions, and kept in their Orbits, respect the Centers of Jupiter, Saturn, and the Sun, respectively, after the same manner, as the Force, whereby the Moon is kept in its Orbit, respects the Center of the Earth; and the Forces whereby they are urged, increase and decrease by the same Law, according to their different Distances from the Centers of Jupiter, Saturn, and the Sun, respectively, as the Force acting upon the Moon, placed in various Distances from the Center of the Earth; as has been demonstrated before at large. Consequently, since this Force in the Moon has been shewn (in *Prop. preced.*) to be the same with the Force, which we call Gravity; that Force, whereby the abovemention'd Planets, both Primary and Secondary, tend respectively towards the Sun, and their Primary Planets, must be the same with the Force of Gravity. Besides, all the Secondaries gravitate also towards the Sun, and their Gravity towards the Sun, is the accelerative Force that is common to each Secondary and its Primary; concerning which, see *Prop. 20.* and 45. Which was to be demonstrated.

COROLLARY.

Hence it is evident, that there is such a thing as Gravity towards the Sun and all the Primary Planets. For since the Moon and all Terrestrial Bodies gravitate towards the Earth, and the Satellites of Jupiter and Saturn, towards Jupiter and Saturn, respectively; and since the three other Primary Planets, Mercury, Venus and Mars, are Bodies of the same kind with Saturn, Jupiter and the Earth, (which also is evident from their Gravity towards the Sun, before demonstrated,) 'tis certain, from the similitude of Causes of the like natural Effects before laid down and established, that there is a Gravity also towards them, whose accelerative Force is reciprocally proportional to the Square of the distance from the Center. And because to any kind of Action, there is a Re-action, equal and contrary, the Sun likewise must gravitate towards all the Planets, both Primary and Secondary. Besides, because it is certain, from Experiments accurately made, that all kinds of Bodies, at the same Distance, and in equal Times, run equal Spaces in their Falls towards the Earth, and that the quantity of Motion of unequal Bodies, equally accelerated, is as the quantity of Matter. All Bodies, of what Nature soever, in equal Distances, gravitate towards the Earth proportionally to their quantities of Matter. And since no doubt but the nature of Gravity towards the Sun and Planets, is the same as towards the Earth, (which is sufficiently manifest from what has been already said, as also from hence, that the Primary Planets and their Satellites, that is, Bodies very unequal, descend toward the Sun with an equal accelerative Force, just as Bodies unequal in Magnitude and Density fall with equal Celerities

ties towards the Earth ;) 'tis evident that all Bodies gravitate towards the Sun, and each of the Planets ; and their Gravities towards any one, at equal Distances from its Center, are proportional to the quantities of Matter in each. And because all Action is mutual, the Sun and Planets gravitate according to the same Law towards all Bodies, and consequently all Bodies towards one another.

PROPOSITION XLVIII.

THE Force or Efficacy of any Virtue propagated from a Center, or to a Center, in Right lines every way round about, in different places, is reciprocally proportional to the Square of the Distance of the place from the Center.

Let S , [Fig. 45.] be the Center, from which, or to which, the Virtue is propagated, round it describe two Spherical Surfaces TE , MA , at any distances, ST , SM . I say the Force or Efficacy of the Virtue in T , is to the Force of the same in M , as SM^2 , to ST^2 .

The same Virtue equally diffused and spread thro' a double Space, will be twice as little in any given part: And if it be diffused thro' a triple Space, it will be three times as little: And, universally, the Efficacy of the Virtue is reciprocally as the Space into which it is diffused ; because 'tis directly as the constipation of the Virtue. But any Virtue, which in the distance ST from the Center, is equally diffused thro' the Spherical Surface TE , in the distance SM , is diffused thro' the Spherical Surface MA , after the like manner. The Efficacy therefore of that Virtue, in the distance ST , is to the Efficacy of the same in the distance SM , as the Spherical Superficies MA , is to the Sph-

Fig: 45
p: 102

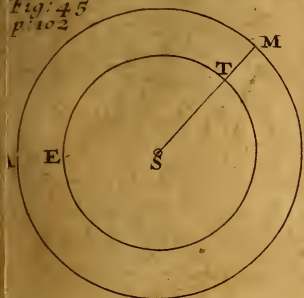


Fig: 46
p: 103

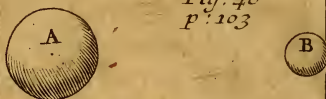


Fig: 47
p: 106
p: 109

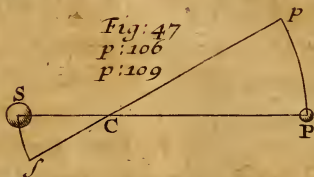


Fig: 48
p: 107

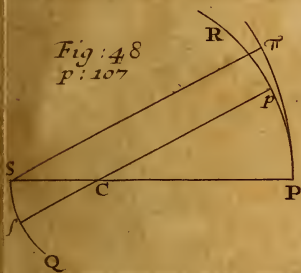


Fig: 49
p: 110

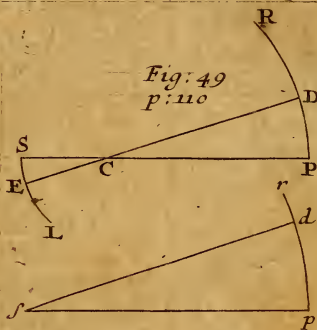
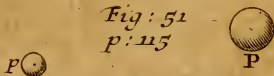


Fig: 50
p: 112



Fig: 51
p: 115



○π



Spherical Superficiēs TE ; that is, as SM to ST . Which was to be demonstrated.

Of this kind are the Virtues or Effects of Light, Heat, &c. and also of Gravity, as is evident from what has been already demonstrated. But some other Virtues, as those of the Loadstone and Amber, are of another kind.

PROPOSITION XLIX.

THE accelerative Gravities towards different Bodies, in equal distances, are as the Bodies themselves toward which they tend.

Let there be any two Bodies A and B [Fig. 46.] in any places A and B ; they (by Corol. Prop. 47.) will gravitate towards one another: And since the Reaction is equal and contrary to the Action, the Gravity or Motive Force of the Body A towards B , is equal to the Gravity or Motive Force of B towards A . But the Motive Force results from the Quantity of Matter or Mass drawn into the Accelerative Force, just as the Quantity of Motion from the Quantity of Matter drawn into the Celerity: The Quantity of Matter therefore, or Mass of the Body A , drawn into its Accelerative Force towards B , is equal to the Mass of the Body B drawn into its Accelerative Force towards A . Consequently the Accelerative Force of the Body A towards B , is to the Accelerative Force of the Body B towards A , as the Mass of the Body B , to the Mass of the Body A : But the Forces (treated of here,) are the Gravities, and the Masses of the Bodies are the Quantities of Matter or the Bodies themselves: And therefore the Accelerative Gravity of the Body A towards B , is to the Accelerative Gravity of the Body B towards A , as the Body B , to the Body A . But (by what has been demonstrated above,) all

Bodies placed in an equal distance from the Body *B*, with that of the Body *A*, have an accelerative Gravity towards *B*, equal to that of *A*; and all Bodies placed in a distance from the Body *A*, equal to that of *B*, have an Accelerative Gravity towards *A*, equal to that of *B*; and the distance of the Body *A* from the Body *B*, is equal to the distance of the Body *B* from the Body *A*, and the Distances and Bodies are taken at pleasure. Therefore the Accelerative Gravities towards different Bodies, in equal distances, are as the Bodies themselves. Which was to be demonstrated.

C O R O L L A R Y

From hence it follows, that if Gravity be consider'd as the Attractive Force of the Body towards which it is directed, propagated as in the preceding Proposition; the Absolute Forces of the attracting Bodies are as the Bodies to which they belong. For the Absolute attractive Force of the Body *A*, is to the Absolute attractive Force of the Body *B*, as the Accelerative attraction of all Bodies towards *A*, is to the Accelerative attraction of all Bodies towards *B*, in equal distances. But, in equal distances, the Accelerative attraction of all Bodies is the same with the Accelerative attraction of any Body, and these Accelerative attractions, by this Proposition, are as the Bodies toward which they tend; that is, as the Attracting Bodies themselves: And therefore, *ex æquo*, the Absolute Forces of Attracting Bodies are as the Bodies themselves, to which they belong. And hence again, and from what goes before, (by *Prop. 23. El. 6.*) we may conclude that the Accelerative Force of the Body *A*, gravitating towards *B*, is to the Accelerative Force of the Body *C*, gravitating towards *D*,
in

in a ratio compounded of the ratio of the Body *B* to the Body *D*, and of the duplicate ratio of the distance between *C* and *D* to the distance between *A* and *B*: And that the ratio of the weight of the Body *A* to the weight of the Body *C*, is compounded of the abovemention'd compound ratio, together with the ratio of the Body *A* to the Body *C*.

SECTION VIII.

Of the Motion of Bodies attracting one another, and the Affections; which are all applied to the System of the Sun and Primary Planets.

PROPOSITION L.

THe Bodies attracting one the other, and revolving about one another, describe similar Figures, both about themselves, and about the common Center of Gravity.

In Prop. 29. and the following ones, we have explain'd the Motions of Bodies attracted to an immovable Center; for it is most proper to begin with the most simple Cases. But because the Attractions are made to Bodies, and Bodies attract one another; as has been before demonstrated: When a Body revolves about another, the attracting Body it self can't be at rest; but both being acted upon by a mutual Attraction, do revolve about the common Center of Gravity. And if there were more Bodies that mutually attracted one another, they would so move among themselves, that the common Center of Gravity might be either at rest, or in an uniform direct Motion. For it is well known to Geometricians, that the common Center of Gravity does not change its state of

Motion

Motion or Rest, by reason the action of Bodies upon one another; as has been demonstrated by Mr. *Wren* and Mr. *Hugens*.

Method therefore requires us to consider the Motions of Bodies, as was said, attracting one another; namely, the Orbits they describe, and the other generals, wherein a System of Bodies acting upon one another mutually differs from the System supposed above; and that in the first place universally, whatever the Law of mutual Attraction be.

Let S and P [*Fig. 47.*] be the two attracting Bodies, which, revolving mutually about one another, describe the Curves Ss , Pp , while their Center of Gravity C , is either at rest, or (which does not disturb the Demonstration,) moves uniformly in a Right line: I say that the four Figures, namely, SsC , PpC , that which S describes about P , consider'd as immovable, and that which P describes about S , consider'd after the same manner, are similar.

Because S and P continuing the same, the ratio of CP to CS , during the Motion, will likewise continue the same; because, from the nature of the Center of Gravity, it is the same with the respect to the Bodies S and P : And since the Angles PCp , SCs , are every where equal, (because C is in the Right line connecting the Bodies,) the Figures SsC , PpC will be similar. For the Figures PpC , SsC , are similar, which the Right line pCs , describes with an Angular Motion about the Point C , consider'd as at rest, provided every moment, the ratio of Cp to Cs , be the same with that of CP to CS .

Again, because the ratio between SC and Cp is always given, by compounding, the ratio of sp to Cp is likewise given, and the Line pCs continues a Right one: And therefore (by what

has

has been said) the Right line Cp about the Point C , and the Right line sp about the Point s , considered as at rest, describe similar Figures. Besides, the Figures whereof one is described by p about s , considered as at rest, the other described by s about p , considered likewise as at rest, are similar and equal, (as is demonstrated in Optics,) because sp is the same in Magnitude and Position with ps . Wherefore the four Figures proposed are similar. Q. E. D.

PROPOSITION LI.

IF two Bodies S and P attract one another by any Forces, and in the mean while revolve about the common Center of Gravity C ; I say, that a Figure may be described, by the same Forces about either of the Bodies consider'd as at rest, similar and equal to the Figures, which the Bodies so moved describe about one another.

Suppose the Orbits PR , SQ [Fig. 48.] to be described first about the Point C at rest, and the Points s and p any places at pleasure, to which the Bodies S and P arrive at the same time. Let the Right line scp be suppos'd to move parallel to its self, till the point s coincide with S , and let this be done in all the corresponding points of the Curves SQ , PR : 'Tis evident, that when s comes to S , the other extreme of the Right line sp , thus moved parallel to it self, will arrive at the point ω , in the Curve $P\omega$, which the Body P would describe about the point S , if (S being at rest) P should revolve about it, being attracted by the same Force as the Bodies did before mutually attract each other: For the parallel Motion of the Right line sp , makes no alteration in the Forces, with which the Bodies attract each other. But the same $P\omega$ is the Line which P describes about S , considered as at rest:
For

For thus it really continues at rest; and the length and inclination of the Right line $S\pi$ to SP , is the same of that with $\int p$. Since therefore these Figures are congruous, they are equal and similar. And what has been demonstrated concerning the Figure, which P describes about S , is true of the Figure, that S describes about P , looked upon as at rest, namely, that it is equal and similar. But if the Center of Gravity C go forward uniformly, by superinducing upon the Systeme of the Bodies S and P , an equal and contrary Motion, all things will be reduced to the former Case: But there is no change made by this uniform Motion along parallel Right lines, in the Forces of the Bodies S and P . Therefore the Proposition is certain in both Cases. Which was to be demonstrated.

COROLLARY 1.

Two Bodies mutually attracting one another, by any Forces, and revolving about the Center of Gravity, describe, by Radii drawn to that Center and to one another, Areas that are proportional to the Times.

COROLLARY 2.

Because a Body revolving about another immovable one, and attracted to it by a Force reciprocally proportional to the Square of the Distance, would describe (by *Prop. 39.*) an Ellipse, or some other Conic Section, one of whose Foci coincides with the Center of the Body that is at rest; it follows, from this Proposition, that two Bodies attracting one another, by Forces reciprocally proportional to the Square of the Distance, describe, both about the common Center of Gravity, and about one another, Conic Sections, having their Foci in the Center, about which they are described. And consequently, if such Figures are described, the Centripetal

tripetal Forces are reciprocally proportional to the Squares of the Distance.

PROPOSITION LII.

IF two Bodies attracting one another by any Forces whatever, mutually revolve about one another, their Motion will be the same, as if they did not attract one another, but both were attracted with the same Forces by a third Body placed in the common Center of Gravity. And the Law of the attractive Forces will be the same, in respect to the distance of the Bodies from that common Center, and in respect to the whole distance between the Bodies.

For the Forces, whereby the Body P [Fig. 47.] tends to S , are directed to any point whatever, in the Right line PS produced towards S . And in like manner, the Forces of the Body S , are directed to any point whatever, of the same Line produc'd towards P . But C is the only point of the Right line SP , produced both ways, that continues at rest, in every situation of the Bodies; and therefore C is that point, to which the Bodies P and S tend, with the same Forces, as they attract one another.

Again, because the Bodies that revolve are given, and the ratio of the distance of either of the Bodies from the common Center, to the distance of the same Body from the other. And consequently the ratio of any Power of the distance of the one, to the like Power of the distance of the other, is given; as also the ratio of any Quantity, which is any way compounded of one distance and any given Quantities, to another Quantity, which is in the like manner compounded of the other distance and as many given Quantities, having that given ratio of the Distances to the former. Wherefore, if the Force, whereby one Body is attracted by
the

the other be directly or inverſly as the diſtance of the Bodies from one another, or as any Power of this diſtance ; or laſtly, as any Quantity compounded any how of this diſtance and the Quantities given ; this ſame Force, whereby the ſame Body is attracted to the common Center of Gravity, will be in like manner directly or reciprocally as the diſtance of the attracted Body from that common Center, or as the ſame Power of that diſtance, or laſtly, as a Quantity compounded after the ſame manner of this diſtance and of the analogous given Quantities : And therefore there will be the ſame Law of the attractive Force in regard of the diſtance from the Center of Gravity, as there is in regard of the diſtances of the Bodies. Which was to be demonſtrated.

PROPOSITION LIII.

THE Periodic Time of two Bodies, *S* and *P*, [Fig. 49.] revolving about the common Center of Gravity *C*, is to the Periodic Time of any other Body *P*, revolving about the other immovable point *S*, (attracting with the ſame Forces,) in the ſubduplicate ratio of the Body *S*, to the aggregate of the Bodies *S* and *P*.

Let the Body *p*, equal and ſimilar to *P*, be imagined to deſcribe the Orbit *p d r*, about *ſ*, at reſt, and ſimilar and equal to *S*. The Figures *C P D R*, *ſ p d r*, (by Prop. 50.) are ſimilar ; and the Forces whereby the Bodies being in any Points (*D* and *d*) ſimilarly poſited, tend toward the Centers *C* and *ſ*, are equal ; becauſe, (by Prop. 51.) the Force whereby the Body *D* tends to *E*, is equal to the Force whereby the Body *d* tends to *ſ*. And, by the preceding Propoſition, the Body *D* tends to the *C*, with the ſame Force as to *E*. And therefore, (by

Corol.

Corol. 2. Prop. 26, and Schol. Prop. 27.) the Square of the Periodic Time of the Body P , moving in the Orbit PDR , is to the Square of the Periodic Time of the Body p , in the Orbit pdr , as the Right line CP to the homologous Right line sp ; that is, as CP to SP : that is, (from the nature of a Center of Gravity) as the Body S to $S + P$. Wherefore the Periodic Time of the Body P in the Orbit PDR , (or of the Body S in the Orbit SEL) is to the Periodic Time of the Body p in the Orbit pdr , in the subduplicate ratio of the Body S , to the aggregate of the Bodies S and P . Which was to be demonstrated.

PROPOSITION LIV.

IF two Bodies S and P , attracting one another by Forces reciprocally proportional to the Square of their distances, revolve about the common Center of Gravity; I say, that the greater Axe of the Ellipse, which either Body P describes by this motion about the other S , (in the manner described in Prop. 50.) is to the greater Axe of the Ellipse, which the same Body P might describe about the other S at rest, (as in Prop. 51.) in the same Periodic Time, in the subtriplicate ratio of the sum of the Bodies S and P , to the quiescent Body S .

If the Ellipses described about the movable and quiescent Body S , by the Body P , were equal, the Periodic Times in them (by Prop. 51, 53.) would be in the subduplicate ratio of the Body S , to the aggregate of the Bodies S and P . Imagine the last Ellipse to be changed, till the Periodic Time of the Body P (describing this Ellipse about the quiescent Body S ,) become equal to the Periodic Time of the Body P revolving about the Body S ; (as we suppose in this Proposition;) that is, till the Periodic Time in
this

this being lessen'd may be in the said subduplicate ratio of the Body S , to the sum of the Bodies S and P . But because (by *Prop. 40.*) in the present case, the Squares of the Periodic Times are as the Cubes of the Transverse Axes, the Periodic Times themselves will be in the sesquiplicate ratio of the said greater Axes. The greater Axes therefore of the latter Ellipse, being lessen'd, is in a ratio, whose sesquiplicate is the subduplicate ratio of P to $S + P$; that is, in the subtriplicate ratio of S to $S + P$: (For $\frac{2}{3}$ of $\frac{1}{3} = \frac{1}{2}$:) and consequently is to the unvaried Axe (equal to this before it was lessen'd) of the other Ellipse, described about the Body S , in the subtriplicate ratio of S , to $S + P$. And therefore inversly, the greater Axe of the Ellipse described about the movable Body S , is to the greater Axe of the Ellipse described about the quiescent Body S , in the same or an equal Time, in the subtriplicate ratio of the sum of the Bodies $S + P$, to the quiescent Body S . Which was to be demonstrated.

PROPOSITION LV.

THE Law of Attraction being the same as before, I say the Orbit PR , (*Fig. 50.*) of the Body P , comes nearer to an Ellipse, whose Focus is C , the common Center of Gravity of the Bodies S and M , than to an Ellipse, whose Focus is the Center of it S ; and that the Areas described by Radii drawn to C , are more proportional to the Times, than the Areas described by Radii drawn to the Centre of it S .

For the attractions of the Body P toward S and M , compose its absolute Gravity, which is directed more towards C , the common Center of their Gravity, than towards the greater Body it self S , and which is more nearly reciprocally proportional to the square of the distance

PC ,

PC , than to the square of the distance PS : For this last would then hold precisely, when M becomes nothing; and the same thing would be true of M , if it should increase till it immensely exceed S . 'Tis certain therefore, that the Forces, in the present Case, are directed to some Point lying betwixt S and M , and nearer the greater of them, as is the common Center of their Gravity C ; which also is that Point where the Natural Forces of any kind of heavy Bodies placed round about it, are united and concenter'd: for the aggregate of several heavy Bodies is most justly looked upon to be there, where the common Center of their Gravity is; as it is well known by Philosophers. What therefore was proposed is evident and certain. Which was to be demonstrated.

PROPOSITION LVI.

Supposing the same Laws of the Attraction, I say the external Body P describes Areas more nearly proportional to the Times, about the common Center of Gravity C , of the internal Bodies S and M , by Radii drawn to that Center, and an Orbit coming up nearer to the form of an Ellipse, having a Focus in that Center, if the most internal and biggest Body S be agitated by these Attractions, just as the others are, than if so be it should be at rest, not being attracted at all, or much more or much less attracted, or much more or much less agitated.

By the Prop. preced. it is evident, that the Center to which the external Body P is urged, is nearest the Center of Gravity of the internal Bodies S and M . If that Center coincided with this common Center, and the common Center of Gravity of the three Bodies S , M , and P , were at rest; P on one side, and the common

Center of *S* and *M* on the other, would describe accurate Ellipses about the common Center of all three, being at rest, and as it were the common Focus, by *Corol. 2. Prop. 51.* And therefore the perturbation of the said Motions (in the Ellipses) will be very small, when the common Center of Gravity of three Bodies, *S*, *M*, and *P*, is at rest: And it is at rest, if the Body *S* be agitated after the same manner as the others are; as was said before. And therefore the perturbation will be the least, when *S* is attracted by the same Law as the others are. Which was to be demonstrated.

COROLLARY.

After the same manner, if more lesser Bodies revolve about the greatest, it may be inferred, that the Orbits described come nearer to Ellipses, and the description of the Areas more equable, if all the Bodies mutually attract one another with accelerative Forces, which are as their Masses directly and the Squares of their distances inversely, (for this is to be agitated, just as the others are, supposing the Law of the attractions to be the same with that in the Proposition;) and the Focus of each Orbit be placed in the common Center of Gravity of all the internal Bodies, than if the greatest Body were at rest, and made the common Focus of all the Orbits.

PROPOSITION LVII.

Several Bodies, whose Accelerative Forces are to one another reciprocally as the Squares of the distances from their Centers, may be moved in Ellipses about a great one, and describe Area's nearly proportional to the Times, by Radii drawn to that Body.

Let

Let the several lesser Bodies P, p, π , [Fig. 51.] be supposed to revolve about a great Body, as S : And because the common Center of Gravity of them all does not change its state of Motion or Rest, by the mutual actions of the Bodies upon one another; (as was explain'd before,) and we suppose them free from all other external agitation or impediment; that Center may be at rest. If now we suppose the lesser Bodies P, p, π , to be so little in comparison to the great one S , as that it is never sensibly distant from the said common Center of Gravity, then that great one S will be likewise very nearly at rest; but the lesser Bodies will revolve about this greater in Ellipses, having the great Body in the common Focus; and therefore they will describe Areas, by Radii drawn to it, proportional to the Times; bating the Errors arising either from the digression of the greatest Body from the common Center of Gravity, or from the mutual attractions of the lesser Bodies upon one another. But these lesser Bodies may be lessen'd till that digression or those mutual attractions be less than any given ones. And therefore several Bodies may move about a great one in Ellipses, and by Radii drawn to its Center, describe Areas proportional to the Times, so as that the Errors committed (or what hinders them from being exact) may be less than any assignable; that is, indefinitely near. Which was to be demonstrated.

PROPOSITION LVIII.

B*Y the mutual action of the Sun and any primary Planet, the Planet describe an Ellipse, whose Focus is the common Center of Gravity of the Sun and the said Planet. But by the action of the primary Planets upon one another, the Orbit of each is nearly*

an Ellipse, whose Focus is the common Center of Gravity of the Sun, and all the inferior Planets.

If the primary Planets gravitated towards the Sun, and not the Sun towards the Planets, each primary Planet would describe an Ellipse about the Sun's Center at rest, as a Focus; as was demonstrated in *Prop. 42.* And if the Masses of the Planets were insensible in comparison to the Mass of the Sun, the same would hold very nearly, by the preceding: And that this is almost the case of the Solar System, is evident from *Prop. 34.* But because (by *Corol. Prop. 47.*) not only the Sun gravitates towards a Planet, but a Planet also towards the Sun, and besides a Planet is not a Mass intirely insensible in regard of the Sun; each Planet (by *Corol. 2. Prop. 51.*) describes an Ellipse, whose Focus is the common Center of Gravity of the Sun and the said Planet: And (by *Prop. 54.*) the greater Axe of this Ellipse, exceeds the greater Axe of the Ellipse which the Planet would describe about the Sun being at rest, in the same or an equal Time, in the subtriplicate ratio of the sums of the Masses of the Sun and Planet, to the Mass of the Sun.

Besides, because (by the abovementioned *Corol. Prop. 47.*) the Planets gravitate towards one another, and the Sun is agitated by the same attractions as the Planets; Mercury, (by *Prop. 50.*) will describe very nearly an Ellipse having the Focus in the Center of the Sun; Venus, an Ellipse whose Focus is in the common Center of Gravity of the Sun and Mercury, and so on; each primary Planet will describe an Ellipse, whose Focus is in the common Center of Gravity of the Sun and all the inferior Planets. And after this manner, the actions of the primary Planets upon one another, are considered, and reduced to a standard.

SECT.



Fig: 52
p: 117

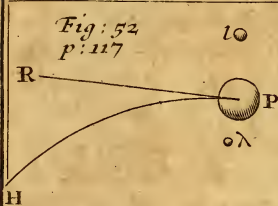


Fig: 54
p: 123
p: 127

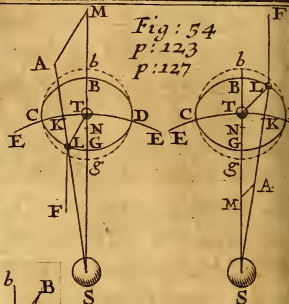


Fig: 55
p: 124

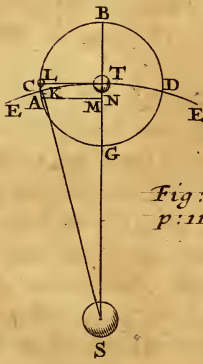
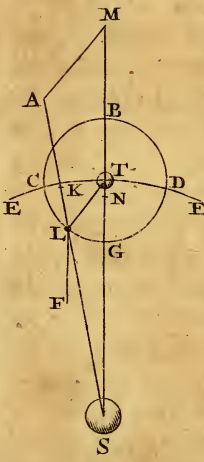
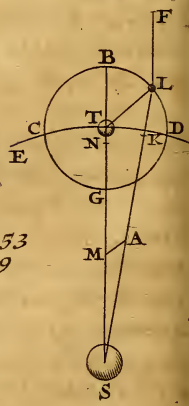


Fig: 53
p: 119



SECTION IX.

Of the Motion of a System of Bodies, which revolves about another Body; all which is applied to the System of the Sun and Planets Primary and Secondary.

PROPOSITION LIX.

THE common Center of Gravity of a System of two or more heavy Bodies, revolving about one another in Ellipses, may move in an Ellipse, or any other Conic Section, about another greater placed in the Focus of the Ellipse, and may describe, by Radii drawn to the Focus, Areas proportional to the Times, indefinitely near.

Let us suppose a System of two Bodies revolving mutually about one another (as in Cor. 2. Prop. 51.) and another System of lesser Bodies L, l, λ , revolving (after the manner described in Prop. 57.) about a great one P , [Fig. 52.] to go on uniformly in a Right line, which is evidently possible: (that is, for its common Center of Gravity to move along Right lines, for instance, PR ;) and in the mean while to be urged laterally, by the Force of another great Body as S , placed at a great distance, and forced to move in the Curve PH , being drawn off from the Right line PR . And because equal Accelerative Forces impressed along parallel Right lines don't alter the situation of the Bodies to one another; but cause the whole System, preserving their Motions in regard of one another, to be transfer'd at the same time together: 'Tis evident that there will arise no alteration in the Motion of the Bodies mutually attracted, from

the attractions, towards the greatest Body S , but from the inequality of the Accelerative attractions, or from the Inclination of the Lines to one another, according to which the Attractions are made. Now the distance of the greatest Body may be increased, till the differences of the Right lines drawn from it to the Bodies of the System in Motion (namely, the great one P , and the lesser L, l, λ) in respect to the Lines themselves thus drawn, and the Inclinations to one another, become less than any given ones. And since all the Accelerative attractions towards S , by the Hypothesis of Gravity, are reciprocally as the Squares of the Distances; these (in this case) differ from equal Forces, by a difference which is less than any given one: And therefore the Motions of the parts of the System P , with its Bodies L, l, λ , will persevere after the same manner as before, when the greatest Body S did not attract that System; and the whole System attracted after the manner of one Body, will describe by its Center of Gravity, (in which the whole System of the heavy Bodies P and L, l, λ , is supposed to be contracted and united, as in other Philosophical cases,) a Conic Section whose Focus is S ; and by a Radius drawn to that greatest Body, will describe Areas proportional to the Times: And the errors which are over and above, may be lessen'd at pleasure, by increasing the distance of the Body S . Therefore the Center of Gravity of a System of Bodies P , and L, l, λ , may move in an Ellipse very nearly. *Q. E. D.*

COROLLARY I.

When a System of two Bodies revolving about one another, or of a great Body with lesser Bodies revolving about it, is moved about a greater; the Motion

Motion of the parts of the System among themselves will be so much the more disturbed, as the greater Body approaches nearer the said System: Because the inclination of the Lines drawn from the greater Body to those parts, is greater, and the inequality of the ratio likewise greater; and, in one word, because there is a greater recess made from that case, wherein there is no perturbation at all; namely, when the Body *S* is at an infinite distance.

COROLLARY 2.

From whence, on the contrary, if the parts of a System of the Body *P* and the lesser revolving Bodies *L*, *l*, *λ*, move in Ellipses or Circles), without any considerable perturbation; 'tis manifest, that the same are only urged very slightly by any Accelerative Forces tending towards other Bodies, or that they are urged equally, and along parallel Lines, indefinitely near.

PROPOSITION LX.

IF a less Body *L* [Fig. 53.] revolve about a great one *T*, (as in Corol. 2. Prop. 51.) and the System of these two revolve about a greater *S*, (as in the preceding,) and in these three Bodies, if the Accelerative attractions of any two, towards the third, be reciprocally as the Squares of the Distances from it; let it be proposed to show in general the Errors committed by the Body *L*, together with the causes and reasons of them; namely, why the Body *L* does not, by a Radius drawn to *T*, describe Areas exactly proportional to the Times, and run in the Perimeter of an Ellipse, one of whose Foci is *T*.

Because the Body *L* is sometimes more, sometimes less distant from the Body *S*, let *SK* be its mean distance; which may likewise express the Accelerative attraction of the Body *L* towards *S*, in that mean distance. Let the place *L*

of the least Body in its Orbit $BCGD$, be taken at pleasure : And in SL , produced if need be, let the Right line SA be taken, so as that it be to SK in the duplicate ratio of SK to SL , which consequently will express the Accelerative attraction of the Body L towards S , when it is in the point L , thus assumed ; because, (by supposition,) the Accelerative Force in L is to the Accelerative Force in K , in the duplicate ratio of SK to SL ; that is, (by construction,) as SA to SK . Connect TL , and through A draw AM parallel to it, meeting ST in M .

The Attraction SA of the Body in L towards S , may be resolved (as is well known,) into the Attractions AM , MS . Wherefore the Body L will be urged by a threefold Accelerative Force ; one tending to T , and arising from the mutual attraction of the Bodies L and T ; by this Force alone, the Body L , by a radius LT drawn to the Center of the Body T , would describe Areas proportional to the Times, and an Ellipse, whose other Focus is in the said Center. The other Force, whereby L is urged, is expressive by AM , and is parallel to the direction of it ; which because it tends from L to T , adding the former Force, will with it compose another, whereby the Areas described, will still be proportional to the Times, (by *Corol. 1. Prop. 51.*) But because this Force, as AM , is not reciprocally proportional to the Squares of the distance LT , the Force compounded of this and the former, will not be such ; but will recede from that proportion, and that so much the more, as (other things being alike) this new Force added to the former, and represented by AM , is greater. Consequently the Body L , which is urged by this compound Force, will not describe the Perimeter of an Ellipse, whose

Focus

Focus is T ; (for to this a Force tending to T , and reciprocally proportional to the Square of the distance from it, is required by *Prop. 39.*) but will cause its Orbit to have some digression from an Ellipse, whose Focus is T ; and that digression will be so much the greater, as this compound Force (whereby L is urged,) recedes from a Force reciprocally proportional to the Square of the distance from T ; that is, (as has been shewn,) the greater the superadded Force represented by AM , is, in respect to the Force, whereby L is attracted to T , other circumstances being alike. Again, the third Force, whereby the Body L is urged, which the Right line MS represents, acting according to the direction of it, (namely according to the direction of LF parallel to MS) superadded to the former Forces, will compound another, whose direction is not from L towards T ; but along a Right line declining from thence towards F , more or less, according to the ratio of this third Force to the sum of the former Forces. And therefore the whole Force, compounded of all three, whereby L is urged, since it does not tend to T , (by *Prop. 40.*) will make the Body L describe Areas not proportional to the Times, by the Radius LT . And the digression from that similitude of the ratios between the Areas described and the Times in which they were described, will be so much the greater, as the ratio of this third Force acting according to the direction of LF , to the former Forces, is greater. Besides, the superaddition of this third Force will induce a deformity in the Elliptic Figure of the Orbit, on a double account, both because it is not directed from L to T , and because it is not reciprocally proportional to the Square of the distance; and the deformity is so much the great-

er, as, other things being alike, this Force MS is greater. In the Right line ST , (from S towards T) take SN , to SK in the duplicate ratio of SK to ST ; the Accelerative attraction of the Body T towards S , will be express'd by SN . If the Accelerative attractions SM , SN , be equal, (as in the 2d of these *Fig.*) they will not alter the situation of the Bodies T and L to one another, by acting upon them equally, and according to parallel Right lines: But the Bodies S and T approaching nearer to one another, the motions of T and L , in regard of one another, will be just the same as if these attractions were intirely absent. But if the Accelerative attraction SN , be less than the Accelerative attraction SM , (as in the first of these *Fig.*) the former, and a part of the latter equal to it, will, (as was shewn in the preceding case,) reduce themselves mutually to that State, as if both of them were absent, and the difference of them, on the part of the Accelerative Force SM , expressed by MN , remain; which therefore disturbs the proportion of the Areas and Times, and the Elliptic Figure of the Orbit, $GCB D$. But if SN exceed SM , (as in the 3d of these *Figures*) their interval MN , will denote that difference of the Accelerative Forces, whereby the proportion of the said Areas described, and Times of description, together with the Elliptic form of the Orbit $GCB D$, are disturbed. And therefore by the attraction NS , is the third attraction SM always reduced to MN , which exerts its Force from M to N , or from L towards F , the former Forces being intirely unchanged. And these are the Forces that generate the Errors of the Body L . Q. E. I.

PROPOSITION LXI.

THE same things being supposed, let it be proposed to give an account of such Errors of a lesser Body L . [Fig. 54.] revolving about a great one T , not in the same plane wherein T revolves about a greater Body S , arising from the inclination of the said Planes; as also of the Causes and Reasons of them.

From the demonstration of the preceding Prop. 'tis shewn, that all the errors of the Body L , arise from the Forces represented by the Right lines AM, MN . But because one of them AM , acts according to the direction LT , which lies in the plane of the Orbit of the Body L , the Force acting according to its direction, will not disturb the Motion of the Body L , as to its Latitude.

If the situation of the Orbit of the Body L , be such, as that the Body S is found in its common Section with the plane of the Orbit ETE , which T describes about S : (that is, to speak in the Language of an Astronomer, if the Nodes of the Orbit of the Body L , be in the Syzygies of the Body S ;) MN also lies in the plane of the Orbit of the Body L ; because in the common Section: And therefore the Force acting according to the direction of it, does not disturb these Motions.

The other things remaining the same, let the Plane $G CBD$ of the Orbit of the Body L , be supposed to be inclined to the Plane SET , so that part of it CGD , may be elevated above it, and part CBD , depressed below it: Upon the same Center T and equal Semidiameter, let the Circle $C b Dg$, be supposed to be described in the Plane SET , and the Nodes will be C and D : Suppose them when seen from T , to be in a Quartile aspect with the Body S ; that is,

sup,

suppose the Angles STD , STC right. In the case of the motion of the Body L , along the Semicircle CGD , (*see the first of these Fig.*) the Force as MN of the Body L , acting with the direction MN , (that is, from L to F) attracts the Body from its Orbit CGD towards CgD ; that is, lessens the Latitude of the Body L , (reckoned from the Plane CgD): And the Body L , in its continual receding from the Plane CGD towards S , till it has arrived to the next Node, does not pass the Plane ETS or CgD in the point wherein it passed before, but a little on this side of it; and makes the Node of the Orbit, now described by it, a little more *in antecedentia*, in regard of the Node D of the Orbit CGD , which it would have described without this Force MN . In like manner in the passage from the Node D to the Node C , through the Semicircle DBC , (to which case the second of these *Figures* is accommodated,) the Force as MN , acting according to the direction of MN , or from L towards F , makes the Body L to go backwards perpetually from the Orbit DBC , and describe another inclin'd to $D b C$, in a lesser Angle, and to pass the Plane $D b C$ sooner than in C ; that is, than it would have done if the Force MN had not been present; and by this means there will be made a new place for the Node tending *in antecedentia*; and again, during its passage, the part CGD of its Orbit towards the conjunction with the Body S , the nearest Node will recede *in antecedentia*, as before.

Besides, (the Eye being placed in the Line of the Nodes produced,) let $SgTb$ [*Fig. 55*] represent the Plane of the Orbit of the Body T revolving about S , GDB the Plane of the Orbit of the Body L , while the latter is inclined to the

the former, by the smallest Angle, namely, GDg or GDb ; which happens (as has been shewn) when the Body L is seen from T in the Syzygies of the Body S . And since it has already been shewn, that the Nodes of the Orbit of the least Body L tends in *antecedentia*; the Orbit which the Body L , departing from G , where it was seen in conjunction with S , further describes, will meet gDb on this side D in respect of S , and will be the Line Gd . And the Orbit which the same L describes, departing from B , where it was in opposition to S , namely, the Line Bs , will meet the Plane gDb in s , viz. beyond D in regard of S . And it is evident that the Line Gd or Bs contains a greater Angle with gDb at d or s , than the Angle made by the Line GD or BD with the same gDb ; being about equal to that which the Orbit of the Body L made with gDb , when L was very near the Node. The Nodes therefore being in the Quadratures, go backwards; but in the Syzygies are at rest; and therefore always whether they are Retrograde or Stationary, upon the whole go backwards; but in the intermediate places, partaking of both conditions, go backwards slower; and so much the slower as they are nearer the Syzygies. But when the Nodes are in the Quadratures, the inclination of the Plane of the Orbit of the Body L , is lessened in the passage of the Body L from the Quadratures to the Syzygies, and augmented in the passage of the same from the Syzygies to the Quadratures: (For the parts of the Lines Gd , Bs , towards d and s , are inclined in a greater Angle to gDb than their parts towards G and B ; and the Orbit of the Body at every moment of time is to be looked upon as a lineola, described by it in that moment; and that is the Plane of the Orbit, which

passes

passes thro' the said lineola, and the Center of the Body T .) And therefore the Body L being in the Syzygies, the inclination becomes the least possible, and returns to its former magnitude, or thereabouts, when the Body comes to the nearest Node. And these are the principal Errors arising from the inclination of the Planes. *Q.E.I.*

SCHOLIUM.

The same holds, tho' the Body L [*Fig. 56.*] revolves not about T ; but both T and L about S , as a Center. For in this case the outmost T , produces the Effects of the Body S ; and S , together with L revolving about it, supplies the places of T and L revolving about it, in the foregoing case. For the Demonstration is the same, whether the disturbing Body moves or not.

COROLLARY.

By the same Laws, as the Body L [*Fig. 54.*] revolves about T , let us imagine several Bodies to move about the same T , at equal distances from it; and a Ring $CBDG$ rigid and concentric with the Body T , made up of them, being multiplied, and at length becoming contiguous: And the several parts of the Ring will perform all their Motions (which are not impeded by the rigidity,) according to the Law of the Body L . And therefore the Nodes C and D of the Ring $CBDG$ are at rest, when they are in the Syzygies of the Body S : and will move from them, *in antecedentia*, swiftly in the Quadratures, and slowly in all other places: And the inclination of the Ring will be changed, (the Axe of it Oscillating,) as has been shewn concerning the Orbit $GCBD$, in the Proposition.

If now the Globe T (having the same Axe with the Ring $CBDG$, and compleating its Revolutions about that Axe in the same times,) be conceived to be increased so big, or the Ring

(other

(other things remaining as before) lessen'd, so as that the Globe touch the Surface of the Ring internally, and cling to it, because closely encompass'd by it: The Globe T (which we suppose intirely indifferent to all impressions,) will partake of the Motion of the Ring, and the aggregate of them both, will oscillate, and the Nodes of the Ring (or the intersections of it with the Plane of the Orbit of the Body T about S ;) will go backwards.

PROPOSITION LXII.

THE Body L , [Fig. 54.] by a Radius drawn to T , will describe Areas more nearly proportional to the Times, and a Figure coming nearer to the Form of an Ellipse, whose Focus is T ; if T be attracted towards S , by the same Law as L is, than if it were attracted a great deal more or a great deal less.

For, by the preced. Prop. the perturbation of the Proportionality of the Areas and Times, and the deformity of the Figure of the Orbit, arise principally from the Force represented by MN . And therefore the Areas and Times will come nearest to the Proportionality, and the Orbit $GCBD$ to an Elliptic Figure, when MN is nothing, or the smallest possible: That is, when the Accelerative attractions SM , SN , (whose difference is MN) of the Bodies L and T , are nearly equal; or, which is much the same, when the Accelerative attractions of the Bodies L and T towards S , approach as near as possible to an equality: (For the other part of the Accelerative attraction of the Body L to S , expressed by SA , namely AM , tho' it does not disturb the said Proportionality, yet it renders the Elliptic Figure deformed; that is, when SN , the Accelerative attraction of the Body T towards S , is not none; (for from what has been shewn above,

above, the Attraction SM is reduced to a lesser MN by the Attraction SN ,) nor less than the smallest of all the Attractions SM , (otherwise MN would be too great in every situation of the Body L ,) but as it were a mean between the greatest and least of all the SM ; that is, neither much greater nor much less than the Attraction SK . And therefore the Body L , by a Radius drawn to T , will describe Areas more proportional to the Times, and a Figure approaching nearer to an Ellipse, if T be attracted towards S , by the same Law as L , than if it were attracted much more or less. Which was to be demonstrated.

The same holds, if (other things remaining as before) the Body L did not revolve about T ; but both T and L about S . For, as was said in the preceding, the Demonstration is the same whether the disturbing Body move or not.

S C H O L I U M.

It may in like manner be infer'd, that if a System of Bodies P and L, l, λ , [Fig. 52.] revolving about S ; (as in *Prop.* 59.) the Orbits described will come nearer to Elliptic ones, if all the Bodies acted upon by the same Laws of Attractions, than if on any account, the same Laws did not hold; and the common Center of Gravity of the Bodies P, L, l, λ , will describe an Orbit whose Focus is in the common Center of Gravity of all the inferior Bodies; as in *Cor. Prop.* 56.

PROPOSITION LXIII.

B*T reason of the mutual action of the Earth and Moon, their common Center of Gravity moves in the Orbis Magnus about the Sun. After the same manner, the common Center of Gravity of Jupiter and its Satellites, and of Saturn and its Attendants,*
are

Fig: 57
p: 129

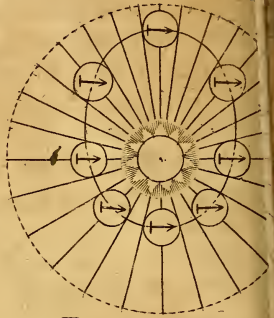
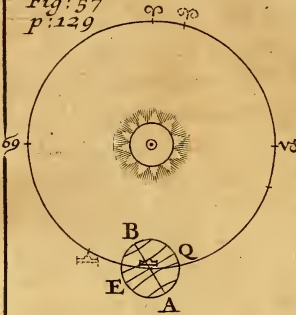


Fig: 59
p: 156

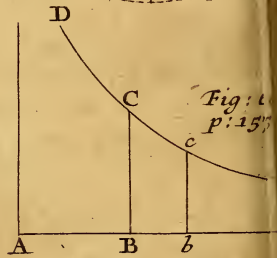
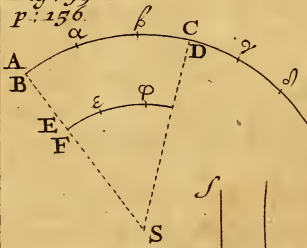


Fig: 60
p: 157

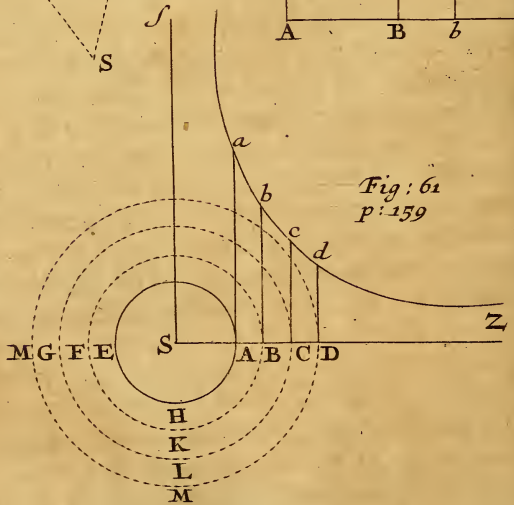


Fig: 61
p: 159

are carried in the Orbits of Jupiter and Saturn; determined in Prop. 58.

Because (by Prop. 46. and 47.) the Moon gravitates towards the Earth, and the Earth towards the Moon; the Satellites of Jupiter towards Jupiter, and Jupiter in like manner towards its Satellites; Saturn's Attendants towards Saturn, and Saturn towards its Attendants; what was proposed is evident from Prop. 59. And because (by Corol. Prop. 47.) the Sun, every Primary Planet, and its Secondary, are acted upon by the same Laws of attractions, the Orbit of each Secondary is very nearly an Ellipse, having a Focus in the Center of the Primary, by the foregoing. But the motions of the Satellites are disturbed, as has been shewn in Prop. 60 and 61. and are affected with various inequalities, that shall be treated of in their proper places hereafter.

PROPOSITION LXIV.

B*T reason of the Figure of the Earth, the Equinoctial Points go backwards, and the Axis of the Earth, in every Annual Revolution, twice changes its inclination to the Ecliptic, and twice returns to its former inclination.*

In Prop. 33. we considered the Axis of the Earth, as remaining always exactly parallel to it self; because then we had only the Phænomena of one Revolution to explain, in which space of time it preserves its Parallelism very nearly. But because of the Earth's Figure, which is an Oblate Sphæroid, arising from the Causes pointed at in Prop. 31. the case is now something different.

Let $\odot$ $\ominus$ --- --- --- --- (Fig. 57.) be the Orbit of the Earth about the Sun $\odot$, AEB the Earth it self, whose Poles are A and B , Equator E . Because (by Prop. 31.) the Earth is an oblate Sphæroid,

(depressed towards the Poles A and B , and elevated towards the Equator $E Q$,) it is like a Globe fasten'd to a Ring; for that excess of Matter, which is about the Equator, supplies the place of a Ring. And therefore, by *Corol. Prop. 61*, the Nodes of this Ring will go backwards: That is, the Earth departing from $\sphericalangle$, (where the common intersection of the Ecliptic and the Equator of the Earth, is directed towards the Sun $\odot$, and consequently the Sun seen in the Equator in the Heavens, makes the Equinox,) thro' $\wp$ towards $\vee$, will arrive at the Node $\vee$ sooner than at $\vee$ the point opposite to $\sphericalangle$: And the Earth going forward from $\vee$ thro' $\mathfrak{S}$, towards $\sphericalangle$, will arrive at the new Node $\sphericalangle$, sooner than at $\sphericalangle$, where the Node was in the former revolution; that is, the plane of the Equator of the Earth produced, will pass through the Sun sooner than the Earth's Center arrives at $\sphericalangle$. But then the Equinox is celebrated, when the Sun is found in the plane of the Earth's Equator; (as has been shewn at large, *Prop. 33*.) and they are to be looked upon as the Equinoctial Points, in which the Sun appears at the time of the Equinoxes. Wherefore it is evident, that the Equinoctial Points, and together with them, all the points of the Ecliptic, that have received any names as such, (because depending upon the Equinoctial Points and reckoned from them,) will seem to go backwards, or move *in antecedentia*; if the Fix'd Stars be taken as immovable. But if the points of the Ecliptic be looked upon as unmoved, then the Fixed Stars will be reckoned to have moved as much forwards or *in consequentia*.

The Regression of the Equinoctial Points, as has been hitherto explained, proceeds from the action of the Sun upon the Ring compassing the

Earth

Earth at the Equator, or the redundant Matter at the Equator, as has been shewn, *Corol. Prop. 61.* But the Force of the Moon also upon this same Ring is considerably great, as is evident from the *Schol. to Prop. 61*, and its *Corollary*: For since she is always in the Plane of the Ecliptic, or not far from it, she will conspire with the Sun in the same Effect.

Besides, in a Semi-revolution of the Earth about the Sun from $\sphericalangle$ thro' ϖ to $\vee$, the inclination of the Equator to the Ecliptic thro' the action of the Sun, is less than when the Earth was in $\sphericalangle$; and it is least when the Earth is in ϖ , (by *Prop. 60.*) the Nodes of the Ring being in the Sun's Quadrature. And when the Earth is arrived at $\vee$, and the Nodes at the Sun's Syzigies, the former Inclination is restored again: In passing from $\vee$ thro' $\oslash$ to $\sphericalangle$, the like lessening of the said Inclination happens again; and the Axis of the Earth Oscillates together with the Equator. The Axis therefore of the Earth, in every Revolution, changes its inclination to the Ecliptic twice, and twice returns to the former inclination; which will happen twice, also in every Periodic Month, upon the account of the action of the Moon. But this Oscillation will scarcely be sensible, whereas the regress of the Points of the Ecliptic is known to every Body, tho' it arise from the same Forces. For in each Equinox, the inclination of the Axis of the Earth towards the Ecliptic, returns to its former bigness, and in a series of Years becomes not more sensible: But the regress of the Points of the Ecliptic is continually onwards, and the Equinoctial Points never return to their ancient Place, till they have compleated an intire Circle. And such a change as is intirely insen-

fible in one Year, or half of a Year, (in which space of time the whole change of the inclination of the Axis of the Earth is finished,) will in a series of Years become sensible; being made perpetually the same way. On the account therefore of the Figure of the Earth, the Equinoctial Points go backwards, and the Axis of the Earth every Year changes its inclination to the Ecliptic twice, and twice returns to its former situation Q.E. D.

PROPOSITION LXV.

THE common Center of Gravity of the Sun and all the Planets and Comets is at rest; and it therefore may be taken for the Center of the Solar System, and even of the World it self.

It has been demonstrated by the Philosophers mention'd in Prop. 50. that the common Center of Gravity does not change its state of motion or rest, by the actions of the Bodies upon one another; but that the Law of a System of Bodies is the same with that of a single Body, as to their continuance in the said state; since the progressive motion of both is to be estimated by the motion of the Center of Gravity. The common Center therefore of the Solar System is either at rest, or moves uniformly forward; for excluding all external Force, this is the state of a solitary Body. Now, the latter can't be asserted, since of the surrounding Fix'd Stars some would become nearer than formerly, and others more remote; and consequently their situation and order in length of time would be sensibly changed, contrary to Observation. And since it has already been shewn, that the Sun, Planets, and Comets gravitate towards one another, and are therefore in continual motion, (only the common Center of Gravity remaining quiet,)

'tis evident that the movable Center of any one of them, can't be look'd upon as the Center of the Solar System, but the above-mention'd immovable common Center of Gravity.

But if not an invisible Point, as the above-mention'd, but some considerable Body very near it, is to be looked upon for the Center of that System, it will be the Sun; because that, by reason of the vast Quantity of Matter it contains in comparison to the others, will scarce ever be sensibly distant from the common Center of Gravity above-mentioned.

And further, if any Figure agree, and bounds be set, to the universal compages of Bodies (or to the World,) no other System more justly claims the middle of it, than our Solar; and therefore no other Center than that above determined, is agreeable to the Universe. Which was to be demonstrated.

SECTION X.

Concerning the Causes and Reason of the Motions of the Planets assigned by Philosophers.

HAVING been describing hitherto the System of the World, and that only in general, so far as was sufficient for our purpose, and having explained the analogous cause of the Motions of the greater Bodies of the World, or rather the same, (as is most fitting,) with that which we daily experience about us; namely the Gravity of all Bodies towards all Bodies, whose Law is the same with that of our Gravity, or of any other Natural Virtue propagated along Right lines

concurring in one Point; that is, having laid down the principal foundations of a Physical Astronomy, regularity of Method requires us to consider the Causes and Physical Reasons of these same Effects as are assigned by others; and at the same time see, whether they solve the Phænomena, so well as those explained before have done. But by the Causes we speak of here, we do not understand the ultimate Causes, or such as have no Causes of themselves; but such as we experience about us, and can calculate their Forces, and Geometrically investigate their Properties, as we do of any other Quantities; such as Light, to the consideration of which, various Sciences, as Optics, Catoptrics, Dioptrics, owe their Original; Gravity, the affections of which are considered in Statics, and in almost all Mechanics: tho' the Cultivators of those Sciences neither understood the ultimate Causes of Light or Gravity, nor will they do so hereafter.

There are two other famous Sects of this Cœlestial Philosophy, that attempt to explain the Causes of the Motion of the Bodies of the Universe, the *Keplerian*, and the *Cartesian*: As for that which brought in Solid Orbs, and fasten'd Intelligences to them to move them, there is no occasion to take notice of it, since there is little of Philosophy in it, and it has been sufficiently confuted by the Astronomers of the last Age. We shall give an account of both these, and annex the Reasons why neither of them belongs to the Heavens, by shewing that the Laws of Bodies moved as these Philosophers would have them, are not such as are found to be observed by the Heavenly Bodies. And first we shall explain the *Keplerian Physics* deliver'd in

Book 4. Epitome of the Copernican Astronomy, and that for the most part in his own Words.

PROPOSITION LXVI.

TO explain briefly the Substance of the Cælestial Physics of Kepler, or the Cause and Reason, which Jo. Kepler has assigned, why the Planets are carried in Orbits about the Sun.

This great Astronomer would have the Sun to be a vast Body, made to move about an Axe given by Position, by the Creating Power of the Divine Being in the beginning of Things; and lest this Motion should languish by degrees on the account of the sluggishness of Matter, that it is constantly maintained by the care either of the Creator, or of some Mind destin'd to it: The same likewise he would have to be observed in the Earth, and the rest of the Globes of the Universe. The Sun by this gyration of his Body, carrying about the immateriated Species of its own Body thro' the vast Spaces of the World, carries about like a Leaver, at the same time, the Planets which are apprehended by this Species. This Species is the Virtue of the Sun emitted in Right lines thro' the whole Mundane Space, which, because it is the Species of a Body, is whirled about together with the Sun, like a rapid Vortex, running thro' the whole Mundane Space, with the same swiftness as the Sun turns it self about its Axis, and therefore carries along with it the Planet, swimming as it were in this River. We are farther to take notice, that besides this Vectory Force in the Sun, there is a natural sluggishness in the Planets to all Motion, by which they are inclined, by reason of their Matter, to continue in the place where they are. So that there is a conflict between the Vectory Power of the Sun, and the sluggish-

ness of the Planet, arising from its Matter. Both have their share in the Victory ; the former moves the Planet out of the place it was in ; the latter in some measure disengages its own, that is, the Planet's Body from the shackles it was laid under by the Sun, when apprehended by some part or other of the circular Virtue ; namely by that, which immediately succeeds the part from which the Planet had just disengaged it self. And because this Virtue that carries along the Planet, and flows from the Sun's Body, has the same degrees of weakness at different distances, as Light has ; this Vectory Power, like the Sun's Illumination, decreases as the distance from the Sun increases : Wherefore it will carry a Planet placed at a greater distance from the Sun more slowly, and consequently will make its Periodic Time, on this score, greater. Besides, since a like Vectory Power is propagated from the Body of each Planet moving about its own Axis ; this also will carry the secondary Planets, which it finds in this River of its immateriated Vortex, after the same manner ; and they will never leave their respective Primary, because they are carried by the Species of the Sun's Body about the Sun, as well as their Primary, and together with their Primary. And upon the simple supposition of these things, 'tis evident that the Primary Planets would be carried about the Sun in Circles concentric to the Sun, in the Plane of a great Circle of the Sun lying exactly betwixt its Poles ; and that the Secondary Planets would be carried after the same manner about their Primary.

PROPOSITION LXVII.

TO explain how Kepler deduces from these Principles, that the Periodic Times of the Planets will be exactly in the Sesquialter Proportion of their Orbits or Circles.

Kepler was the first who observed that this Proportion held in the Cœlestial Bodies, to the no small advantage of Astronomy and true Physics; namely, that in the Revolution of the Primary Planets about the Sun, and of the Secondary about their respective Primary, the Squares of the Periodic Times had the same Proportion as the Cubes of the Semidiameters of their Orbits or Distances from the Sun; taking the Mean distances, and abstracting from the Excentricity. And this he inferred from his own Principles after the following manner. Since the Vectory Power of the Sun, (whereby the Planets are carried about by the immateriated Vortex or River of the Solar Species,) decreases like the Illumination, the distance from its fountain or the Sun increasing; (that is, in the duplicate ratio of the distances,) and only exercises its Virtue according to one dimension, namely, in Length. This Vectory Power in producing this Effect, (namely, carrying the Planet round) will be lessen'd only in the simple ratio of the distance from the Sun increased: And since (by his Archetypical Proportions) he makes the bulks of the Planetary Bodies to have the same ratio as their distances from the Sun; the Vectory Power of the Sun, will employ equal Forces in the carrying round each Planet; namely, so much greater on the account of the greater size or magnitude of the Planet, that the Moving Species lights upon, as the Moving Virtue in that distance is weaker, and in the same ratio; namely, of the distance

distance from the Sun. Again, induced by the Geometrical concinnity, he settles the Quantity of Matter in the Planets compared with one another, in the subduplicate ratio of the Bulk or Magnitude; (and so the Density will be in the same subduplicate ratio of the Bulk or Magnitude reciprocally,) that is, as was said before, in the subduplicate ratio of the distance from the Sun.

Since therefore the Periodic Time in a Planet, that is more distant from the Sun, is greater than in a Planet that is nearer, because the Way, (that is, the Orbit) is greater, and because the Quantity of Matter to be transported is greater, (but in other respects equal, because equal moving Forces are employed upon them, as was shewn above,) and since the greatness of the Way is as the distance from the Sun, and the Quantity of Matter as the subduplicate of that distance; on both causes conjoined, the Periodic Time of the remoter Planet will be greater, in the ratio compounded of these: But the ratio compounded of the simple and subduplicate, is the sesquialter or sesquiplicate: Wherefore the periodic Time of the remoter Planet will be greater than that of the nearer, in the sesquiplicate ratio of the distances from the Sun; that is, the Squares of the Periodic Times are as the Cubes of the distances from the Sun. Which things are to be understood after the like manner concerning the Secondary Planets revolving about their Primary.

PROPOSITION LXVIII.

TO explain the Causes assigned by Kepler, why the Orbits of the Planets are Excentric.

Since each Planet is a Body cognate with the Body of the Sun, having one part (like a Loadstone)

stone) that is, friendly to the Sun, and the other the quite contrary. In the conversion of the Body of the Sun, this Virtue thereof, whereby it attracts or repels the Planet, according as its friendly or contrary part is turn'd to the Sun, is also turned about; just as in the conversion or turning a Load-stone round, the attractive or repelling Force is transferred toward different parts of the World. And when the Sun lays hold of a Planet by this Virtue of its Body, whether attracting or repelling, or acting dubiously between both, it carries it round along with it; for it retains it both by attracting and by repelling it, and by retaining it, it carries it round. Each of our Load-stones indeed has two Poles or Places of different Virtues, by the one of which, one Load-stone attracts another after a certain direction; but by the other it repels the same. But in the Heavens things are something otherwise. For the Sun possesses this active or energetic Faculty of attracting or repelling a Planet, not in this or that place or part, but in all the parts of its Body. Therefore it is probable, that the Center of the Body of the Sun answers to one extremity, place, or pole of the Load-stone; and the whole surface to the other; since it attracts or repels a Planet according to a certain direction, not by this or that place on its surface or part of its Body, but by the whole.

These things being thus settled; 'tis evident, that the Body of the Planet will not be carried about the Sun at the same distance from the Sun, every where; but while it turns [*Fig. 58.*] its friendly part to the Sun, it will come nearer to the Sun, till the Right lines drawn according to the direction of this part, (that is, the Fibres along which this attractive Virtue is propagated

pagated from the Sun,) being almost every where parallel to themselves, are no more inclined to the Sun, but a Right line drawn from the Sun to the Planet becomes a perpendicular to these Fibres; in which case, since the Sun does attract no longer, the effect of the attraction, (namely, the access of the Planet towards the Sun) will cease, and the Planet will then be in its nearest approach to the Sun or in its Perihelion. The Planet being still carried about, by the above-described Vectory Power of the Sun, its unfriendly part, or that which shuns the Sun, will by little and little be directed towards the Sun, and hereupon the Planet will be continually repelled from the Sun; and that will be continued till neither of these parts or places be inclined towards the Sun, but a Right line connecting the Sun and Planet, would again become perpendicular to the Fibres of the Virtue; and then the Planet will be found in its Aphelion, because at the greatest distance that the repelling force of the Sun can remove the Planet to, while it is carried about. And the Planet being librated by these Fibres, so as to be attracted by the Sun in one part of its Orbit, and repelled by it in the other, describes an Orbit about the Sun indeed, but not about it as its Center; but Excentric to the Sun. Again, because the Species of the Sun's Body communicating a Motion to a Planet, is thinner and weaker in a larger Circle in the ratio of the distance from the Sun, as it produces this effect, (as was shewn in the preceding Proposition) and the Planet in the mean while continues of the same Bulk and Density, because the same Planet: 'Tis evident, that, other circumstances continuing the same, the Celerity of the Planet is lessen'd, in the same ratio with the constipation

tion and density of the Vectory Power of the Sun; that is, reciprocally as the distance from the Sun. Farther, the Virtue of the Sun that attracts and propels the Planet issuing along virtuous lines proceeding from the Center of the Sun, and going round together with the Sun, has its strength proportional to the Co-sine of the Angle, made by the Radius of the Sun and the Fibre that admits the Solar Virtue into the Planet: And therefore this Excentric Orbit will not be a perfect Circle, but an Ellipse, narrower and more depress'd on the sides, and having the Sun for one of the Foci.

Now, from the same Principles he shews, that the Ellipse is described about the Sun in the Focus, in such a manner as that equal Areas are described in equal Times, by a Radius drawn to the Sun, which he calls the *Vector*; and universally, that the Areas, which this same Radius sweeps over, are proportional to the Times wherein they are described.

But since these libratory Fibres, by the help of which the Planets are librated upwards, always continue as it were parallel to themselves while the Planet is carried intirely round, and (by what was shewn before) is so inclined in respect of the whole Universe, as that the line of the Apfides of each Planet is perpendicular to those Fibres, which are placed either in the plane of the Orbit, or of the Planet, or in a situation parallel to it; and since the Axis of no Primary Planet, about which it is turn'd as it were by a diurnal Motion, has this Position, and no other Line but this Axis, and Right lines parallel to it, can keep the same situation in regard of the Universe, if the intire Globe of the Planet be solid and united; 'tis evident that *Kepler* was obliged to suppose within this exterior Crust of the Planet,

net, (whose Axis keeps a given Position, namely, that about which the Planet is turn'd in its Diurnal Motion,) a Globe intirely separate and divided from this Crust, and no ways yielding to its Diurnal Motion, whose Axis and Right lines parallel to it may sustain the Places of the said *Libratory* Fibres, and conserve the Position necessary to this purpose described above; while the exterior Crust, according to the nature of each Planet, revolves by a Diurnal Motion peculiar to it self.

PROPOSITION LXIX.

TO explain the Causes assigned by Kepler, why the Orbits of the Planets are inclined to the *Ecliptic*.

In this *Keplerian* Physics, which we are now upon, if there were no Fibres (such as we described in the preceding Proposition) in the Planets, by which they are librated upwards; or if these Fibres turned their side to the Sun; that is, neither that part which was friendly to the Sun, nor that part which was the contrary; each Planet remaining at the same distance from the Sun, would describe a perfect Circle concentric with the Sun. After the like manner, all the Planets (being acted upon only by Forces arising from hence) would go round in the same Plane in an uniform and direct Stream as it were, in the middle of the Channel, (or in a Circle lying exactly between the Poles of the Revolution of the Sun, called by him the *Via Regia*, or *Royal Way*.) But because they wander variously on this or that side of the *Via Regia*, the Cause of this Effect must be somewhat like that, whereby a Ship under sail is thrust to one side of the line, according to which it would have been moved by the Wind alone, namely, by an Oar held

held obliquely to the Stern, or by the Rudder kept in an oblique position. In each Planet there are Fibres (which he calls from their Office, the *Fibres of Latitude*) always placed in the Plane of its Orbit, or parallel to it, and which remain always parallel to themselves in every conversion or motion of the Planet; so that when a Planet is in a Node or common intersection of the Plane of the *Via Regia*, with the Plane of the Orbit, the middle of the Fibre of Latitude that passes thro' the Center of the Planet, touches the Orbit; and therefore at every one of these Fibres, one part towards one side of the *Via Regia*, the other part towards the other side, stands without that Plane, inclined to it at the same Angle, as the Orbit of the Planet is to the *Via Regia*. When therefore the Species of the Sun's Body meets with that half of the Fibre, which is behind and stands upwards, it will impel the Planet that way, as the half, which is foremost, tends towards; that is, according to the Plane of the Orbit proper to that Planet: But since these Fibres of Latitude continue in a position parallel to themselves thro' the whole Revolution, hence it comes to pass, that in a Planet placed in its Limit or greatest distance from the *Via Regia*, neither extremity of the Fibres go foremost, but the Fibres directed as it were towards the bottom of this River, (that is, towards the Sun,) and receiving the impulse on it's side directly opposed to it, give no cause for an ejection farther any way; 'till a change be made in these points of the Limits, so as that the extremity of the Fibres, which went foremost before the arrival of the Planet to the Limit, may now go behind, the other going before; and for that reason the Planet may begin to approach again towards

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the *Via Regia*. For instead of the Power of the Rudder in a Ship, there is in the Planet another Force of the Fibres much more suitable; because, as the Libratory Fibres have a natural Power to retain a parallel situation in the transportation of the Body, so also there is in the Fibres of Latitude, besides a like Power of retaining a parallel situation, another natural Power of agility, or of preserving the same Line, and of deriving according to the direction of it, a Motion impress'd upon themselves, so far as the Motion tends the same way with that other extreme of the Fibre. Because these Fibres of Latitude are entirely different from the Fibres described above, by which the Sun attracts or repels a Planet, as well in regard of their nature and office, as in respect to their situation; for the Fibres of Latitude are everywhere parallel to a Right line touching the Orbit at the Node in the *Via Regia*: But those others (by preced. *Prop.*) are parallel to a Right Line, which is a Tangent to the Orbit in the Aphelium: These Fibres, according to *Kepler*, ought to be found in different Bodies, since (in *Kepler's* judgment) the Apfides go forward, and the Nodes go backward, tho' it be but slowly in the Primary Planets; and 'tis certain that the same happens in the Secondary. Wherefore, besides an exterior Crust furnished with Fibres, by which the Axis, about which the Planet revolves by its Diurnal Motion, is kept parallel to it self in its whole circuit about the Sun, he found it necessary to suppose an interior Crust (which might be like a Nucleus, in regard of the exterior,) intirely separate from the former, and agitated by other Forces; in which there are Fibres, by which the Planet may be librated upward and downwards: And within this Nucleus,

cleus, or rather Crust, he is forced to suppose again another interior Nucleus separate from the former middle one, and having a different Period, in which the Fibres of Latitude are expanded ; since it is impossible that these different Fibres having different Periods, should be spread in one and the same Solid.

PROPOSITION LXX.

TO point out and refute very briefly the mistakes that are to be found in the Keplerian Celestial Physics explain'd above.

Tho' we undertake to oppose Kepler's Celestial Physics, yet we don't do it with such a temper, as if we rank'd him a Person, whose Fame will be Immortal among the common System-makers or hunters after Physical Causes. For he was so far from this, that on the contrary, he is the only Man (excepting perhaps some of the Ancients, as *Pythagoras*, &c.) who has treated of the Celestial Physics in a Mathematical manner. His *Archetypical Ratios*, *Geometrical Concinnities*, and *Harmonic Proportions*, shew such a force of Genius as is not to be found in any of the Writers of Physical Astronomy before him : So that *Jeremiah Horox*, a very competent judge of these matters, tho' a little averse to Kepler in the beginning of his Astronomical Studies, after having in vain tried others, entirely falling in with Kepler's Doctrine and Physical Reasons, thus addresses his Reader, *Kepler is a Person whom I may justly admire above all Mortals beside ; I may call him Great, Divine, or even something more ; since Kepler is to be valu'd above the whole Tribe of Philosophers. Him alone let the Bards Sing of----. Him alone let the Philosophers Read ; being satisfy'd of this, that he who has Kepler, has all Things.*

For what greater, or more, like something Divine, could happen to a Mortal in this matter than by these Harmonic Speculations, exactly to determine the proportion of the Orbits of the Planets; than to be the first that should find out the true way of the Planet about the Sun (namely the Elliptic;) and lastly, than to shew the true manner of a Planet's Motion in this way, viz. that by a Radius drawn to the Center of the Sun, placed in the Focus of the Ellipse, it describes Areas proportional to the Times? But there are some things which I can't approve of in *Kepler's* Mechanical cause of these Motions and they are these following.

1. If neither the Friendly nor the Opposite part of the Planet be turned to the Sun, or if the Planet (on the account of the weakness of the Fibres, whereby it is librated upward and downwards) be so disposed, as neither to be attracted or repelled by the Sun, (as it happens in Venus, according to *Kepler's* judgment;) then he would have it, that the Planet describes a perfect Circle, by the Species of the Sun which is carried about; whereas 'tis certain, that a Body which is not urged by a Force tending to a Center, will not describe a Circle, but a Right line. Since therefore the Orbits of Planets are Circular even in this case, 'tis evident that they are impelled towards a Center by some other Force, besides that which arises from the obverting the Friendly part.

2. During the Planet's ascent from the Perihelium to the Aphelium, that part which is Unfriendly is turned to the Sun, and consequently is repelled from the Sun, and in this case it is urged only by that Force, which is directed along the Line that connects the Sun and Planet. But 'tis well known in Mechanics, that

Plane

Planet acted upon by this Force alone, would be so far from describing an Ellipse, having the Sun for a Focus, that on the contrary, it would describe a Curve convex towards the Sun. Wherefore a Planet, whose Orbit is an Ellipse, having the Sun for a Focus, must necessarily be urged, even in its ascent from the Sun, from the Perihelium to the Aphelium, by a Force tending to the Sun, since an Ellipse is concave towards both Foci. *Kepler* therefore, must be very much out of the way, when he would have a Planet repell'd by the Sun, in its ascent to the Aphelium, or when he would have a Planet neither attracted nor repelled, when it describes a perfect Circle about the Sun concentric to it: For unless in both cases it be attracted or otherwise impelled towards the Sun, it will not describe a Curve concave towards it: For it can't be retain'd thus, without some kind of attraction.

3. In his enquiry into the proportion of the Planetary Orbits, he assumes some things plainly contrary to Observation, how much soever it is that they agree with his imagined Archetypical Ratios: As that the Magnitudes of the Planetary Bodies are as their Distances from the Sun; and that their Densities are reciprocally as the Square Roots of their Magnitudes. From whence it follows, that the Quantity of Matter in the Planets is directly as the Square Root of their distances from the Sun: Because the quantity of Matter is as the Magnitude and Density conjunctly. But every Body knows that Mars is less in bulk than Venus, which notwithstanding ought to be as big again as Venus, according to *Kepler's* reasoning; because distant as far again from the Sun. Observation informs us, that the Globe of Jupiter is bigger than Saturn, tho' it ought by the aforesaid Rule to be as little again.

gain. Notwithstanding he might deduce from the things that he knew, and first discovered, (namely, that the Path of a Planet is the Perimeter of an Ellipse, one of whose Focus's is in the Center of the Sun, which it so describes, as that its Radius that carries it, sweeps Areas proportional to the Times,) without the consideration of the Magnitude and Density of the Planet, that the Periodic Times of the Planets revolution about a common Center, are in the sesquiplicate ratio of their greater Axes, or mean distances from the Sun. But it is probable that *Kepler* deduced the Proportion of the Planetary Orbits, and their Number, from the intervals necessary for the interposition of five regular Bodies; and that he afterwards assigned to them such Magnitudes and Densities, as he could draw this same proportion from by another Method.

4. Not to speak of his Virtue, which has too little of Mechanism, and too great an affinity to an Animal Power, which he gives to the Fibres of Libration and Latitude in the Planets; (to the latter, natural power of Agility, but to the former, in one part, an inclination towards the whole Body of the Sun; in the other, a flight from it;) and the division arising from hence, of every Planet into a Shell, and a Kernel separate from the Shell, and librated in its cavity, in a different Period, and in some another Nucleus within the former.

Not to mention, I say, these things, which have but little of Mechanics in them, and foreign to the Method which Nature uses, *Kepler's* System about the manner of the Sun's carrying round, attracting, or repelling the Planets, is founded upon what is contrary to one of the chief Laws of Nature and Motion, whereby

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care is taken that there be a Reaction always contrary and equal to any Action. For according to Kepler, in Epit. Book IV. *This Virtue of Expulsion and Attraction is not in the Planets themselves, but in the Sun only. And tho' the attractory and repulsory Power of the Sun be extended to, and affect the Planets, yet like power of a Planet does not reach to the Sun and affect it.* For were it so, he confesses, that the Sun would be moved, out of the place it has in the Center of the World, by the Planets, in the reciprocal ratio of their Bodies; or at least it ought to titubate or reel as it were, being sometimes attracted more this way, sometimes more that way, according as more Planets happen to come together on the same side of the Sun, having similar Powers. But this he entirely gets rid of, by denying a mutual attraction. Kepler here had forgot the true Doctrine of Gravity, that he had laid down about 11 Years before, in his *Introduction to his Commentaries upon Mars*, built upon these Axioms: *That Gravity is a mutual affection of union or conjunction in Bodies that are a-kin to one another. That two Bodies, if they are not hindered, will meet in the intermediate Space, each having moved one towards the other a length reciprocally proportional to the magnitude of the other: So that if the Moon and Earth were not retain'd each in their Circuit, the Earth would ascend towards the Moon the Fifty fourth part of the distance between them; and the Moon would descend towards the Earth about Fifty three parts of the interval, and they would both meet there; supposing the substance of both to be of the same density. That the Moon draws up the Waters of the Terrestrial Globe, from whence arises a flux, where the Channels of the Ocean are broadest, and afford room for a reciprocation. And if the Earth should cease to attract towards it self its own fluids, the Water of the Sea would be raised and flow towards the*

Moon. Nor did he fear himself, answering the difficulties above-mentioned, about the titubation of the Sun, and its removal out of the Center of the World; where he speaks after the following manner: *For so great is the bulk, so great the density of the Matter of the Body of the Sun, and so great the attractive and repulsive Virtue of it; on the contrary, such is the smallness both of the Planet and its renitence, that the Sun is in no danger of being removed out of its place.*

From what has been deliver'd, 'tis evident, how near this great Philosopher and Astronomer was, to the finding out the true Cœlestial Physics, (who even in *Epit. Astron. Book 5. § 1.* had said, *As the Sun attracts the Planet, so the Earth attracts Bodies; on the account of which attraction, the Bodies are said to be heavy,*) and how much assistance he has given in this matter, and how rightly he Prophesied, when he said, *The true Causes, Quantities, and Directions of the Motions of the Apfides and Nodes of the Planets, and other matters of this kind, remain in the Pandects of the following Age, not to be understood before the Divine Being, the disposer of Times and Seasons, has open'd this Book to Mortals.*

PROPOSITION LXXI.

TO explain briefly and summarily the Cartesian Cœlestial Physics, or the Cause and Reason, why the Planets are carried round the Sun, which Renatus des Cartes invented.

Such as have endeavored to find out the Physical Causes of the Cœlestial Motions, have all of them sought them in the Sun. For the Rotation of the Sun in his Space about an immovable Axis, the same way that all the Planets take and follow in, and that in a shorter Period than that of Mercury, which is the nearest to him, and every other Planet in a greater period,

as it is farther from the Sun ; and the Rotation of the Primary Planets about their proper Axes ; and of the Secondary again about these Primary, just as those Primary revolve about the Sun ; all towards the same coast of the Heavens, and almost in the same Plane , have the highest degree of probability. And these Phænomena do plainly bespeak, either that all this motion of the Primary Planets arises from the Sun's motion, or at least, that both of them depend upon one common Cause. Hence it came about, that several Philosophers thought the Planets were carried by the Æther, whirl'd about the Sun by a Vortex or Whirl-pool ; and that long before *Cartes*, or even *Epicurus's* Time. But *Kepler*, who was better acquainted with the motions of the Stars, saw well enough that they could not be carried about by any material Vortices : For if the Planets swimming in the Matter of the Vortex, and as it were at rest in it, were carried about together with the parts of the Vortex, he saw that they would describe perfect Circles, and have other periods in regard of their distances from the Center, than we find by Observation agree to them. He therefore rather chose to substitute the immaterial Species of the Sun's Body, instead of Matter whirl'd about like a Vortex, yet calling it by the name of the *Cæstial Vortex and River*. But afterwards *Cartes*, making light and taking no notice of these niceties and Astronomical Observations, and being resolved to frame a World, brought in Vortices again ; together with a popular way of representing the motions of the Stars. And tho' Vortices were introduced into the Heavens by others before *Cartes*, and have been made use of by others since him, and pressed into other service, yet they retain

Cartes's Name. We shall therefore briefly describe this System, so far as serves our purpose, in *Cartes's Words*, out of the *Third Part of his Principles of Philosophy*.

Cartes supposes then, That all the Matter of which the visible World is made, was, in the beginning of Things, divided by God into Particles very nearly equal, of a moderate size, and equally moved, not only each about its proper Center, and separately from one another, so as to compose a Fluid ; but several also about some other points, in like manner remote from one another, and so they composed various Vortices. Afterwards he resolves these equal Particles, worn a little by their intestine motion, into Globules of different Sizes, all which he calls Particles of the second Element, and a part of a Fluid form'd out of the irregular minute parts rubbed off from the corners of the Globules, and moved always with a very great rapidity, he calls the Matter of the first Element. Now, there being more of this sort of Matter than was necessary to fill up the interstices of the Globules, what remained was thrust down towards the Center by these Globules receding from their Center, by reason of their circular motion, and being there collected into a Spherical Body, constitutes the Body of the Sun in the Center of each Vortex. Besides this equal Motion impressed originally upon these Particles, about the Center of the Vortex, the Sun, just form'd in the center of the Vortex, and revolving about its own Axis the same way with the rest of the Matter of the Vortex, and always emitting some of it self along the narrow passages that are within the Globules of the second Element towards the Ecliptic, or Circle lying just in the middle between the Poles, (receiving as much at its Poles

Poles from the neighbouring Vortices, as it emits,) has a power of carrying those Globules along with it; those nearer, quicker, but those farther off, slower. But since these lower Globules of Cœlestial Matter move quicker than the superior ones, they ought also to be less. For if they were greater or equal, they would upon that account have more Forces, and therefore would become uppermost by their centrifugal Force. And all these particulars will be the same in any Vortex whatever, within certain limits, beyond which the upper Globules are moved quicker than the lower, being as to their Sizes equal. These limits in the Solar Vortex he places about the Orb of Saturn, or a little beyond.

These things being supposed, if any Star, which being placed in the center of its Vortex; does the office of a Sun, should happen to be covered with Spots, and by that means weakened so as to be overcome and carried along by the neighbouring Vortex of the Sun; if this Star be but capable of a less degree of agitation, or have less solidity than the Globules of the second Element that are near the circumference of our Vortex, yet something more than some of those which are nearer the Sun; we are to understand that this Star, so soon as 'tis arrived into the Vortex of the Sun, ought to descend continually towards its Center, till it arrive to such Cœlestial Globules as it is equal to in solidity or aptitude to persevere in its motion along Right lines. And when it has at last got thither, it will neither approach nearer to the Sun, nor yet farther off from it, unless it be something propelled from thence, by some other Causes afterwards to be related: But being in *equilibrio* with those Cœlestial Globules, will
revolve

revolve about the Sun with them, and become a Planet. Every one of them therefore is at rest, in that part of the Heaven where it is; and all the variation of situation observed in them, only arise from the motion of the matter of Heavens or Vortex that contains them. According to this System then, nothing hinders us from imagining the Space, in which at present the single Vortex of the Sun is contained, to be at first divided into seventeen or more Vortices; and that they were so disposed, that the Stars, which they had in their Centers, were by little and little at length covered with several Spots, and then these destroy'd by other Vortices, the one quicker, the other slower, according to their different situation. So that since these four, in whose Centers were the Sun, the Earth, Jupiter, and Saturn, were bigger than the rest, the Stars which were in the Centers of the five lesser that stood about Saturn, descended towards Saturn; and those in the Centers of four others about Jupiter, towards Jupiter; in like manner the Moon near the Earth, towards the Earth: Afterwards Mercury, Venus, the Earth, with the Moon, and Mars, (each of which Stars also formerly, had a Vortex,) towards the Sun; and at length Jupiter and Saturn, together with the lesser Stars about them, sunk together towards the Sun, much greater than themselves, after their own Vortices were destroyed: But the Stars of the other Vortices, if they were ever more than seventeen in this Space, pass'd away into Comets, moving almost in a Right line out of one Vortex into another, and never returning. So that now, since we see the Primary Planets carried at different distances from the Sun, we are to judge, that this happened, because the solidity of such of them as
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are nearer the Sun, is less than that of those that are farther off: And that from hence it came to pass, that the same part of the Moon is turned towards the Earth, or at least does not defect much from it, because its other part, which is turned from the Earth, is something more solid, therefore in moving round the Earth ought to describe a greater compass. And since we see the inferior of those higher Planets to move round quicker, we still judge it to arise from hence, that the Matter of the first Element, which composes the Sun, by whirling round very rapidly, carries along with it the nearer parts of the Heaven, and the Planets swimming in them, which are there relatively at rest, better than the more remote. Jupiter, the Earth, &c. revolve about their own Axes, because they were formerly lucid Stars placed in the Centers of some Vortices, and no doubt gyrating there like our Sun, and even now the Matter of the first Element collated in the Centers of them, still retains the same Motion, and impels them.

Lastly, we are not to think that all the Centers of the Planets are always accurately in the same Plane, nor the Circles they describe perfect and compleat: But, as we see it happens in all other natural things, only nearly so, and in time perpetually changing. For since all the Bodies that are in the Universe, are contiguous, and act upon one another, the motion of each depends upon the motion of all the rest, and is therefore varied innumerable ways. On which account we find, that tho' all the Planets affect a Circular Motion, yet none of them ever describe Circular ones, but always err something from them, both in Longitude and Latitude.

PROPOSITION LXXII. LEMMA.

IF two Bodies revolve about the same Axe, in the same distance from the Axe, and two other Bodies likewise in any other distance; I say, the ratio of the difference of the Angular Motion of the former Bodies, to the difference of the Angular Motion of the latter, is compounded of the ratio of the translation of the former Bodies from one another, to the translation of the latter Bodies from one another, and of the ratio of the distance of the latter from the Axe, to the distance of the former from the same.

The Angular Motion is that by which the Angle is formed at the Axe about which a motion is: Hereupon they are said to have the same or equal Angular motion, that being placed at any unequal distance from the Axe, make equal Angles at the Axe, in the same or equal Times: And the difference of the Angular Motions is the difference of the Angles formed at the Axe in the same Time by the Bodies.

At the same distance from S [Fig. 59.] the Axis of Motion, let there be two contiguous Bodies A and B ; likewise two others C and D . Let them afterwards be moved any how, and separated from one another; namely, A in α , and in the same time B in β , C in γ , and D in δ . 'Tis evident (from Prop. 33. Elem. 6.) that the difference of the Angular Motions of the two Bodies A and B , (namely the Angle $\alpha S \beta$) is to the difference of the Angular Motion of the two other C and D (that is, the Angle $\delta S \gamma$) as the translation of the two former from one another, (or the way, whereby the one is transferred from the other, namely $\alpha \beta$,) to $\delta \gamma$, the translation of the two latter; that is, if the distances are equal, the translations are as the differences of the Angular Motions. Again, if at any unequal distances from the Axe S , the two contiguous

guous Bodies A and B , and the two others E and F , be so moved into the places α, β , and ϵ, ϕ , that the differences of the Angular Motions (*viz.* the Angles $\alpha S \beta, \epsilon S \phi$) be equal; then will (by *Prop. 33. El. 6.*) the Translations $\alpha \beta, \epsilon \phi$, be as the distances SA, SE . And therefore (by *Prop. 23. Elem. 6.*) if neither the distances from the Axis, nor the differences of Angular Motion be equal, the ratio of the Translations will be compounded of the ratio of the differences of the Angular Motions, and of the ratio of the distances from the Axis: And therefore (taking away from both sides the ratio of the distances from the Axis,) the ratio of the differences of the Angular Motion is compounded of the direct ratio of the translations of the Bodies from one another, and the reciprocal ratio of the distances from the Axis of Motion. *Q. E. D.*

PROPOSITION LXXIII. LEMMA.

IF the Curve DCE (Fig. 6.) be so related towards the Right line AE , produced infinitely towards E , that a Perpendicular CB , let fall from any point of the Curve to AE , is reciprocally as the Cube of the Right line AB , intercepted between B and A a point given in the Right line EA , the infinite space $BC EE$, contain'd between the Right lines CB, BE , and the Curve CE , will be reciprocally as the Square of the Right line AB .

Call AB, x ; BC, y : and because y is as $\frac{1}{x^3}$

then will $y = \frac{1}{x^3}$ drawn into a certain constant or invariable Quantity; let that be a^4 , that the Law of Homogeneity may be observed: that is,

$y = \frac{a^4}{x^3}$ or $y = a^4 x^{-3}$. And therefore, the sum

of.

of all the y 's (or the Flowing Quantity whose Fluxion is y , or its value a^4x^{-2}) will be $-\frac{1}{2}a^4x^{-2}$: That is, as is known to Geometers, the Area that lies on one side of the Line BC , namely the Area $BCE E$, is equal to $\frac{a^4}{2x^2}$. And, because $\frac{1}{2}a^4$ is a given quantity, the Area $BCE E$ will be as $\frac{1}{x^2}$; that is, reciprocally as the Square of the Right line AB : or the Area $BCE E$ is to the Area $bce E$, as Ab^2 to AB^2 . Q. E. D.

PROPOSITION LXXIV. LEMMA.

IF two contiguous Bodies slide one upon the other, and the same be done in two other Bodies, pressed by the same Force as the former towards one another; I say the impression made by the friction of the former upon one another, is to the like impression of the latter, in a ratio compounded of the ratio of the translation of the former to the translation of the latter, and of the ratio of the Superficies, wherein the former touch one another, to the Superficies, wherein the latter touch one another; namely, wherein the impressions are made.

If the contiguous Surfaces were perfectly smooth, there would be no impression of the Bodies upon one another, while they slide. 'Tis evident that this supposition is very natural, that the resistance arising from the defect of smoothness in the parts, is (other things being alike) proportional to the Velocity whereby the parts are separated from one another; that is, if the contiguous Surfaces are equal, that the resistance and the impression of the Bodies upon one another arising from thence, is proportional to the relative Velocity, or to what is in a given Time proportional to it, the translation of the Bodies. But if the Translations (or Velocities wherewith
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the parts are separated from one another) are equal, the Resistances or Impressions, are as the Superficies wherein the Bodies two by two touch one another.

And therefore, since neither the Superficies, nor Translations are equal; tho' all the rest are, (as the Force pressing the Bodies towards one another, the want of Smoothness, &c.) the Impressions which the Bodies two by two make upon one another, are in a ratio compounded of the ratio of the contiguous Surfaces, and of the ratio of the translations from one another.

Q. E. D.

PROPOSITION LXXV.

IF a solid Sphere revolves in an uniform and infinite Fluid, about an Axe given by position, with an uniform Motion, and by the sole impulse of it a Fluid be moved circularly, and each part of this Fluid persevere uniformly in its motion; I say, the Periodic Times of the parts of the Fluid, are as the Squares of the Distances from the Center of the Sphere.

Let $A E H$ [Fig. 61.] represent a Sphere revolving uniformly about the Axis S . Let the infinite Fluid of the Sphere spreading about every way, be conceived to be divided into innumerable concentric Orbs of the same thickness, every one of which considered alone is Solid; let these be expressed by the Circles $B F K$, $C G L$, $D M N$, &c. Because the Fluid thus divided is supposed to be homogeneous; (that is, all its parts equally compress'd;) the impressions of the contiguous Orbs upon one another (by the preceding Lemma) are as their translations from one another, and the contiguous Superficies in which the impressions are made conjunctly. If the impression upon any Orb is greater or less on the Concave part, than on the Convex part,

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the stronger Impression will prevail, and either accelerate or retard the velocity of the Orb, according as it is directed towards the same way with its Motion, or a contrary. Wherefore, since (by supposition) a Fluid perseveres uniformly in its motion, the Impressions made on both parts of each Orbit towards the contrary ways, are equal; and therefore the ratio which is equal to the ratio of the Impressions, (namely the ratio compounded of the ratio's of the translations and contiguous Surfaces) is also a ratio of Equality. Consequently the Translations are reciprocally as the contiguous Superficies; that is, reciprocally as the Squares of the distances from the Center: But (by *Prop. 72.*) the differences of the Angular Motions are as the Translations directly, and the distances from the Center of Motion reciprocally: And therefore the said differences of the Angular Motions of these concentric Orbs, are reciprocally as the Squares of the distances from the Center, and reciprocally as the distances from the Center; that is, reciprocally as the Cubes of the distances from the Center conjunctly. If, therefore, upon all the points of the Right line *SZ* drawn from the Center, where the contiguous Superficies of the concentric Orbs cut that Line (namely, *A, B, C, D, &c.*) perpendiculars to *SZ* be erected, *Aa, Bb, Cc, Dd, &c.* reciprocally proportional to the Cubes of *SA, SB, SC, SD, &c.* these Ordinates will be respectively as the said differences of the Angular Motions of the respective concentric Orbs, and the sum of these Ordinates, as the sum of the said differences; that is, (since the outmost or infinitely distant Orb is not moved at all,) as the whole Angular Motions. If now the thickness of those Orbs be lessened in infinitum (by which means the Medium spread round

round about becomes so far uniformly fluid,) the Sums of the foresaid Ordinates will become the Areas $AZ a$, $BZ b$, $CZ c$, $DZ d$, &c. and so the intire Angular Motions of the Orbs $B F K$, $C G L$, $D M N$, &c. are as the Areas $BZ b$, $CZ c$, $DZ d$, &c. respectively. But in the Figure $AZ a$, the Ordinates $B b$, $C c$, $D d$, &c. are reciprocally as the Cubes of the Right lines $S B$, $S C$, $S D$, &c. from whence (by Prop. 73.) the Areas $BZ b$, $CZ c$, $DZ d$, &c. are reciprocally as the Squares of the Right lines $S B$, $S C$, $S D$, &c. And therefore the Angular motions of the Orbs, $B F K$, $C G L$, $D M N$, &c. are reciprocally as the Squares of their Semidiameters $S B$, $S C$, $S D$, &c. But the Periodic Times are reciprocally proportional to the Angular motions: Therefore the Periodic Times of the Orbs $B F K$, $C G L$, $D M N$, &c. are in the direct (because the reciprocal of the inverse) ratio of the Squares of their distances from their Center S ; namely, of the Right lines $S B$, $S C$, $S D$, &c. And therefore the Proposition is evident, if the medium of the Sphere circumscribed be supposed to consist of innumerable thin concentric solid Orbs. But if there be supposed to be drawn a vast number of infinite Right lines making with the Axis Angles mutually exceeding one another by equal excesses, and these Right lines being revolved about the Axis, and describing Conic Superficies, Orbs be imagined cut into innumerable Rings; each of these Rings will have four others contiguous to it, one internal, another external, and two lateral, cut out of the same Orb. By reason of the attrition of the interior and exterior, the intermediate can't be moved any otherwise than formerly, before the Orbs were cut: otherwise the parts of the fluid would not persevere in their motion uni-

formly; but the middle one would be accelerated and retarded in their motion, (as was shewn before in the intire Orbs,) contrary to the supposition. And therefore any series of Rings going on directly from the solid central Sphere *in infinitum*, and comprehended between the two next Conic Superficies, will be moved after the same manner as these were moved before the division of the Orbs into Rings; bating what hindrance each Ring in this Series meets with from the attrition of the Rings laterally. But here the attrition is nothing at all, because all the Rings equally distant from the middle solid Sphere, (that is, cut off from the same Orb) are revolved in the same Time: For if it were not so, but those which are towards the Poles, finished their revolution quicker or slower than those which are towards the Circle that lies in the middle between the Poles; the slower, by reason of the mutual attrition, would be quickened in their motion, and the quicker would be retarded, contrary to the supposition, which was, that the Fluid persevered in its motion uniformly. Since therefore the Periodic Time of all the Rings, at the same distance from the Center, is the same, they will revolve after the same manner as they would do, were they not as yet cut off from the solid Orb; that is without any attrition: And therefore the same Law holds in this case also of Orbs cut into Rings, as held before any division into Orbs; that is, the Periodic Times of each of the Rings will be as the Squares of their distances from the Center of the middle solid Sphere.

Let each Ring be divided by transverse Sections, into innumerable Particles constituting a Fluid absolutely and uniformly such: And because these Sections have no reference to the

Law

Law of Circular Motion, but conduce only to the constitution of a Fluid; the circular Motion will persevere as before. By these divisions all the infinitely small rings will either not change the Asperity and Force of mutual Attrition at all, or will change them equally: and the proportion of the Causes continuing the same, the proportion of the Effects will likewise remain the same; that is, the proportion of the Motions and Periodic Times as before. And therefore the Periodic Times of each of the parts, in the above-described Vortex, will be as the Squares of the Distances from the Center of the Vortex. Q.E.D.

COROLLARY 1.

If the resistance of the Parts of the Vortex be greater in a greater distance from the Center than in a less, either on the account of the greater thickness or less fluidity of the Particles which compose the Fluid, or on the account of any other Cause; then the more remote parts of a Vortex will move slower than in the ratio settled above in the Proposition: that is, the Periodic Time of the parts of the Vortex more remote from the Center, will be to the Periodic Time of the parts nearer the Center, in a greater than in the duplicate ratio of the Distances of these Particles from the Center.

COROLLARY 2.

If the Vortex be not extended *in infinitum*, but the Fluid moved after the manner of a Vortex, be contained in a rigid Vessel held by force, of a different Figure from a spherical one concentric to the central Globe; the Particles of the Vortex will be moved, not in Peripheries of Circles concentric with the central Globe, but in Lines almost conformed to the Figure of the Vessels; and the Periodic Times will be as the Squares of the Mean Distances from the Center very nearly.

PROPOSITION LXXVI.

TO point out and refute, in few words, the Errors committed in the above-described Cartesian Physics.

1. Since from the preceding Prop. 'tis evident, that every Vortex made by a solid Sphere, revolved uniformly about an Axis given by Position, will be propagated *in infinitum*, if not hindred; if the World was made up of such kind of Vortices, (as it is according to *Cartes's* System,) there would be as many Vortices going forwards *in infinitum*, as there are Fix'd Stars, every one of which, (according to *Cartes*) like our Sun, makes a Vortex. Nor would these Vortices be bounded by certain limits, but creep on, and by little and little run into one another. And, by this means, each part of the infinite Fluid making up the Universe, would be agitated by that Motion which results from the actions of all the Central Spheres or Suns. But how foreign this uncertainty is, from that certain Order, Situation, and Motion of the Fixed Stars, which are firmly connected by a very simple Calculus, let such as are acquainted with Astronomy judge.

2. Further, because the parts of the Vortex, generated in the manner above-described, that are nearer the Center, being (by the preced. Prop.) moved quicker, urges the exterior, and constantly communicate Motion to them by that action; and these exterior Particles transfer at the same time the same quantity of Motion into others that are still more exterior; because (by Hypothesis) it is a condition of the Vortex, that each part of the Fluid persevere in its motion uniformly, and is not incited, sometimes more, sometimes less, in the same distance from the Center:

Center: 'tis evident that Motion is perpetually transferred from the Center to the Circumference of the Vortex, and absorbed by the infinity of the Circumference. And therefore to the conservation of a Vortex in the same state, there is required some active Principle, from which the Central Sphere (or Sun of that Vortex) may always receive the same quantity of Motion, as it impresses on the matter of the Vortex. But from whence the *Cartesians* can get such an active Principle residing in each Sun that moves the whole Vortex, I cannot see; since *Cartes* himself, in *Parag. 146. third Part Princ. Phil.* boldly says, *There is nothing to be met with in the Phenomena of the Planets, that can't be accounted for from what has been already said.* The penetrating *Kepler* saw well enough the necessity of such an active Principle in the central Spheres of the Vortices: *And therefore lest the Motion should languish by degrees, which Creating Omnipotence impressed upon them in the beginning of Things, he would have it maintained either by the perpetual care of the Creator, or by the assistance of a Spirit destin'd to that employment.*

For without such an active Principle, the central Spheres and the interior parts of the Vortices, always propagating their Motion into the exterior, and receiving no new Motion, must necessarily move slower by degrees, and cease to move circularly. And consequently this World of *Des Cartes*, without such an active Immechanical Principle naturally tends towards rest and its dissolution.

3. *Des Cartes* and the *Cartesians* acknowledge, that the Bodies which are carried in this Vortex and return again, are of the same density with the parts of the Vortex, where they are placed, and move by the same Law with them, as to

the Velocity and determination of the course : the deferent Fluid, and that Body that is carried by it, differ only in this, that the parts of the Bodies thus carried by the Vortex, keep the same situation in regard of one another; and that they are as it were congealed and solid parts of the Vortex, and can't be mingled together by an intestine Motion, as the fluid parts of it are; being, in other respects, intirely equal. Since therefore the parts of the Vortex (by the preced. *Prop.*) revolve by this Law, that their Periodic Times are in the duplicate ratio of their distances from the Center, the Planets also carried about the Sun, and at relative rest in the fluid Matter of the Vortex, in which they swim, will revolve by the same Law. Since therefore the Earth, for instance, compleats its Period in a Year, the Period of Saturn, whose distance is $9\frac{1}{2}$ times the distance of the Earth, would be 90 Years, when it is not quite 30 Years: After the like manner the Period of Saturn would be 27 Years, whereas it is not 12; And all the superior Planets would have Periods greater than the truth; and the inferior less. If the Patrons of Vortices answer, that the Vortices are not uniformly fluid, (as was supposed in the preceding *Proposition*;) but the parts constituting them in the recess from the Center are grosser, as *Cartes* says in *Parag.* 82. *Part* 3; then (by *Corol.* 1. *Prop.* preced.) the more remote Planets from the Sun are still slower, whereas they are too slow already, (according to this System;) and it is evident from this constitution of the Vortices, according to *Cartes*, there will be a greater distance from Observation, than if the Vortices were made up of an uniform fluid. Since therefore it has been demonstrated, that the parts of the Vortex revolve by a very different

rent Law from that which is observed in the Planets; namely, that the Periodic Times of the parts of the Vortex are as the Squares of the distance from the Center, if the matter of the Vortex be homogeneous; or as some higher Power of the Distances, if the matter of the Vortices be grosser at a greater distance; whereas in the mean while (by *Prop. 22.*) in the Planets, the Squares of the Periodic Times are as the Cubes of the Distances; (that is, the Periodic Times of the Planets are as those Powers of the Distances, whose Exponents are $\frac{3}{2}$, which are much lower than those whose Exponents are 2, or a number greater than two;) 'tis evident therefore, that the Planets are not carried by any such corporeal Vortices.

4. If the Vortex were extended *in infinitum*, 'tis evident (from the preced. *Prop.*) that each part of the Fluid ought to describe a perfect Circle, and (by *Corol. 2.*) that the aberration from a circular Path proceeded from the rigid Vessel (or what is equivalent to it,) in which the Matter of the Vortex is contain'd. From whence it evidently follows, that the greater the Vortex is in respect of the Path described by its fluid part, (or by the Planet carried together with it,) the more ought the Path of the Planet to come up to a perfect Circle; that is, the excentricity of a Planet nearer to the Sun will be less than that of one more distant from the Sun; whereas Astronomical Observations testify and assure us, that the Orbit of Mercury is much more excentric than that of Saturn. Again, because (by *Corol. 2. preceding Prop.*) the Particles of the Vortex, and the Planets carried along with them, will be moved in lines almost conformed to the figure of the Vessel, in which the fluid is contain'd, the Aphelia of all

the Planets viewed from the Sun will be posited towards the same Fixed Stars ; whereas the Aphelia of the Planets are at a great distance from one another, as Jupiter's and the Earth's, by a quarter of a Circle ; Mars's and the Earth's, by a third part of a Circle ; those of Mars and Venus are almost opposite. Besides, the matter of the Vortex (like a River) being within narrower bounds, (by the Laws of Mechanics) ought to move quicker than when it moves freely in a broader Channel ; and that in the reciprocal ratio of the breadths of the Channels, or what is equivalent to them : But in the beginning of ϖ , the distance between the Orbs of Mars and Venus is to their distance in the opposite Point, (namely in the beginning of $\times$) almost as 3 to 2 ; and therefore the intermediate fluid of the Vortex, that carries the Earth along with it, will be carried swifter in the same ratio in the beginning of $\times$ than in the beginning of ϖ ; because contain'd within narrower limits ; (for the paths of the Fluid, which carries Mars and Venus, serve instead of boundaries or a Channel ;) that is, the Earth seen from the Sun in the beginning of $\times$, will be carried half as swift again as when it is seen from the Sun in the beginning of ϖ , or the Sun seen from the Earth in the beginning of ϖ , will seem to move half as swift again among the Fix'd Stars, as in the beginning of $\times$: But just the contrary happens by the Observations of all Astronomers : and the Sun seen in the beginning of ϖ from the Earth, seems to move in its diurnal Motion only 58 Minutes ; whereas in the beginning of $\times$ it seems to move a whole Degree in the same time.

5. A Body carried by a Vortex, and of the same density with the parts of the Vortex in which

which it is, in its return round, if it be not otherwise hindered, describes a Circle; to whose Plane the Axis of the central Sphere, whereby the Fluid is turn'd about in the form of a Vortex, is perpendicular: and if the Path of the Body be inclined to this Plane, that inclination will be continually lessen'd, and they at last will coincide. For it is evident from the demonstration of the preced. *Prop.* that each part of the Fluid composing the Vortex, describes such a Circle; wherefore even the Planet, which is carried only by these Particles being relatively at rest between them, will also describe such an one; and if by any external Force impress'd it deflect a little from hence, the inclination will be lessened by degrees by the encounter of the Particles of the Fluid describing the above-mentioned Circles, and the Planet it self will describe such a Circle, being thus at last relatively at rest in the fluid of the Vortex, and carried about together with it. But there is not one Planet to be found having the Axis of the Sun perpendicular to the Plane of the Orbit; nor during these two thousand Years, wherein Observations have been made, has the Inclination of the Orbit of any Planet to that Plane been found lessened; nor are there any fibres of Latitude assigned by these Philosophers in the Planets, for the preserving the same Inclination. But the Apology that *Cartes* brings for the various excentricity of the Planets, and the various inclination of the Orbits to one another, and the various situation of the Aphelias, which is, in *Parag.* 34. *Part* 3. *Princip.* (namely this: *But as we see it happens in all other natural things, that they are only nearly so, and in succeeding Ages are continually changed,*) shews plainly that he was unacquainted with the Calculatory

culatory part of Astronomy. For tho' it can't be denied that Physical Causes have a place in the motion of the Stars, yet no Person used to these matters will judge that such casual and almost irregular inequalities, (such as *Cartes* seems to insinuate,) are to be admitted in the motions of the Stars; but will rather say, with that most acute and best fitted for Astronomical Speculations *Fer. Horrox*, who labour'd to defend the contrary Opinion, *We may justly wonder at some Slingers or Enginiers, that can throw Bodies out of Slings or Cannons with such incredible certainty of aim. But if we see so great exactness in those momentaneous things, that are moved thro' the thick unsettled Air; shall we not expect a much greater in those perpetual Bodies as are revolved in a quiet and pure Aether by an eternal Law?*

6. Since the Globules composing the fluid of the Solar Vortex are lesser near the Sun, and consequently their Mass, contained in a given Space and filling it, is less dense than in a greater distance from the Sun; it follows, that even the Planets which are of the same density with the particles of the Vortex, in which they swim, are less solid the nearer they are to the Sun; which *Cartes* also acknowledges in *Parag. 147. Part 3. of Princip.* But the contrary to this we shall demonstrate in its proper place hereafter; namely that each Planet, the nearer it is to the Sun, the solidier or more dense it is; that is, it contains more Matter under an equal bulk. Which also agrees much better with the Archetypical Ratio's, Geometrical Concinnities, and Final Causes: From the consideration of which, 'tis evident, that the Divine Being has placed the Planets at different distances from the Sun, so that each, according to its degree of density, may enjoy a greater or less degree

degree of Heat; and consequently the more dense Planet is nearer the Sun, since all more dense sort of Matter require a greater heat for the performance of natural Operations. And tho' the ratio of the density of the Planet be not the same with what *Kepler* makes it, namely the reciprocal of the Roots of the distances from the Sun; but the reciprocal of the distances from the Sun drawn into the roots of the apparent distances: Yet here (where he was most of all mistaken,) *Kepler* came at this general Proposition, by his Architypical ratio's, that the denser Planets are nearer the Sun; whereas *Cartes*, who was since him, from a neglect, (or rather contempt) of the Architypical ratio's and final Causes, cherished opinions entirely contrary to these Final Causes, and true Physics or Mechanics, (which generally lead us to the same thing;) and amongst others, this opinion, that the solidity of the Planets near the Sun, is less than that of those farther off.

7. *Tycho* formerly, from the motion of the Comets, demonstrated, that the Planets were not moved in solid Orbs; because the Comets passed thro' them. The same Comets do now with equal evidence shew, that the Planets are not carried by corporeal Vortices; seeing the Comets move in Paths very oblique, and sometimes cutting the Plane of the Zodiac at Right Angles, and sometimes directly contrary to the course of the Planets, and they preserve these motions with the greatest freedom and length of time, being subject to this universal Law, that by a radius drawn to the Sun, they describe Areas proportional to the Times; which would be altogether impossible, if the intire matter of the Vortex were carried about the Sun with a force sufficient to carry along with them the
vast

vast Bodies of the Planets. Nor does it help *Cartes* at all, that he has banished the Comets beyond Saturn, contrary to the credit of all Observations, (which *Tycho* and the Astronomers never forc'd to serve their Hypotheses, as *Cartes* would insinuate;) since the matter of the Solar Vortex beyond Saturn (according to himself) has a greater moment to carry them along with it, than near Venus and Mercury; both because the Globules composing the Vortex are greater there than here; and especially because they are moved much swifter, since they run a Circle immensely greater *within a few Weeks*, as *Cartes* has it, *Parag. 62. Part 3. Prin.* From all together, and each of these in particular 'tis evident, that this Hypothesis of Vortices is entirely disagreeable with the Phenomena of Astronomy, and conduces not so much to the explaining as the disordering of the Celestial Motions.

PROPOSITION LXXVII.

TO enumerate and explain the Causes of the Celestial Motions brought by Mr. Leibnitz.

'Tis so natural in enquiring after the Causes of the Celestial Motions, to seek them in the ambient fluid *Æther*, that the Philosophers have looked upon the Planets to be carried about the Sun, after the same manner as so many Straws swimming in Water, are carried about by the Water put into the motion of a Vortex, by a Stick whirled about its Axis in the middle of a Vessel at rest. This ancient opinion lay neglected for a long time: But in the last Age was revived by some great Men; for before *Cartes*, (of whom above,) it was approved of by *Torricellius* and *Gallilæo*; for the reasons without doubt, summarily proposed about the beginning of *Prop. 71.*

This

This opinion pleas'd so well, that that most acute Philosopher Mr. *Leibnitz*, (even after the publication of *Newton's Mathemat. Princip. of Nat. Philos.*) adopts it; and judges that there remains now no more, than to shew how the Causes of the Celestial Motions arise from the motions of the *Æther*, or (speaking Astronomically) from deferent Orbs, but then such as are not solid, but fluid. A name so celebrated among Geometricians certainly deserves, to have his *Essay upon the Causes of the Celestial Motions* (which he communicated to the Learned World in the *Acts of the Learned*, Published at *Lipsic* in February 1689.) exactly considered.

For Mr. *Leibnitz* (if any Man) is able to accommodate Vortices to the Celestial Motions, and assign reasons for the Laws of the Heavenly Bodies discovered by Kepler. Especially since he has attained some peculiar Light in that matter, and the enquiry he made, seemed to himself, who is so penetrating a Person, to succeed very well and agreeable to Nature, so that hereupon he had good hopes, of having arrived at the true Causes of these Motions. We will therefore lay down the Method this Philosopher uses in explaining these great Works of Nature, in as few Words as possible, and, as near as we can, in his own.

First, *Leibnitz* takes for granted, that all Bodies which in a fluid describe a Curve line, (and consequently that the Planets themselves,) are acted upon by the motion of the Fluid itself. For all Bodies describing a Curve, endeavour to recede from it along a Right line, which is a Tangent, (from the nature of Motion;) there must therefore be something that keeps them in. But there is nothing contiguous except the Fluid, (by supposition,) and no endeavour to move thus can be restrained but by a contiguous Body

Body likewise in Motion, (from the nature of Matter;) the fluid itself therefore must necessarily be in Motion.

Since from the most exact Observations that have been made, every Planet describes an Orbit about the Sun, according to this Law, that by radii drawn from the Sun to the Planet, Areas are always cut off proportional to the Times; 'tis reasonable enough that the Æther or Fluid Orb of each Planet should move about the Sun after such a manner as may agree with the said Law of Motion. And this will be done, if each of the innumerable circular concentric Orbs of smallest thickness, into which the fluid Orb of the Planet is divided by the Mind, have its proper Circulation, so much the swifter in proportion, as each of them is nearer to the Sun; that is, if the Velocities of circulating, in the parts of the fluid, are reciprocally proportional to the radii or distances from the Sun. By which means it will happen, that whether the Planet be more or less distant from the Sun, the Sectors that are described in equal Times by the Radius, are equal. For the said Sectors are in a ratio compounded of the direct ratio of the radii or distances from the Sun, and of the reciprocal of the Arcs or Circulations, and therefore (from the nature of the Circulation) in the ratio of equality. The Circulation above describ'd (where the Velocities of Circulating are reciprocally proportional to the distances from the Center) he calls *Harmonic*, because supposing the Distances from the Center to increase equably or Arithmetically, the Circulations decrease in an Harmonic Progression; being reciprocally proportional to the Quantities that are in an Arithmetic Progression, which is the known property of Harmonic Proportionals.

This

This Great Man therefore supposes the Planet to be moved by a double Motion, or one compounded of the Harmonic Circulation of its deferent fluid Orb, and of a *Paracentric* access to and recess from the Sun. The Circulation of the *Æther* therefore makes the Planet circulate Harmonically, not by any Motion of its own, but as it were by a perfectly quiet swimming in a deferent fluid, whose Motion it obeys. Consequently neither retains an impetus of circulating swifter, than it had in an inferior or nearer Orb, but one that languishes and lessens, as it passes through the superior (resisting a greater Velocity than their own) till at last it insensibly accomodates itself to the Orb it approaches; and on the contrary, while it moves from the superior to the inferior Orbs, it receives their Impetus. Consequently the Harmonic Proportion holds not only in Arcs of Circles, but in describing any other Curve.

For the Area described in an Element of Time, cut off by radii drawn from the Sun to the Planet, moved in any other Curve, doth not comparably differ from the Sector of a Circle, whose Angle at the Center is the same, and radius any one of the preceding.

The other Motion compounding with the former, the intire Motion of the Planet (namely the *Paracentric*,) arises from a double Curve, namely, the *excussory impression of Circulation and the Sun's attraction* (or what is equivalent) compounded together. For since every Body endeavours to recede from the Curve line, it describes, along a Tangent, by this Motion alone, the Planet carried by the Vortex circulating Harmonically, will recede from the Sun. And these Centrifugal Forces are shewn by the Author, to be reciprocally as the Cubes of the radii.

radii. The other component part of the Paracentric Motion is the Sun's Attraction, or what supplies the place of it, the Planets Gravity: And tho' he call this Force Attraction, he does not question but it is derived from the impulse of the ambient Fluid, like the actions of the Magnet.

Because it is certain by Observation, that each primary Planet describes an Elliptic Orbit, in one of whose Foci is the Sun, so as that by radii drawn from the Sun to the Planet Areas are always cut off proportional to the Times; and there is no Law of Circulation in a Vortex that answers this condition, but the Harmonic: It remains that a Law of Gravity be sought after, that join'd with the Centrifugal conatus of the Body circulating Harmonically, may compose a paracentric Motion, which in conjunction with an harmonic Circulation makes the Body to move in the Perimeter of an Ellipse, which (after the premising of a Lemma shewing the ratio of a Circulation in a Conic Section to the paracentric Velocity) he defines, by demonstrating the following Theorem: *If a heavy Body be carried in an Ellipse (or any other Section of a Cone) with an harmonic Circulation; and the Center both of Attraction and Circulation be in the Focus of the Ellipse; the Attractions or Sollicitations of Gravity will be as the Squares of the Circulations directly, or as the Squares of the radii or distances from the Focus reciprocally.*

He has found therefore that there is such a thing as Gravity in the Planets, tending to the Center of the Sun, the Law of which is, that it is reciprocally proportional to the Squares of their distances from the Center. And he farther acknowledges that this Law was known before to Mr. Newton; his *Principles of Philosophy* being pub-

published before that time, and an account of them given in those *Leipſic* Acts.

After having annexed ſome Corollaries naturally flowing from his chief Theorem, he cloſes his *Phyſico-Aſtronomical Eſſay*, with a confeſſion that there remained two things to be done on this Argument: The one is to explain what motion of the *Æther* it is, that makes the Planets heavy, or impels them towards the Sun, and that reciprocally as the ſquares of the Diſtances, or (as he expreſſes it) as the ſquares of the Vicinities: The other is, what is the cauſe of the compariſon of the Motions between the different Planets of the ſame System, that the Periodic Times ſhould be in the ſeſquialter ratio of the mean Diſtances. And ſince (according to the Author) they both ought to depend upon the contiguous Bodies, all which he calls by the name of the *Æther*, theſe two end to explain more diſtinctly the motion of the Vortex or *Æther* that conſtitutes each System. But theſe things requiring a deeper ſearch, , and therefore not to be contain'd in ſo ſmall a compaſs as a Sheet, he ſays, 'tis more proper for him to publiſh his Thoughts upon that Matter ſeparately.

PROPOSITION LXXVIII.

TO point at ſuch Things as are not agreeable to Natural Philoſophy, in the above recited *Cauſes of the Celeſtial Motions*; aſſigned by Mr. Leibnitz in his Eſſay.

There are ſome of the Reaſons againſt Corporeal Vortices carrying Planets, that likewise make againſt the above deſcribed *Eſſay of Leibnitz about the Cauſes of the Celeſtial Motions*; namely, that the Paths of ſome Comets are very oblique to the Zodiac, ſometimes they cut this

Plane at Right Angles, nay, they are even directly contrary sometimes to the course of the Planets. And since the Comets (all the while they are observed by us) describe Areas about the Sun proportional to the Times, by parity of reason, an Harmonically circulating Vortex ought to be supposed, such an one being a necessary for the motion of a Comet as of a Planet; that is, there ought to be one Vortex contrary to another.

In considering indeed but one Planet carried about the Sun (or any other Center) if it be to be carried by a Vortex, without any proper motion of its own, only swimming quietly along in a deferent Fluid, circulating Harmonically, and obeying its Motion, (which the Author supposes;) the Vortex ought to circulate Harmonically, so as that the Areas cut off by Radii drawn from the Planet to the Sun, be proportional to the Times. No other constitution of a Vortex can make the ratio between the Area and Times to be the same. If therefore the Vortex of our Author will not solve the Motions of the Planets, certainly no other will. And this being once supposed, all the rest, which our Author draws from thence by Geometry, proceeds very well and justly, as things usually do under his Hand. To which I add, that such a Vortex as this (whose Fluid circulating Harmonically in a Plane thro' the Center, to which the Axe is perpendicular,) may be produced Mechanically, by a solid Sphere, if it be revolved in an uniform and infinite Fluid, about an Axe given by position, with an uniform Motion, and the Fluid be made to move round by the sole impulse of it, and every part of the Fluid persevere in its motion uniformly. And this is a third thing, which remains for the Author to explain.

explain concerning the motion of the Solar Vortex or Æther; namely, how it might be impelled, and put into an Harmonic circulation. For, since universally the Times of describing or running any Space, are directly as the Spaces run, and reciprocally as the Velocities; and in a Circular motion the spaces run in one revolution are as the radii, and in an Harmonic circulation the Velocities or Circulations are reciprocally as the radii; 'tis evident, that the Periodic Times of the parts of a Vortex circulating Harmonically, are as the Squares of the radii. But we have demonstrated, *Prop. 75.* that this is the property of a Circulation in a Plane thro' the Center, to which the Axis is perpendicular in the Vortex, put into its motion by a central Sphere.

But, if the motions of two or more Planets be compared together, it will appear, that the fluid Matter of the Solar Vortex is not moved in an Harmonic Circulation. For we have already demonstrated, that the Periodic Times of the parts of a Fluid circulating Harmonically (and consequently of Bodies that are carried by a quiet swimming in the Fluid) are as the Squares of the radii. But the Periodic Times of the Planets are not as the Squares, or in the duplicate, but in the sesquuplicate ratio of the distances from the Center. And therefore different Planets are not moved by an Harmonic circulation.

The Author no doubt will say, that he did not suppose that the whole Fluid of the Solar Vortex was moved by a continual Harmonic circulation, no where interrupted, from Mercury to Saturn inclusively; but that the Fluid carrying about Mercury moved in an Harmonic Circulation from its Perihelium to its Aphelium; then (this Harmony being interrupted,)

again Circularly harmonical from the Perihelium of Venus to its Aphelium, but not in an Harmony continued with the former circulation near Mercury ; that is, so as that the circulation of the Fluid carrying Venus, is to the circulation of the Fluid carrying Mercury, as the distance of Mercury from the Sun, is to the distance of Venus from the same : And again, (the former Harmony being interrupted,) the Fluid carrying the Earth is moved Harmonically from the Perihelium of the Earth to the Aphelium, if this Harmony be considered solitarily, and not compared with the Harmony of the Fluid to Venus or Mercury ; and so on in the other Planets : that is, that by reason of the thickness of each Orb, there holds a particular Harmony separate and distinct from the rest ; and in like manner in the Satellites of Jupiter and Saturn, ascending from the Center of Jupiter and Saturn. But it must be confessed, that the Author does seem likewise to extend his Harmonic circulation also to different Planets, in *Parag. 17.* where he says : *In double the distance, only a fourth part of the Angle seen from the Sun, is finished in the same Element of Time ; in triple the distance only the ninth part :* For certainly these numbers have no place in the same Planet. But if, (as he ought principally to do,) he speaks expressly concerning the interrupted Harmony of the circulating parts of the Fluid, he will hardly persuade Philosophers that things are thus ; namely, that by reason of the thickness of the Orb of Mercury there is an Harmonical circulation from the Perihelium to the Aphelium, or the Velocities of the Fluid are as the distances from the Center reciprocally : But comparing this Circulation with the Circulation of the Æther at any other Planet, the one is to the other reciprocally

ciprocally as the Squares of the Radii: For we have demonſtrated (in *Prop.* 27 and 28.) independantly from all Phyſical Cauſes, that the Celerities of the Planets are reciprocally as the Square Roots of their diſtances from the Sun. Certainly the Vortex of the Sun would be monſtrous, and theſe deferent fluid Orbs more abſurd than the deferent ſolid ones, if by reaſon of the thickneſs of each of the ſix deferent Orbs the Harmonic circulation has a place, and yet this Harmony in the mean while is five times interrupted; namely, at the interſtices of the deferent. And it makes contrary to this interruption, that a Comet carried thro' the Zodiac *in conſequentia*, and paſſing thro' theſe interſtices of the Orbs carrying the Planets, is moved juſt as if it were carried by an Æther circulating Harmonically; that is, ſo as that the Areas deſcribed by the radius that carries it, are proportional to the Times. It may be obſerved likewise, how diſformly diſform the Solar Vortex would be, in which the thickneſs of the Orb of Saturn, where the Harmonic circulation of the Æther holds, is greater than the diſtance of Mercury from Venus, or of Venus from the Earth, or of the Earth from Mars, in every one of which the Harmony is interrupted; and where, having laid aſide the reciprocal ratio of the radii, which they keep in the Harmony, the circulation of any two of theſe compared together, are reciprocally as the ſquare roots of the Radii, being about to reſume the former Harmony. The famous Author himſelf ſeems to have diſcovered this fault of his Harmonically Circulating Vortex, when he confeſſes, *that this likewise was one thing remaining, namely, to explain what ſhould be the cauſe of the compariſon of the motions between different*

Planets of the same System, so as that the Periodic Times may be in the sesquiplicate ratio of the Mean Distances: For he saw plain enough, that this could not be the effect of an Harmonic Circulation.

Perhaps some Persons may wonder, that Mr. *Leibnitz*, who deduced the Motions of the Planets from Causes (at first sight) so different from those established before, should notwithstanding light upon the same Law of Gravity, whereby every Planet is kept in its Elliptic Orbit. The reason of that coincidence is thus: Tho' *Leibnitz* makes use of the Matter of a Vortex Circulating Harmonically for carrying the Planet about; yet because in the Planet thus carried about, and describing any Curve, he confesses an Excussory Conatus, whereby a Planet would be made to move in a Right line touching the Curve, if not hindered. He does the same as if he had made the Planet to move equably in a Right line, if not hindered: For these two Motions together make it so; that is, all one as if it were carried in free Spaces, acted upon by no external force at all. For in both cases it will describe a Right line with an equable Motion: For the Body describing the Right line with an equable Motion, runs in it after such a manner as (by *Prop. 1. Elem. 6.*) that the Areas cut off by radii drawn to any given Point, are proportional to the Times. Since therefore the Harmonic Circulation and Excussory Conatus together perform the same as an equable Motion in a Right line does, he could not but find the Law of Gravity the same with that found out before; namely, that which would force the Planet from the same Right lined Tangent which it would run after the same manner, into the same Conic Section towards the Sun, posited after the same manner.

SECTION



Fig: 62
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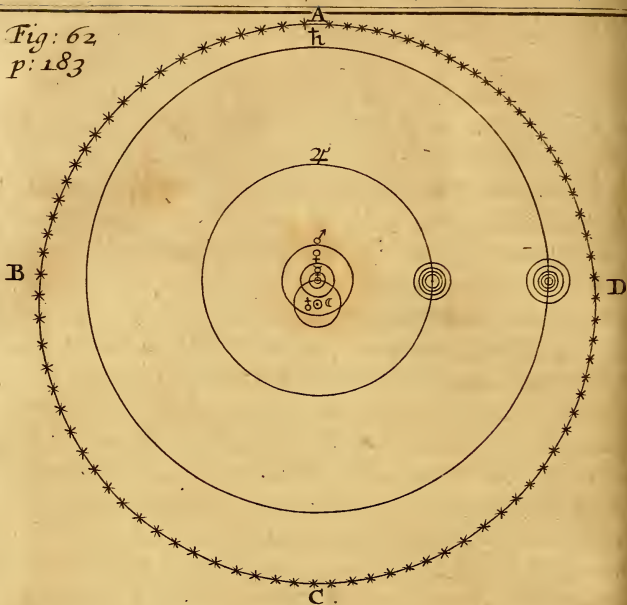
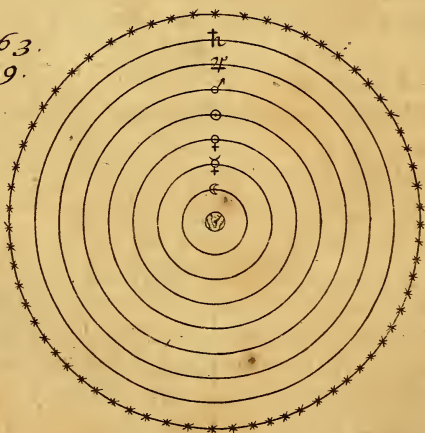


Fig: 63.
p: 189.



SECTION XI.

Of other Systemes of the World, and Forces necessary to preserve them.

Hitherto we have been explaining the old *Philolaic* System of the World, by shewing what Phænomena would happen in it, and that they agree with Observation. We largely treated also concerning the Direction and Law of the Forces, whereby the System of the Sun is kept in that state; and after what manner the Philosophers, who were Maintainers of it, thought it was done. It remains that we dispatch briefly the Order and Motion which other Philosophers ascribed to the Bodies of this System, which Opinions are now-a-days called the *Systems* of their Assertors; and we will at the same time shew the Direction and Law of the Forces, whereby those Systems might subsist.

PROPOSITION LXXIX.

TO describe the Semi-tichonic System of the World.

The maintainers of this Opinion, place the Earth, which is not moved progressively or out of its place, in the Center of the Sphere, in whose Superficies *ABCD*, [Fig. 62.] they will have the Fixed Stars to be placed. But grant that the Earth is moved about its own Axis from West to East by a Diurnal Motion. Besides, in this System the Sun is moved about the Earth in the space of a Year, at a Distance equal to that at which the Earth is about the Sun in the *Philolaic* System: And about the Sun, as a Center,

the five Planets, viz. Mercury, Venus, Mars, Jupiter and Saturn, are revolved towards the same way, and at the same Distances as they were in that. Moreover, the Moon revolves about the Earth, and about Jupiter and Saturn, their proper Satellites, in the same Periods as was said before, at *Prop. 15.*

This Order, and these Motions of the great Bodies of the World please most of those Philosophers, that the other System explain'd at large before displeases, and which, from the reviver of it, *Copernicus*, is commonly call'd the *Copernican* System, as by *Longomontanus* and others. For in this the apparent Diurnal Revolution of all the Stars from East to West, and the proper or second Motions of the Planets are intirely the same as in the *Copernican* System, and the Earth continuing in the mean while in the Center of the Sphere of the Fixed Stars, is only moved about its own Axis. And tho' they place the Fixed Stars in the Superficies of a Sphere, whose Center is the Earth, that is to be looked upon rather as the Astronomical than the Physical one.

For the Fixed Stars scattered as it were thro' the whole Mundane Space, may be placed at different Distances, without any prejudice to this System; and their Motion *in consequentia*, explained by ascribing a very slow Motion to the Axis of the Earth, as we have explained it in *Prop. 64.*

PROPOSITION LXXX.

TO explain the Direction and Law of the Forces whereby the Semi-tychonic System may be preserved.

'Tis evident first of all, that the same Forces and the same Direction are necessary here for the

the keeping the five Primary Planets, Mercury, Venus, Mars, Jupiter and Saturn, in their Orbits about the Sun, and the Attendants of Jupiter and Saturn in their Orbits about Jupiter and Saturn, as in the *Copernican* System. The Law therefore of the Centripetal Force, whereby in this System, the Secondary Planets of Jupiter tend to Jupiter, the Satellites of Saturn to Saturn; and the Primaries Mercury, Venus, Mars, Jupiter, and Saturn, and their Satellites, towards the Sun, is known: For it is that Force which, in *Prop.* 47. we demonstrated to be the same with our Gravity, which is reciprocally proportional to the square of the distance. 'Tis farther evident, that this System being supposed, the Moon is kept in its Orbit by Gravity propagated towards the Earth by the same Law: For *Prop.* 46. where this was demonstrated, holds as much, whether the Earth continue in the same place unmoved, or be carried by an Annual motion about the Sun. But since the difference between the *Copernican* and *Semi-tychonic* System is, that in the latter, the Sun (as it were, with its five attending Primary Planets and their Secondaries,) is moved about the Earth in an Orbit similar and contrarily posited to that, in which the Earth with its attendant is carried about the Sun in the former; besides the Force of Gravity necessary in the *Copernican* System, that the Earth and its Attendant might be carried about the Sun, there is occasion for another Force equal to it, but with a contrary Direction, that the Sun, together with the attending Planets, may be moved about the Earth at rest in the *Semi-tychonic* System: For these two equal and contrary Forces impressed upon the Earth, keep it at rest. (Nay, if one contrary impetus and not

to be repeated, do but put a stop to the progressive motion of the Earth, and its attendant the Moon, along the Tangent of the *Orbis Magnus*, and afterwards that other Force now described, act upon the intire Solar System, the *Copernican* System will be turned into the *Semi-tychonic*. Therefore to preserve the *Semi-tychonic* System, there is occasion for both the above explained accelerative Gravity of all Bodies towards all others, which is, in a given distance, as the Body towards which it tends, and the Body being given towards which it tends, is reciprocally as the square of the distance; and also of another certain Gravity, equal to that whereby the Earth tends towards the Sun, acting equally upon the Sun, Mercury, Venus, the Earth, Mars, Jupiter and Saturn, and their Attendants, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth. For I do not reckon among these Forces, that Force whereby the Earth is revolved upon its own Axe; because only once to be impressed upon the Earth, as also upon the other great Bodies of the World revolving about their own Axes: Nor do I take in (for the same reason) that Force, whereby the Planets would describe the Tangents of their Orbits: For we are now treating of those Forces and their Law, necessary to preserve the System, and not of the Forces whereby it may be produced.

PROPOSITION LXXXI.

TO describe the *Tychonic* System of the World.

This differs from the System described in *Prop. 79.* only in this, that the Earth being deprived of all Motion, is absolutely at rest; and there-

therefore all the Stars, as well Fixed as Wandring, are not only carried by their second Motions, but also by that common or first Motion from East to West, in the space of a Natural Day.

'Tis said, that the great *Tycho* liked this System best; tho' others deny, that *Tycho* allowed expressly a Diurnal Motion to the Stars. But it must be confessed, that he disputes against the Diurnal Motion of the Earth as well as the Annual: And consequently it may be safely concluded, that he ascribed a Diurnal Motion to the Heavens and all the Stars.

PROPOSITION LXXXII.

TO explain the Direction and Law of the Forces, whereby the Tychonic System may be preserved.

The same Forces that are required in the *Semi-Tychonic* System, are required also in the *Tychonic*, since the same motion of the Sun and Planets are supposed in both. And besides these, some other Forces, whereby all the Stars, both Fixed and Wandring, are drawn from their Rectilineal motions and retained in their Orbits, which they describe by their Diurnal motion about the Earth. But these Forces, since the Periods in all are the same, are (by *Corol. I. Prop. 26.*) as the Semi-diameters of the Circles they describe. And therefore, besides the Forces explained in *Prop. 80.* there will be need of another Force tending towards the Earth, that increases in the recess from the Earth as the distances increase; contrary to what happens in Gravity thro' the whole Solar System, and in all Natural Virtues propagated from a Center, or to a Center, in Right lines, every way thro' the Regions round about, which (by *Prop. 48.*)

in

in its recess from the Center, is lessen'd in the duplicate ratio of the distance increased. And that this Force, whereby the Stars are retarded in their Orbits, described by their Diurnal Motions, may be the more easily estimated, let it be compared at the Moon with the Force of Gravity, whereby (by *Prop.* 46.) the Moon is retained in the Orbit, she describes in her Monthly Course round the Earth. And since the Circles described by the Moon in her Monthly and Diurnal Course, are equal, the Forces whereby the Moon is retarded in them, (by *Corol.* 3. *Prop.* 26.) are reciprocally as the squares of the Times; that is, the Force whereby the Moon, (when she is carried in the Equator by a Diurnal Motion about the Sun) is kept from flying off in a Right line, which is a Tangent to the Orbit, is to the Force, whereby it is kept in the Orbit that it describes about the Earth in a Periodic Month, as at 27×27 to 1; or as 729 to 1. But this Force tending towards the Earth, holds only in those Stars, which are placed in the Equator: For the Force, whereby any Star declining on one side of the Equator, towards either of the Poles, is retracted from the rectilineal Motion, and kept in an Orbit parallel to the Equator, (by *Prop.* 12.) tends to the Center of that Circle, and not to the Earth, which is situated without the Plane of it; and it is, (by *Corol.* 1. *Prop.* 26.) as the Semidiameter of that Circle; that is, in a given distance from the Axe, equal or the same, and in a given distance from the Earth, as the Cosine of Declination.

To preserve therefore, the *Tychonic* System, there is required 1st. That accelerative Gravity of all Bodies towards all others, which in a given distance, is as the Body towards which it tends; and

and the Body being given towards which it tends, is reciprocally as the Square of the Distance. *2dly.* Another accelerative Force equal to that whereby the Earth, by a superior Force, tends towards the Sun, and acting equally upon the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and their Attendants, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun to the Earth. *3dly.* An accelerative Force, tending towards the nearest Point of the Axis of the World, proportional to its distance from that Point, and reaching to the farthest Fixed Stars. *4thly.* Another Force like it tending to the nearest Point of the Axis of the Ecliptic, and proportional to the distance from that Point. This is the Force, whereby the Fixed Stars are retain'd in the Orbits, which they describe in their proper motion from West to East, upon the Poles of the Ecliptic, and does not act at all in the Places that lie between the Center and the Fixed Stars, but begins to act at the Fixed Stars themselves; and is (by *Corol. 3. Prop. 26.*) to the like Force in the Axis of the World, in the duplicate ratio of the Natural Day, to the Period of the Fixed Stars upon the Poles of the Ecliptic. And these four will suffice.

PROPOSITION LXXXIII.

TO describe the Ptolemaic System.

In this System the Earth continues absolutely at rest in the Center: Next to the Earth, the Moon, then Mercury, afterwards Venus; above Venus the Sun, next Mars, then Jupiter, and Saturn surrounding Jupiter just as in the Scheme, [*Fig. 63.*] finishing their Revolutions under

der the Zodiac from West to East. Above Saturn the intire Sphere of the Fixed Stars, (whose Center is the Earth,) is also revolved very slowly from West to East, upon the Poles of the Ecliptic. But both the Fixed Stars and all the Planets, revolve upon the Poles of the Equator from East to West, in the space of a Natural Day. We don't reckon up the several Orbs, which the assertors of the *Ptolemaic* System, place above the Fixed Stars; since they are only of use in the *Theory*, and don't belong to the Aspectable World. And for the same reason, we omit likewise the Epicycles of each Planet: For in the Orbits here delineated, the Centers of the Epicycles themselves are carried about, in whose Circumferences the Bodies of the Planets themselves are moved.

In this Scheme of the Planets, we have not delineated the Orbits according to the proportion they have in the Heavens; as well because *Ptolemy* himself has not declared it in any, but the Sun and Moon; and because that tho' the Circles, that carry about the Centers of the Epicycles, are at some distance, yet the Heavens of the Planets are, in this Scheme contiguous, the inferior sometimes in the Apogæum, ascending from the Earth, to a distance equal to that, to which the next superior descends in its Perigæum: as because the Orbits of Mercury and Venus, (not to mention the Moon) taken from the Theories of *Ptolemy*, do almost vanish in respect of the Orbit of Saturn, and therefore are hardly to be described in this Scheme: For in this System, the distance of Saturn from the Earth, is an hundred and sixty times greater than the distance of Mercury from the Earth: But in the *Copernican* System, the distance of Mercury from the Sun, is only four and twenty

times less than the distance of Saturn from the same.

Tho' the Order of the Planets described above pleased the generality of Philosophers and Astronomers, yet there were some, who placed the Sun next above the Moon, then Mercury, and then Venus; and this Opinion is usually ascribed to *Plato*: But *Porphyrus* placed Venus immediately above the Sun, and Mercury above Venus, the three others (Mars, Jupiter and Saturn) continuing as before: Perhaps induced by this reason, that it was better for the two Luminaries to be nearest the Earth; whereas the *Ptolemaeans* minding their motions more, placed the Sun in the middle of the Planets, being satisfied that the motions of the other Planets in their Epicycles depended upon the motion of the Sun.

PROPOSITION LXXXIV.

TO explain the Direction and Law of the Forces whereby the *Ptolemaic System* may be preserved.

First, for the keeping the Stars within the Orbits they describe, by their Diurnal Motion about the Earth; the same Forces are necessary here, as were in the *Tychonic System*, for the producing the same Effect; namely, such as may tend from any Point of the World to the nearest Point of the Axis, and be in the ratio of the distances from it. Secondly, to contain the Fix'd Stars within the Orbits which they describe, by their proper motion from West to East, Forces like those made use of in the *Tychonic System*, are necessary. Then to keep the Luminaries (the Sun and Moon) in the Orbits which they describe by their second Motions; since these Orbits are similar to their Orbits

bits in the *Tychonic* System, the like Forces will be necessary in this System, to retain them in the said Orbits, as in that ; namely (by *Prop. 80.*) the accelerative Gravity of the three Bodies, Sun, Earth and Moon, towards one another ; (that may not affect any other Mundane Bodies, tho' they lie between them,) which, in a given distance is as the Body towards which it tends, and towards a given Body, is reciprocally as the Square of the distance from it ; and besides, another sort of Gravity equal to that, whereby the Earth would tend towards the Sun, acting equally upon the Sun, Earth, and Moon, by Lines parallel to a Right line, connecting the Sun and the Earth, and with a direction from the Sun to the Earth, but such as affects no other Planet. For that Force, whereby the Sun is retained in its Annual Orbit, is not the same with that whereby the Moon is retained in its Menstrual Orbit, propagated unto the Sun: For the Force, whereby the Moon is kept in its Orbit, is the same with Gravity, with us, (by *Prop. 46.*) But if the Sun were retained by the same Force, propagated so far as itself, the Cubes of the Distances of the Sun and Moon (by *Prop. 27.*) would have the same ratio as the Squares of their Periodic Times ; and therefore supposing the distance of the Moon from the Earth to be 60 Semidiameters of the Earth, the Sun would not be above 340 Semidiameters of the Earth distant ; and its Horizontal Parallax would be above 9 minutes, contrary to all Observations ; and besides, there would be no room for the Epicycles of Mercury and Venus, between the Heavens of the Moon and the Sun, where they are placed according to this System.

Some of this Sect of Philosophers are not willing to put any more than a mental distinction between the Motions of the Sun and Moon under the Zodiac, and the Diurnal Motions upon the Poles of the Æquator, so that the motion of each is really in the same simple Spiral Line, arising from the composition of the above-mentioned motions, into which these Celestial Motions are imagined to be divided by Astronomers, for the greater ease; and the most simple Motion in a Right line, may be compounded of two or more Motions. But because that Spiral is not a Line described in a Plane, a Body can't (by *Prop. 11.*) be retained in it by a single Force tending to an immovable Point. Either therefore, there is need of the several different Forces described above, or of a single Force equivalent to them, tending to a Point moved in the mean while forward and backward, and that in a very compounded Line.

Besides, to keep the other Planets (Mercury, Venus, Mars, Jupiter, and Saturn) within the Orbits they describe by their proper Motion, the Forces tending to the Earth are not sufficient, (as was demonstrated, *Prop. 13.*) nor are the Forces tending to the Sun, since the Orbits of Venus and Mercury do not surround the Sun in this System, and for that reason they are not concave toward him in all their Parts: Wherefore, from *Prop. 11.* the thing proposed is certain. And there is no Point in the System, (much less a Body worthy to be taken notice of,) that can be a Center, to which the Forces may tend, whereby the Planets are kept from going off in the Tangents; one of the above-mentioned reasons contradicting it.

But if this *Ptolemaic* System be understood literally, that is, if the motions described by *Ptolemy*, are not look'd upon as motions only separated by the Mind, but as such as are really in the Planets; then there will be need of a Force in the center of the Epicycle of every Planet, (Saturn, Jupiter, Mars, Venus and Mercury,) tending to the center of the deferent; that is, in an imaginary Point, (or where there is no Body at all,) to an imaginary Point: And besides, there is need of a Force in the Planet itself, that may tend to the center of the Epicycle; that is again, a Force in a Body, that may tend to an imaginary Point. But even those Forces whereby each Planet, and the center of its Epicycle are urged, are not propagated beyond its proper Point and Planet, nor do they affect any other Planets; and there is still need of a peculiar Force for the keeping every center of an Epicycle, and every Planet in its proper Orbit; so that there is no Law which respects all the Planets: And the ordinary respective Gravity of the Satellites of Jupiter and Saturn, towards the centers of Jupiter and Saturn, continue in the mean time untouched.

After that the Phases of Venus and Mercury were observed by the help of Telescopes, the *Ptolemean* System of the World was soon thrown aside by Philosophers: because 'tis evident from them, that the Orbits of Mercury and Venus, must encompass the Sun; as was shewn in *Schol. Prop. 6.* Such therefore as still maintained, that the Earth was Immovable, made Mercury and Venus move in Epicycles round the Sun as a Center; thus far embracing the *Philolaic* System of the World, and bringing in again the old *Egyptian* one; that *Vitruvius* in his *Book of Architecture*

ecture, and Venerable Bede in his *of the Nature of Things*, approved of.

Ricciolus in his *Almagest*, adds Mars to the Attendants of the Sun; so that such as have no Satellites of their own, are made Satellites to the Sun, and only Saturn and Jupiter that have Satellites, besides the Luminaries, revolve about the Earth, as their immediate Center; and this System, which is almost the same with the *Tychonic*, he calls his own.

There is no need of explaining the Forces and their direction, that are requisite to preserve the *Aegyptian* System, or that other, which is little different from it, the *Ricciolic*: For that appears very easy, from what has been said above. As they are composed of the Systems already described, so the Forces necessary to preserve them, are parts of the respective Forces explained before.

SCHOLIUM.

This Section was subjoin'd, to the end, that not only Astronomers, (who are no ways afraid of a multiplicity of the Celestial Motions, and make it a Postulate, that any Star may be moved with any motion,) weighing the several different Systems, might embrace that which agrees best with the Phænomena: But that the Philosophers may likewise chuse such a System as best agrees with Nature, namely, either the Old *Philolaic* or *Copernican* System, which agrees with the Phænomena, and may be preserved by Gravity alone, propagated in the same tenor thro' the Universe; or by an accelerative force of all Bodies towards all others, that is, in a given distance, as the Body towards which it tends, and reciprocally as the square of the distance, when the Body is given towards which it tends:

Or the *Semi-tychonic*, described in *Prop.* 79. that performs the same, as the *Copernican*, in regard of the Phænomena; but then besides the same Gravity, as is requir'd in the Copernican System, (by *Prop.* 80.) it stands in need of another Force, whose Magnitude and Direction are perpetually changed; for this Force is equal to that, whereby the Earth would tend towards the Sun, and acts equally on all the Bodies of the Solar System, according to Right lines parallel to a Right line that connects the Sun and Earth, and with a direction from the Sun towards the Earth: Or (if it be resolved upon, that the Earth shall be or is intirely at rest) the *Tychonic System* described in *Prop.* 81. doing the same as to the Phænomena, in which (by *Prop.* 82.) besides the two aforesaid Forces, there is need of two more; namely, one equal in every point of the Axis of the World, that encreases as the Distances, and is at the Moon to the Force of Gravity in the duplicate ratio of the Periodic Month, to the Natural Day; and another Force like it, and similarly propagated in every point of the Axis of the Ecliptic, that acts only upon the Fix'd Stars, and is at the Fix'd Stars to the former Force in the Axis of the World, in the duplicate ratio of a Natural Day, to the great Year, or period of the Fix'd Stars upon the Poles of the Ecliptic: Or the *Ptolemaic System* itself describ'd in *Prop.* 83. But this does not agree with the Phænomena of the Heavenly Bodies, as has been shewn in *Prop.* 84. But if, setting aside the Phænomena of the Phases of Mercury and Venus, we only attend to such Phænomena as were known by *Ptolemy*, and his System be taken for true; (by *Prop.* 84.) besides the two sorts of Attractive Forces, equally diffused thro' the Axis of the World, and the Axis of the Ecliptic, there

is need of an accelerative Gravity of the Sun, Earth and Moon towards one another, without affecting any of the other Planets, which if the distance be given, is as the Body towards which it tends, and towards the given Body, reciprocally as the square of the distance from it; and besides, there is occasion for another Gravity, (equal to that whereby the Earth would tend toward the Sun,) acting equally upon the Sun, Earth and Moon, along lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth. Over and above, it stands in need of a particular Force in the center of the Epicycle of each of the Planets, Saturn, Jupiter, Mars, Venus, and Mercury, that tends to the center of its Deferent; and still of another particular Force in the Planet itself, tending to the centre of its proper Epicycle, without any prejudice in the mean while to the ordinary Gravity of the Satellites of Jupiter and Saturn. But if the *Ægyptian* (or *Ricciolic*) System be most fancied, (which indeed may be made to solve the Phænomena,) then, (besides the Forces residing in the Axes of the World and the Ecliptic,) there is need of a mutual Gravity of the Sun, Mercury, Venus, the Earth with the Moon, (and Mars,) as in the *Copernican* and *Tychonic*; but such an one as does not reach so far as Jupiter and Saturn; and besides, of a Force equal to that, whereby the Earth would tend towards the Sun, which acts equally upon the Sun, Mercury, Venus, the Earth together with the Moon (and Mars,) along Right lines parallel to a Right line connecting the Sun and Earth, and with a direction from the Sun towards the Earth; and besides of a particular Force in

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the Center of the Epicycle of both Planets, Jupiter and Saturn, that tends to the Center of its Deferent, and of a particular Force in Jupiter and Saturn, tending to the Center of their proper Epicycle.

T H E



THE
ELEMENTS
OF
Astronomy,
Physical and Geometrical.

The SECOND BOOK.

Of the First Motion.

IN the preceding Book, we have laid down the Principles of a True and Physical Astronomy; that is, we have explained, in general, the motions of the Stars, as are really in nature: But some notice must be taken of the Appearance of them; that is, the Elements of an Astronomy fitted to them, are to be formed. For as it is out of all doubt to any Philosopher that considers the Reasons of Things, that the Earth is carried about among the Planets, so 'tis likewise certain to every one that enjoys his Sight, that the Earth appears at rest, in the

middle of the Universe, and that the Heavens, and all contained by them, are moved as to Sense, round about it. Now not only the reason of this Phenomenon is to be assigned, (which has already been done, in the foregoing Book,) but the Methods must be explained, that the Astronomers have used, to define the places of the Stars, seen from the Earth, who either thought, with the common People, that things were really so as they appear, or who thought that this System of Appearances ought to be retained, tho' they knew for certain, that things were not after this manner; and that 'tis according to this System that we must speak with the common People; and not make our Reason and Philosophy perpetually offer violence to our Sight and other Senses.

To this end is this Second Book designed; namely, the explaining the Words us'd in Astronomy, and the giving the description and use of Spheres, Globes, and other Instruments, made to set before our Eyes principally the Diurnal Motion, and in general to search out in this Physical System, and to handle according to the apparent System, and Calculate all those matters as are usually said by Astronomers to belong to the *Doctrine of the Sphere*: And tho' the Doctrine of the Sphere be usually first treated of, by all Authors, we have not been afraid to invert the Order; since by this method better care is taken against young Persons in learning Astronomy, their taking the System where the Earth is at rest for true, to which they are first accustomed, and to which they are more inclined by their Senses; and that the Doctrine of the Sphere, in Kepler's judgment, *stands in need of several things, that are borrowed of the Theoric part of Astronomy* (or Doctrine of the proper Motion of

of the Planets, a faint Draught of which you had in the preceding Book,) *by anticipation, whereas the Theoric Part may be deliver'd alone, without needing the assistance of the Diurnal or First Motion, or of the Doctrine of the Sphere, which treats of it.*

SECTION I.

Of the Generation of the Circles of the Sphere, and of the Terms used in Astronomy, depending thereupon.

PROPOSITION I.

TO explain the generation and nature of the Ecliptic and Zodiac, and the division of them, also of the Secondaries of the Ecliptic, and their uses, and the Terms used in Astronomy depending upon them.

An Observer placed upon the Earth, by extending his Sight every way equally, if not hindered, bounds the World by a Spherical Superficies concentric with himself, (that is, with the Earth) and judges all remote Objects, whose distance is unknown, tho' really very unequally removed from one another, to be placed as it were in the same concave spherical Superficies; as for instance, he does the Fixed Stars, Sun, and Moon itself; as was shewn in *Prop. 32. B. I.* This Sphere, concentric to the Earth and surrounding it, defined by the Sight, and bespangled with the twinkling Fixed Stars, supplying the place of an absolute Space, is consider'd as at rest; while we abstract from the Diurnal Motion, whereby this intire Sphere, or rather the whole Heavens, are revolved from East to West in the same Space of 24 Hours. But since by reason of the Annual Motion of the Earth

about

about the Sun, the Sun seen from the Earth seems to move forward daily towards the more Eastern Stars (as was shewn in *Prop. 2. B. 1.*) the Sun will seem to move in this immovable Sphere or Mundane Space; and its Path marked among the Fixed Stars is call'd the *Ecliptic*, because the Eclipses of the Luminaries happen only in this Line; as was shewn above.

Having once settled the *Ecliptic*, 'tis evident that as all the Orbits of the Planets about the Sun, are not in one Plane, but each of the Planets seen from the Sun, makes an excursion, sometimes on this side, sometimes on that side of the Way of the Earth, mark'd among the Fix'd Stars; so the same Planets seen from the Earth, make an excursion sometimes on this side, sometimes on that side of the *Ecliptic*, and even in the same points of their Orbits seem to run off variously, according to the different situation of the Earth, (as was shewn in *Prop. 5. and 8. B. 1.*) But their greatest excursion is contained within the boundaries of ten Degrees; the Astronomers therefore have called that tract which the Sun, Moon, and Planets seem to adorn with their motions, and which is a Zone or Belt, twenty Degrees broad, (ten on each side of the *Ecliptic*,) by the name of the *Zodiac*, from the images of Animals, which the Fixed Stars, in that tract, seem to represent. The number of the images, pitched upon by them, is twelve; either because this number seems the fittest, for its being divisible into parts without a remainder, as two, three, four, six, and twelve; or because, while the Sun seems to run thro' the *Zodiac*, in the space of a Year, there happen near upon twelve Lunations, or the Moon runs thro' all her Phases twelve times. These parts of the *Zodiac* are called *Signs*, and their

their marks are, Aries γ , Taurus δ , Gemini π , Cancer ϵ , Leo α , Virgo m , Libra u , Scorpius m , Sagittarius z , Capricornus w , Aquarius w , Pisces x . Every one of these Signs is again divided into 30 equal parts, so that the whole Circle (like all others) is divided into 360. Tho' the Zodiac or any other Circle has no beginning nor ending, properly speaking; yet Astronomers begin their reckoning from the first point of Aries, which the Sun is seen in when the Days and Nights are equal, and Spring begins to the Inhabitants of the Northern Hemisphere, which also is the point of the common intersection of the Ecliptic and Æquator; as was shewn in *Prop. 33. B. 1.* The reckoning is made from γ to δ , π , and so on, till a return is made to γ , the beginning again towards the same way that the Sun seems to move about the Earth. And the motion or progress this way is said to be *in consequentia signorum*, or according to the order of the Signs; namely, that according to which the Sun, Moon, and almost the rest of the Planets seem to move: But that Phænomenon, which is carried in the contrary order, or from δ to γ , or from γ to x , is said to be moved contrary to the order of the Signs, or *in antecedentia*.

Because the Axis of the Earth after several revolutions about the Sun, goes a little off from a Site parallel to that it formerly had, describing the Superficies of a Cone, the images of the Stars have removed from the Signs of the Zodiac, to which they originally gave names. And this happens not upon the account of the motion of the Stars or the Zodiac, but because the Æquator of the Earth is moved together with the Axis of the Earth, so as that the intersections of the Celestial Æquator, with
the

the Ecliptic, or Equinoctial points, (and consequently all the points of the Zodiac, as they are expreffible by numbers) remove *in antecedentia*; and the images or the Fixed Stars feem to be transferred, in refpect of them, *in confequentia*, as was fhewn in *Prop. 64. B. 1.*

If innumerable Circles be fuppofed to be drawn thro' the Poles of the Ecliptic, cutting the Ecliptic at right Angles, which are called its *Secondaries*, any point in the Heavens may be referred to the Ecliptic, by the help of them; that is, any Phænomenon is underftood to be in that point of the Zodiac or Ecliptic, in which fuch a Semicircle, paffing thro' the Phænomenon, cuts the Ecliptic. And the Phænomena that are after this manner referred to the fame point of the Zodiac, are faid to be conjunct, or in *Conjunction*; but fuch as are referred to the oppofite points, are in *Oppofition*. If a Quarter of the Zodiac lie between the points to which the appearances are referred, they are faid to be in a *Quartile* Afpect, but if a third part of the Zodiac, in a *Trine* Afpect; laftly, if a fixth part of the Zodiac lie between them, they are faid to have a *Sextile* Afpect.

Befides, an Arc of the Ecliptic intercepted between the beginning of Aries and the faid point of interfection, and reckoned according to the order of the Signs, is called, the *Longitude* of that Phænomenon or Point, as the Arc of a Secondary, intercepted between the Ecliptic and the faid Phænomenon or point of the Heavens, is called its *Latitude*; on which account thefe Secondary Circles are called *Circles of Latitude*. The Latitude is either *Northern* or *Southern*; for the Heavens are divided according to Aftronomers, by the Ecliptic, (a Celeftial Circle, becaufe originally confider'd in the Hea-

Heavens) into the Northern and Southern Hemisphere.

PROPOSITION II.

TO give an account of the genesis, nature, and uses of the Celestial Equinoctial and its Secondaries and Parallels, and to explain the Terms used in Astronomy, depending thereon.

If the Plane of the Earth's $\text{\AA}$ equator or Circle, lying exactly betwixt the Poles, upon which the Earth is revolved by its Diurnal Motion, be produced every way, it will intersect the Spherical Surface of the Heavens, concentric to the Earth, in a corresponding Circle in the Heavens, and the Axis of the Earth produced will mark out the Poles in the same Sphere. And tho' the Earth, with its Axe and $\text{\AA}$ equator, be carried about the Sun in the Annual Motion, yet the Circle described by the Earth, is so small, in regard of the Fixed Stars, that the Axis and $\text{\AA}$ equator of the Earth (every where parallel to themselves) being produced, falls upon the same Fixed Stars, as to Sense, and therefore all the other Fixed Stars, that retain the same situation in regard of one another, retain also the same situation in regard of the $\text{\AA}$ equator and Poles of the Heavens, excepting so far as the Axis and $\text{\AA}$ equator on the Earth do not continue parallel to one another; concerning which we shall speak in the following Proposition. And because the Earth revolves upon its own Axis, in the space of a natural Day, from West to East, any Body separate from the Earth will seem to describe a Circle in the Concave Sphere, parallel to the Celestial $\text{\AA}$ equator in the same time from East to West, the Eye being placed in the Earth, and judging its own Habitation to be unmoved; as was explained at large in *Prop. 32. B. I.*

But

But because the Earth is carried about the Sun in its Annual motion, the Earth's *Æquator* continuing always parallel to itself, applies itself only twice a Year to the Sun, so as that the Sun may be found in the Plane of it produced, and consequently that the light of the Sun may reach to both the Poles of the Earth, and every point of the Earth turn'd about in the Diurnal motion, may be as long in the Light as in the Dark, (as was shewn in *Prop. 33. B. I.*) that is, the Sun is seen only twice a Year to describe the Celestial Equinoctial Circle in its Diurnal motion, in which case the Days will be equal to the Nights to all the Inhabitants of the Earth: For the Equinoctial intersects the Ecliptic in the Point in which the Sun appears among the Fixed Stars, when its Light reaches to both the Poles of the Earth, and cause the Equinox through all the Globe of the Earth. For in every other case, the Sun, in its Diurnal motion (like the other Celestial Bodies) seems to describe Circles parallel to the Equinoctial, among which they are the most considerable, that the Sun seems to describe in the Heavens, when the Poles of the Earth do most incline towards the Sun; that is, when the Sun is seen from the Earth in the beginning of the Signs of Cancer and Capricorn among the Fixed Stars; which therefore are called the *Tropics of Cancer* and *Capricorn*, because the Sun returns immediately from thence towards the Equator. There are two other Circles also parallel to the Equinoctial Circle (namely those which the Poles of the Ecliptic seem to describe in their Diurnal motion; and by consequence are as far distant from the Poles of the Equinoctial as the Tropics are from the Equinoctial,) called the *Polar Circles*, either because they

they are described by the Poles of the Ecliptic by the first or Diurnal motion, or because they are near the Poles of the Equinoctial, which are called the *Poles of the World*; because all the World besides the Earth, seems to be moved about them in the space of 24 Hours. The Equinoctial Circle, as usual, is divided into 360 Degrees, and these Degrees are reckoned towards the same parts with the Degrees of the Ecliptic or Zodiac, namely, from West to East, according to the order of the Signs, and from the same beginning; namely, the first Point of Aries. As each Point in the Heavens is referred to the Zodiac by the Secondaries of the Ecliptic, so by the Secondaries of the Equator, it is referred to the Equinoctial. And the Arc of the Secondary comprehended between the Point and the Equator, is called the *Declination* of that Point; *North* or *South*, according as it is towards this or that Pole. For a Point is said to decline just so much as it is distant from the Primary Circle, being the middle and principal one of the Diurnal motion which most affects our Sight. This Distance or Declination, belongs to the Secondaries of the Equator, which are therefore called, the *Circles of Declination*: The chief of which are the two *Colures*, one of which passing thro' the intersections of the Ecliptic and Equator, or points of the Equinoxes, is called the *Equinoctial Colure*; and the other at right angles with the former, is called the *Solstitial Colure*, because it meets the Ecliptic in the remotest Points from the Equator, where the Solstices are celebrated.

PROPOSITION III.

TO give an account of the nature and uses of the Terrestrial Equator, and its Secondaries and Parallels, and explain the Terms used in Astronomy and Geography depending thereon.

The Equator is properly a Terrestrial Circle, because existing originally in the Earth, and transferred to the Heavens only upon the account of the First Motion, (which is only an apparent one in the Heavens.) Its four principal Parallels, the two Tropics, and as many Polar Circles, may be understood to be on the Earth, either as so many Circles lying directly under the Circles in the Heavens of the same name, the generation of which we have shewn in the preceding Proposition; or (which is more natural) as originally generated in the Earth itself. And then the Tropics will be Circles on the Earth, lying directly under the apparent Course of the Sun, when the Pole of the Earth, which is nearest, inclines most to the Sun: But the Polars, such as bound those tracts of the Earth, as have at that time perpetual Day or Night, as was explain'd in *Prop. 33. B I.*

These four Circles parallel to the Equator, divide the Globe of the Earth into five Zones; of which, that is the *Torrid*, which is contained between both Tropics, because lying directly under the course of the Sun, and receiving its direct and, by consequence, most powerful Rays. The Inhabitants of this Zone are called *Amphiscians*, because the Shadow of a Person standing upright, moves as well towards the right as the left hand of him that observes it, and because the Noon Shadow at certain different times of the Year, is projected towards both the Poles. The tracts of Land included within the two
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Polar Circles, make two other *Zones*; both of them *Frigid*; because they receive only the oblique and consequently the weakest Rays of the Sun, that are not therefore able to thaw or dissolve the cohesion of the Ice, that froze during the Winter and long Night-time, in the following Summer, or as long a Day-time. The Inhabitants of these *Zones*, in regard of the Shadow, are called *Periscians*, because the shadow (the Sun not setting) moves round about them. Between the *Torrid*, which is the middle, and the two *Frigid*, which are the extreme *Zones*, lie two others, call'd the *Temperate Zones*, because partaking of the affections of both the adjacent extremes; the *Northern* of which is bounded by the Tropic of Cancer and the Arctic Polar Circle; and the *Southern*, by the Tropic of Capricorn and the Antarctic Polar Circle. And tho' the Ancients said that these only were temperate and habitable, yet experience now informs us, that both the *Frigid* and *Torrid* Zones are fit for the nourishment of Animals, and Vegetables; several other Causes, partly physical, and partly Astronomical, making compensation for the too direct or oblique incidence of the Rays. The Inhabitants of the *Temperate Zones* are called *Heteroscians*, because their Noon shadow is thrown only towards one Pole, viz. the nearest, and carried about only towards one quarter, as it is with us, who inhabit the Northern Temperate Zone, and see our own shadow projected towards the right and.

Again, such as live upon the Equator, have their Days and Nights always equal, because as well the Equator as the Circle distinguishing the enlightened part of the Earth from the obscure are great Circles of the Sphere; and therefore, (by Prop. ii. Book I. Theodos.)

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they biseſt one another; conſequently, each Point in the Equator carried round about equally in its Diurnal motion, is as long in the light as, in the dark. But to an Inhabitant living without the Equator towards either Pole, the Days are longer than the Nights, when the Pole of the Earth, that is neareſt inclines towards the Sun, and are then the longeſt, when that Pole inclines moſt towards the Sun: And the farther an Inhabitant lives from the Equator, the longer are the longeſt Days to him, till at laſt at the Polar Circle in Summer, there is no Night at all; as has been ſhewn at large in *Prop. 33. B. 1.* This is the reaſon why *Ptolemy* and other Geographers, divided the Earth by Circles *Parallel* to the Equator, diſtant from one another, and their beginning (*viz.* the Equator,) ſo much as that the longeſt Day is a quarter of an Hour longer in one *Parallel* than in the other next *Parallel*. They take no notice of any diviſion into leſſer parts, becauſe they would be ſcarce ſenſible; and they looked upon this ſpace (tho' properly a little Zone) as indiviſible in regard of Latitude, and therefore they call'd it a *Parallel*, Circle being underſtood. Making the Equator then to be the firſt *Parallel*; the ſecond paſſes thorough thoſe parts of the Earth, where the longeſt Day is $12\frac{1}{4}$ Hours; the third thorough them, where the Day is $12\frac{1}{2}$ Hours, and ſo on; and in the other Hemisphere after the ſame manner. Two ſuch ſpaces make a *Climate*, which therefore differ from one another the length of half an Hour. Each Climate receiv'd a name from ſome conſiderable Place lying in it near its middle: and therefore there muſt be 24 of them reckon'd from the Equator to either Polar Circle, ſince the greateſt or longeſt Day is 12 Hours longer. But the Ancient

cients did not begin this reckoning from the Equator itself, but made the first Climate to pass thro' the Island *Meroe*, where the longest Day was found to be an whole Hour longer than 12 Hours, either because they left this Region near the Equator for a right Sphere, in regard of which, the other Regions on the sides of it, are called *Κλίματα*; or because perhaps they judged the interior parts to be uninhabited: Tho' *Ptolemy* says, there were several in his time, who maintain'd, that there were habitations under the Equinoctial itself, just as it were in a temperate Region; but the People of his Age, had not to that Day penetrated thither. *Ptolemy* has not given us an exact computation of the Parallels, so far as to the Polar Circle itself, they being more numerous thereabouts, and he not thinking it worth his while to be over nice in his enquiries in the more Northern Parts. As for the Climates on the other side of the Equator towards the Antarctic Pole, he has given each of them such a name, as signifies that this unknown Climate is as far distant from the Equator towards the South, as certain known one is from the same Equator towards the North; as the Climate *Ἀντι-δι-α-με-ρῆς*, *Ἀντι-δια-με-ρῆς*, &c.

As the Places of the Stars, or any Points in the Heavens, are computed as to their Longitude and Latitude, by the means of the Ecliptic (which is properly a Celestial Circle) and its Secondaries; so the Longitudes and Latitudes of Places on Earth, are computed in the Equator (which is properly a Circle on the Earth,) and its Secondaries passing thro' the Poles of the Earth. Any such Secondaries drawn thro' any place upon the Earth, is called the *Meridian* of that Place, because when the Earth has in its

Diurnal revolution arrived to such a situation, as that the Sun is in the Plane of that Circle produced, it is Noon Day; as shall be more largely explain'd hereafter. The *Latitude* of any Place is an Arc of the Secondary of the Equator, or of a Meridian, intercepted between the Equator and the Place: And it is either *North* or *South*, as the Place is distant from the Equator towards the North or South Pole. But the *Longitude* of a Place is computed from the West towards the East in the Equator itself, because the boundary of the Land towards the West is better known than that towards the East. And because the Equator has no beginning, nor any Cardinal Point, as there is in the Heavens, marked out by its intersection with the Ecliptic, the first Meridian, from whose intersection with the Equator the beginning of the reckoning is to be made, was left to the liberty of the Geographers to be settled. Now, they allow of no dry and habitable Land besides the Continent they themselves inhabited, together with the adjacent Isles; they feigned therefore a first Meridian passing thro' the most Western Place of the Earth, that was then known, (as the *Azores* or *Fortunate Islands*; and from thence towards the East, they disposed of all Places on the whole Surface of the Earth, in regard of Longitude; and called the *Longitude* of the Place, that Arc of the Equator which was intercepted between the *First Meridian* and the *Meridian* of the Place. But when they found the Globe of the Earth was really Inhabited all about, and there was no such thing as a most Western Place, the above-mention'd way of reckoning the Longitude of Places was by degrees neglected, which would have been of great use in understanding the Monuments of the Ancients; (tho' it might still

still be made use of;) and every one made the Meridian of his own City to be the chief, and took notice how the Meridians of other Places situated towards the East or West stand in regard of this, or how many Hours there are between the more Eastern Meridians leaving the Sun, and the more Western Meridians overtaking of it, or between the Sun's seeming to leave the more Eastern Meridian, and arrival at the more Western.

Again, the Inhabitants of the Earth being compared together in regard of the Meridians and Parallels; some of them are called by Geographers the *Periæci*, and they inhabit in the same Parallel of the Earth, but in opposite Meridians. (The opposite Meridians are the opposite halves of the same Circle computed from the Poles, because such Places as are in such opposite Semicircles are referred to opposite points of the Equator of the Earth.) On this account the *Periæci* enjoy the same Seasons of the Year, by reason of the inclination towards the Sun of that Pole of the Earth which is nearest to both, or its declination from the Sun; or speaking according to the apparent rest of the Earth, because the Sun approaches after the same manner towards the Parallel of both Places, (because the same) or recedes from it: But they have alternate changes of Noon and Midnight, according as the Meridian of the one or the other is turned to the Sun by the diurnal revolution of the Earth, or (which is all one,) as the Sun carried about us in its apparent diurnal revolution, approaches to the Meridian of the one or the other of them; unless they live in a Frigid Zone, where they enjoy the Day together. Again, other Inhabitants of the Earth are called *Antæci*, and they live in the same Meridian,

dian, but in opposite Parallels, so that they have the same Noon and the same Midnight; because they have the same Longitude. But the Summer of the one is the other's Winter, according as the Earth in its annual motion turns sometimes its North Pole, and afterwards its South Pole, more towards the Sun; or as the Sun declines from the Equator to this or the other Pole. And lastly, others are call'd *Antipodes*, because they live in opposite Meridians and Parallels, and walk with their Feet diametrically opposite to ours. These have exactly the contrary things happening to them: The one has Summer and long Days or no Nights, at the same time as the other has Winter and short Days or perpetual Nights; 'tis Night here when 'tis Day there, and Night there when 'tis Day here. 'Tis evident an Inhabitant of the Equator, is an Antœcian to himself; and that the same Person is both Periœcian and Antipode to him; but that an Inhabitant of both the Poles is a Periœcian to himself, and that his Antœcian is the same as his Antipode.

PROPOSITION IV.

TO give an account of the generation and nature of the Horizon and its Secondaries and Parallels, and to explain the Terms in Astronomy depending thereon.

Besides the lately mention'd Circles of the Equator and Ecliptic, in respect to which, Astronomers have determined the Places of the Stars viewed from the Earth, there is another called the *Horizon*, which is the great Circle, that every one standing in an open Plain, defines by turning of his Sight round about, and that divides the visible from the invisible part of the Heavens. This Circle is the most considerable

rable of all in a Sphere to appearance; because immediately taken notice of and determined by any Observer, tho' the most unskilful. Its genesis in the true System, has been deliver'd above in *Prop. 32. B. 1.* But because the Spherical Superficies, to which we referred all Celestial Phænomena, is supposed to be concentric with the Earth and not the Eye, a Plane that passes thro' the Eye and touches the Earth, will not divide it into equal Segments; therefore the Horizon there described, generated by the Section of the said Plane and Sphere, is properly called the *Sensible Horizon*; because defined by Sense: And the *Rational Horizon* of this Spectator, is that which is made by the Section of the said spherical Superficies, by a Plane parallel to the Sensible Horizon, and passing thro' the center of the Earth. These two parallel Planes produced, will mark out the same Circle in the Superficies of that very great Sphere, wherein the Fixed Stars are seen, because the Earth compared to the Sphere of the Fixed Stars, is but like a Point. From whence 'tis evident, that the Horizon considered among the Fixed Stars is a great Circle in a Sphere, every way equidistant from the Point, exactly over the Head of the Person (whose Horizon it is,) called the *Zenith*, and from the opposite to it the *Nadir*: Which Points therefore, namely the Zenith and Nadir are the Poles of the Horizon, in which the Secondaries of the Horizon, drawn thro' all the points of the Heavens, cross one another, which are therefore called *Verticals*, and sometimes *Azimuths*; but the Parallels of the Horizon, whether supposed to be above or below, towards the Zenith or Nadir are called *Almicanters*. There are two of these Vertical Circles, that are most considerable, the one passing thro' the Zenith (the com-

mon Node of all the Verticals) and the Poles of the World, call'd the *Meridian*; namely the *Celestial*, because being directly over the Terrestrial Meridian: This Circle intersects the Horizon in the Cardinal Points of South and North, and marks them out. The other chief Vertical is at Right angles to this, and intersects the Horizon in the points of East and West; and because the former, tho' a Vertical, is called likewise by another name, *viz.* a Meridian, this latter has the name of the *Primary Vertical* left intirely to itself. By the help of the Horizon and its Secondaries, any point of the Heavens whatever is disposed of according to its Altitude above or Depression below the Horizon, and Azimuths. That is, the *Altitude* or *Depression* of any Point is an Arc of a Vertical Circle intercepted between the same Point and the Horizon: And the *Azimuth* is an Arc of the Horizon intercepted between the North or South Cardinal Point and that Point wherein the Vertical drawn thro' the Phenomenon, meets the Horizon; which is *Eastern* or *Western*, according as you reckon from the Meridian towards the Eastern or Western part of the Heavens. Sometimes the Azimuth is reckoned from the Eastern or Western Cardinal Point towards the North or South. But the *Eastern* or *Western Amplitude* is always reckoned from these Points, which is an Arc of the Horizon, reckoned from the East or West to the Rising or Setting point, and therefore both may be *Northern* or *Southern*; tho' this way of naming them seems to have flown first from intire Constellations; and the question was, how large a space of the Horizon any Constellation took up in Rising or Setting.

That portion of the Convex Surface of the Earth, that the Spectator stands upon, being taken as a Plane parallel to the Horizon; the several Quarters of the Winds are consider'd in it, and the Cardinal ones are the North, South, East and West; lying under the Meridian and Prime Vertical, described above, and are the Sections of the Horizontal Plane with the Planes of the said Circles. The Motion of the Heavens will point out to us these four Quarters; namely the East, where the Sun rises in the Equinoxes; the West, where the Sun sets that Day: The North, where the Pole of the World is seen by us the Inhabitants of the Northern Hemisphere of the Earth, and the Stars call'd the *Septem Triones*, always appear: And lastly, the South, whence the Sun shines upon us at Noon. For the Terms in Astronomy are accommodated to such tracts of the Earth as are situated in the Northern Temperate Zone, because Astronomy was first cultivated in those parts. There are as many lying exactly between the four Cardinals, that have names (in the *English* and other Tongues a-kin to it) made up of the Cardinals that are on their sides, so as that the name of the chief Cardinal Point is set first. Betwixt these eight are placed eight more, having Names made up of the Names of the eight preceding, so as that each of them is made up of the two laterals next it, and the principal Cardinal set first: Hence it comes to pass, that the name of the chief Cardinals is presently doubled, and the names of the other Cardinals in the beginning and end of the Word thus compounded. Between the sixteen Points named, there are sixteen others having compound Names, each from one of the eight first, to which the name of the Cardinal Point towards

which

it declines, is connected by a Proposition ; and thus we have two and thirty Wind's Ways, or Points, which are the common interfections of as many Verticals with the Plane of the Horizon, and serve to distinguish the Winds accurately enough. But we use the Degrees of the Horizon, in reckoning the Azimuths of the Celestial Phænomena, beginning from one of the four Cardinal Points, as was said above. Besides these Points, (while they are considered in the Plane of the Horizon) are looked upon as Right lines ; but if the surface of the Earth be looked upon as Spherical (as it really is,) none of them produced upon the surface of the Earth are Right lines, and only the Cardinals are Circles ; namely, one pointing out the North and South, being a great Circle, and the same with the Meridian of the Place, from whence it takes its beginning ; the other shewing the East and West, being a lesser Circle, passing thro' the Place, and parallel to the Equator ; excepting when the Place is in the Equator itself, in which case it is a great Circle ; because the Equator itself. The other Points of the Compass being produced upon the surface of the Earth are Spirals *sui generis*, cutting all the Meridians at given Angles, and are called *Rhumbs* or *Loxodromics*.

PROPOSITION V.

TO explain the generation and nature of the Celestial Meridian and other Hour Circles.

In *Prop. 3*, we explain'd what sort of a Circle the Meridian of each Place considered upon the Earth was ; namely, a Circle compassing the Earth and passing thro' the said Place and both the Poles ; and consequently a great Circle, because passing thro' the opposite Points. And the Celestial Meridian of the same Place is that
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in the Heavens, which lies directly over and answers it. And because the Observer continuing in the same place, looks upon the Earth, and therefore the place where he stands, and the point over his Head as unmoved, he likewise conceives of the Celestial Meridian passing through the Poles and the Vertex of the Place, which are unmoved, as a Celestial Circle unmoved: And seeing the Heavens and all the Stars seem to move round in the Diurnal motion, he conceives of the Meridian of the Place where he is, as what has no share of that motion, and as it were, without the movable Heavens, and of the Heavens, as if they were revolved within it. And it is always Noon at that place, when the Sun, by the Diurnal revolution of the Heavens, seems to have arrived at the above-mentioned immovable Celestial Meridian above the Horizon; and Midnight, when the Sun has reached the other part of it below the Horizon. For since this Celestial Meridian passes thro' the Poles of the Horizon and Equator, and (by *Prop. 15. B. I. Sphæric. of Theod.*) intersects the Horizon and the Equator together with its parallels at right angles; 'tis evident that the Meridian divides the Segments of all the Circles parallel to the Equator made by the Horizon, into equal parts; and therefore since the Sun describes in its Diurnal motion, one of these Parallels, (the Arc above the Horizon in the Day-time, and the Arc or Portion of it below the Horizon in the Night,) 'tis evident that Mid-day or Noon is then made, when the Sun comes to the Meridian above the Horizon, and Mid-night when it arrives at its opposite part, lying concealed below the Horizon: from whence it has its name. And for the same reasons the middle point of the continuance of any Star above or below

below the Horizon happens, when that Star comes to the Meridian: Where, at the same time, it has its greatest elevation, call'd, its Meridian Altitude.

Again, since the space of time between the two nearest Noons is suppos'd to be divided into 24 equal parts, (called *Hours*,) and by reason of the equable Revolution of the Earth about its own Axis, the Sun seems to describe equably the Equator, or some parallel to it upon the Poles of the Celestial Equator; besides the Meridian, there are eleven other *Hour Circles* to be conceived of, passing thro' the Poles of the Equator, and together with the Meridian, dividing the Equator into 24 equal parts; and these are look'd upon (in regard of the same place) as immovable, and like the Meridian, placed without the Sphere, the Heavens in the mean while revolving equably under them. From whence it is evident, that an Arc of the Equator intercepted between any two next of these Circles is 15 Degrees, that is, 24th part of an intire Circle. But when the Terrestrial Meridian is changed, the Observer changes all his Hour Circles with it: They are called Hour Circles, because the Sun, seen to have arrived at any of them, by its apparent Diurnal Motion, causes the Hour of the Day to be that before or after Noon, which this Circle is in order, more Eastern, or more Western than the Meridian. An infinite number more of these sort of Circles may be imagined, according as we suppose the Hour to be divided into 60 Minutes, each of which is again divided into 60 Seconds, and so on. These Hour Circles are the same in position, with the Circles of Declination; (of which in *Prop. 2.*) because they are Secondaries to the Equator: But they differ in

in this, that the Circles of Declinations revolve, together with the Stars and Points of the Heavens, whose Declination they measure; but these Hour Circles (as was said before) are look'd upon as immovable. Hour Circles also in the Heavens, answer to Meridians on the Earth; and indeed the Meridians on the Earth, are the real Hour Circles, and are only apparent ones in the Heavens. For as 'tis Noon in a given Place, when the Earth, revolving by its Diurnal Rotation, arrives at such a situation, as has the Plane of the Meridian produced falling upon the Sun: so it is such or such an Hour, before or after Noon, as that Terrestrial Meridian, in order from the Meridian of the Place, is, in whose Plane produced the Sun is then found.

PROPOSITION VI.

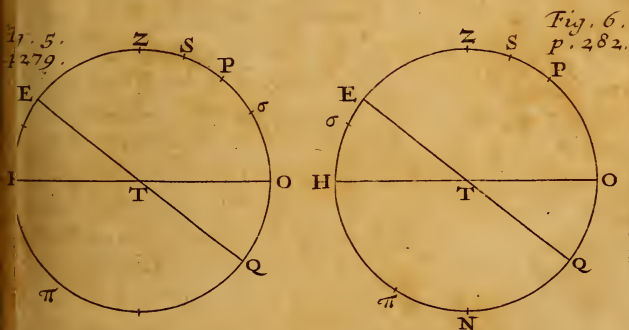
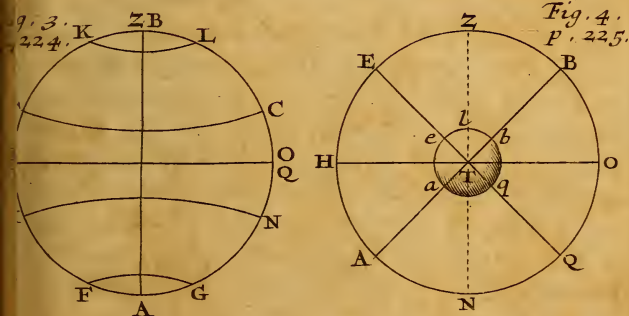
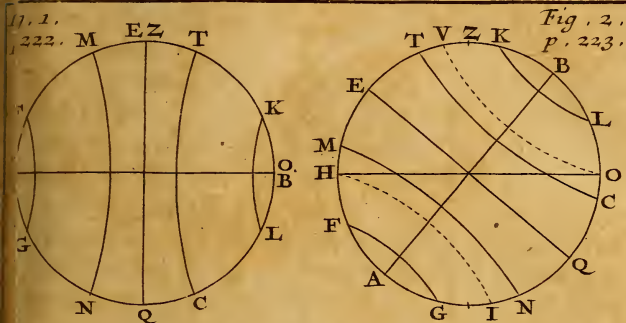
TO explain the various appellations of the Sphere of the World, and other Terms used in Astronomy, depending upon the various inclination of the Horizon to the Equator.

Since according to the diversity of Places upon the Earth, the Horizon, and consequently the Face of the Heavens, (that is, the Sphere of the World,) is different; Astronomers make this diversity threefold, according to the three different kinds of Position of the Horizon to the Equator. For either the Horizon of the Place is right to the Equator, cutting it at right angles, or it is oblique, or else does not cut it at all, but coincides with it. Such as have the first Position of the Sphere, are said to inhabit a *Right Sphere*; such as have the second, an *Oblique*; and such as have the third, a *Parallel Sphere*, because each Star seems to describe a Circle, parallel to the Horizon in its Diurnal Motion.

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In a right Sphere, (namely, where the Equator EQ is erect or perpendicular to the Horizon HO , as in the 1st Figure,) the Equator (by *Prop. 15. B. I. Theodosius*) will pass thro' the Poles of the Horizon, the Zenith, and Nadir; the Place therefore itself, on the Surface of the Earth, will be in the Terrestrial Equator; for such only have the Zenith and Nadir, in the Celestial Equator; all such therefore, inhabit a Right Sphere, and only such as are spread along the length of the Terrestrial Equator. Their Horizon passes thro' the Poles of the World A and B (by the abovesaid Proposition of *Theodosius*); and every Point in the Heavens will seem to ascend right or perpendicularly above the Horizon; because it describes a Circle parallel to the Equator, (which is right or perpendicular to the Horizon,) in its apparent Diurnal Motion. And the Points, that rise together, come to the Meridian also together, and set together; because the Poles, about which the Diurnal Motion is made, by which the Stars seem to rise, come to the Meridian, and set, are as well in the Horizon as in the Meridian of this Sphere. And from this situation of the Sphere, the way of reducing the points of the Heaven to the Equator, derives its original. That is, the *Right Ascension* of any point in the Heavens is an Arc of the Equator, reckoned towards the East, intercepted between the beginning of Aries, and the point of the Equator that rises, together with the said point in the Heavens, in a Right Sphere: After the same manner we are to understand what the *Right Descension* is: and instead of the Horizon of a Right Sphere, the Meridian of any Place may be assumed. Now, because only the Inhabitants of the Terrestrial Equator have a Right Sphere,

Plate 1, Book 2.





Sphere, what the affections of this Sphere are, is evident from what has been said above; (especially in *Prop. 33. B. 1.*) namely, that the Nights are always equal to the Days, and every point of the Heavens is as long above as below the Horizon.

In an Oblique Sphere, where the Horizon HO (*Fig. 2.*) cuts the Equator $E\mathcal{Q}$ at oblique Angles; neither of them passes thro' the Poles of the other, and therefore one Pole of the Equator or World is above the Horizon, the other below it; the former therefore is always visible, but the latter always invisible: Nor will the Equator pass thro' the Vertex Z , but the Vertex will lie between the Equator and the visible Pole. There are two kinds of this Sphere; for either the North Pole B is elevated above the Horizon HO , and the South lies unseen; or on the contrary, the South Pole is elevated, and the North depressed. All such have the Sphere of the former kind, as live between the Terrestrial Equator and the Arctic Pole, and such as live between the Equator and Antarctic have the latter. The chief Phenomena of both of them, as to Summer and Winter, may be seen, already explained, in *Prop. 33. B. 1.* according to the true System of the World. And in the apparent System, since each point in the Heavens, by its equable diurnal Motion describes either the Equator or a Parallel to it; and only the Equator, but not one of the Parallels is divided into two equal parts, by an oblique Horizon; the Sun, and all the Stars, that decline towards the visible Pole, continue longer above the Horizon than below; and on the contrary, such as decline towards the invisible Pole, continue hid longer below the Horizon, than they appear above the Horizon.

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And this holds to a certain limit towards both Poles. For if the declination be increased till the distance of the Star from the visible Pole, is less than the elevation of the Pole above the Horizon, the Star does not set at all. But if its Distance from the invisible Pole be less, it will not ascend above the Horizon; that is, it will not rise at all. And the Parallels OV , HI , bounding the always visible and the invisible Stars in a given Place, and touching the Horizon, are called, by some of the Ancients, (as *Euclid* in the *Phænomena*, and *Manilius* in his *Astronomy*) the *Polars* of that Place, which are therefore greater and more remote from the Celestial Pole, according as the Place, whose Polars they are, is farther off from the Terrestrial Equator; that is, as the Sphere is more oblique. All the Stars that lie between these Polars have the vicissitudes of Rising and Setting, Ascension and Descension. The *Oblique Ascension* of a Star or any Celestial Point, is an Arc of the Equator, reckoned towards the East, intercepted between the beginning of Aries, and that point of the Equator that rises together with the Star, in a given oblique Sphere: and this is various according to the variety of obliquity of the Sphere. And the *Ascensional Difference* is the difference betwixt the right and oblique Ascension. And the same things are to be understood concerning the *Oblique Descension*.

In a Parallel Sphere, where the Horizon HO , [*Fig. 3.*] coincides with the Equator EQ , and makes one of the Parallels of the Diurnal motion, their Poles also (namely, the Zenith and Pole of the World) will coincide; and therefore this position of the Sphere agrees only with the Inhabitants of the two Poles of the Earth; and all the Celestial Bodies or Points,

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that describe Circles parallel to the Equator by the diurnal motion, will likewise describe Circles parallel to the Horizon. Upon this account, there will be no rising or setting at all by the diurnal motion, and therefore no Ascension or Descension in this Sphere, nor any Meridian, since it passes thro' the Pole and Vertex of the Place; and since these two coincide in this Sphere, they can't determine the Circle, each Vertical having an equal right to claim the name of a Meridian. The Phenomena of the Sun arising from the Annual motion of the Earth, visible in a parallel Sphere, are explained in *Prop. 33. B. 1*; namely, such as agree to the Inhabitants of either Pole. In the apparent System 'tis evident, that the Sun, during its declining from the Equator towards that Pole, which is in the Zenith, is always seen, and makes the Day half a Year long; as on the contrary 'tis half a Year Night, while the Sun declines towards the other part of the Equator, and consequently of the Horizon; and that the beginning of the Day and Night falls upon the Sun's ingress into the Equinoctial Points.

PROPOSITION VII.

THE Latitude of the Place is an Arc similar to the Arc of the Elevation of the Pole above the Horizon.

Let *aebq* [Fig. 4.] be the Earth, its Center *c*, Poles *a* and *b*, Equator *eq*. Let any Place on the Surface be taken, as *l*, whose Latitude is *le*, an Arc of the Meridian intercepted between *l* and the Equator. Let the Celestial Sphere be concentric, and its Poles, Equator and Meridian, *B, A, EQ*, and *BEAQ*, answer to the Poles, Equator and Meridian of the Earth. Let *Z* be the Zenith, and *N* the Nadir of the

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Place

Place l ; namely in the intersections of the Right line TL produced, with the Superficies of the Celestial Sphere. Therefore the great Circle HO , in the Sphere, described upon the Poles Z and N , is the Horizon of the Place; and the Arc of the Meridian OB , is the height of the visible Pole B above the Horizon; and this, I say, is similar to the Arc le , namely, to the Latitude of the Place. For, since A and B are the Poles of the Circle $E Q$, the Angle BTE is right; (by *Prop. 10. B. 1. of Theod. Sphar.*) and for the same reason, the Angle ZTO is right. If therefore from the equal Angles ETB , ZTO , you take the common Angle ZTB , the remaining Angles ETZ , BTO , will be equal. And therefore (by *Prop. 33. Elem. 6.*) the Arcs $e l$, BO , they stand upon at the Center, are similar.

PROPOSITION VIII.

TO explain the Causes of the Crepusculum or Twilight; and to define its Limits.

The Twilight is that dubious light, which we have before the Sun rises and after the Sun sets. If there were no Atmosphere about the Earth, nor any brightness from the *Aura Etherea* that is near the Sun, so soon as by the diurnal motion of the Earth any place upon its surface comes within the shadow of the Earth by the diurnal motion of the Earth, or so soon as the Sun descends below the Horizon of that Place, there would be nothing but mere Darkness; because the Spectator being forsaken by the Rays of the Sun, is left destitute of Light. But not only the Sun, but the *Ethereal Aura* that is spread about the Sun very nearly to it (and its inflamed Atmosphere as it were) do also shine and give some light: Now, this spend

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ing more time than the Sun does in rising or setting, before Sun rise the Aurora shines out in a manifestly Circular-figure, rising into the brightness of the same Figure with that of the Segment of the Circle, that is already risen of the Atmosphere of the Sun, and intirely different from that, which proceeds from the illumination of the Atmosphere of the Earth, made by the Sun: Which is to be understood in like manner concerning the Twilight after Sun-set. Because the matter that thus shines, by reason of the nearness of the Sun, shines sometimes more, sometimes less, the boundaries of the Twilight, which arises from thence are not so certain, especially when it acts in conjunction with the other and more powerful cause of this dubious Light. For after that the Inhabitant of the Earth standing upon its surface something beyond the bounds of Light, is revolved into the shadow of the Earth, the Atmosphere that surrounds the Earth and is extended to a considerable distance above the Earth, is still inlighten'd, and does also inlighten the Place where the Observer stands with its reflected Rays. This cause is often upon the change; according as more or fewer Particles fit for reflecting or other ways conveying the rays of the Sun to us, are found suspended in the Air; and according as these Particles get up to a greater or less height, which depends upon the gravity of the Air (it being the Fluid in which they swim,) that is shewn by the Barometer. For if they hang low and very near the surface of the Earth; even these also soon cease to receive any of the Rays of light, being revolved presently afterwards, together with the Place over which they hang, into the Earth's shadow. If they are ei-

ther very rare as they float in the expanded space, or unfit to reflect the Light, they will return to us either such a Light from the Sun as is nothing to speak of, or at least so weak and thin as not capable of affecting our Sight, as we really experience in all the expanded space that lies without the Earth's shadow; for tho' it lies perpetually open to the Sun's Rays, yet it sends back to us so weak and faint a Light, that it scarce deserves that name.

Tho' the duration of the Twilight depending upon both these causes is various, yet 'tis certain, that the beginning of the Morning Twilight happens generally about the time that the Sun is not above 18 Degrees below the Horizon; and that the Evening Twilight ends about the same time, or when the Sun is got to the like degree of depression below the Horizon. *Tycho* would have this depression of the Sun that bounds the Twilight to be 16 Degrees. Others extend it to 19; that is, till the least Fix'd Stars become visible. *Cassini* from his own Observation, extends it only to 17 Degrees. *Ricciolus* finds by his Observations, that 'tis not the same in the Morning as it is in the Evening and that it is different in different Seasons of the Year.

SCHOLIUM.

There is a Light that seems very near akin to the Twilight, which was first observed by the Quick-sighted Mr. *Cassini*, in the Year 1683; a little before the Vernal Equinox in the Evening, and extended along the Ecliptic from the Sun towards the East. The Observations made afterwards by Mr. *Cassini* and *Fatio*, evince that this Light is diffused from both sides of the Sun almost along the Ecliptic; but ordinarily deflects a little from it towards the North rather than

than the South. Its form is pointed on both sides; its two points being sometimes more, sometimes less distant from the Sun, at first about two Signs or a little more; but three Years after, its distance from the Sun encreased to three Signs or 100 Degrees: Its breadth is above 30 Deg. near the Horizon. But it can't be seen where it is broadest; namely, just at the Sun it itself along the Circle of its breadth. Its sides are streight, bating its being gibbous a little sometimes about the middle, between the Sun and either Point, and they are inclin'd to one another, sometimes at a greater and sometimes a less Angle. This Angle in its mean bigness is about 21 Degrees. From the whole, it is evident, that this Luminous Phænomenon moves together with the Sun, just as that does, thro' the Ecliptic; which is also true of both the Points, allowing for their greater or less distance from the Sun during the increase or decrease of the whole. The brightness of it is much like that of the Milky Way, or the Tail of a Comet, and is pellucid like the latter; 'tis greater in the middle, less towards the extremes, and sensibly decreasing, till it vanishes in the surrounding Blue of the Heavens: From hence it appears to different Observers, according to the sharpness of their Sight, at the same Time and Place, of a different magnitude, and according as the Heavens are clear or cloudy, and the true Twilight and brighter Stars are present or absent, it is more or less extended and variously terminated, and appears fainter always in the Morning than in the Evening. This light, by reason of the Twilight lasting all Night long, can't be seen in the midst of Summer in the Regions about either Pole; but in the Morning and Evening of the the same Day

it may be seen about the middle of Winter (if the Moon be away.) In places near the Equator it may be seen at any time of the Year. And in any place it is brighter, the more erect it is, in regard of the Horizon; because it is then more out of the Twilight and Vapours that hang about the Horizon: And therefore it is best seen in these Northern Regions in the Morning after the Vernal Equinox, in the beginning of *October*; and in the Evening, at the end of *February*; the Ecliptic along which it is spread, being then more erect in regard of the Horizon, (in the beginning and end of the Twilight, which is then shortest) the Equinoctial Points being at that time in the Horizon.

The Body (or rather collection of Corpuscles, whose appearances these are, seems to surround the Body of the Sun in the form of a Lens; as Mr. *Fatio* conjectures. The Plane thro' the edge of the Lens is in the Plane of the Ecliptic or at least not far from it: The Edge itself is between the Orbits of Venus and the Earth, but nearer to the latter. The Particles which fill up this lentiform space (like the particles of the Sun's Atmosphere, of which we spoke but just now) seem to produce the abovemention'd appearance, by reflecting the light of the Sun.

For a Ring whose opposite Faces are plane and parallel, would appear in the form of an Ellipse, more obtuse towards the Points than this Phenomenon. Mr. *Cassini* takes these particles reflecting the Sun's light, contained in this space, for an infinite number of Planets, since being separated, they would, like the Planets, exert their motions about the Sun; by which method, this morning and evening light owes its original to innumerable Planets, just as the Milky Way does to innumerable Fix'd Stars.

Mr. *Fatio* suspects that this light is coeval with the World : *Cassini* on the contrary, that it was produced a little before the first Observation he had made of it; and that it was not in being two Years before, since he had often viewed a Comet very intensely with his Eyes, in the same place as this Phenomenon ought to possess. That it has appear'd formerly, and afterwards disappeared, he thinks is very probable from some ancient Histories that seem to give a description of it; but especially from the Observation of Mr. *J. Childrey*, who no doubt saw it, as is evident from his description, pag. 183, 184. in the Advertisement at the end of a Book, published Anno 1661. intitled *Britannia Baconica*; because it appears even now in the same place, and near the same Constellations, as it did at that time of the Year. His Words are these: *There is something more, that we would recommend to the Observation of the Mathematicians; namely, that in the Month of February, and a little before and after it (as I have observed for several Years) about six a Clock in the Evening, when the Twilight has entirely left the Horizon, a Path of Light tending from the Twilight towards the Pleiades, and touching them as it were, presented itself very plainly to my Sight. This Path is to be seen whenever the weather is clear; but best of all in a Night when the Moon does not shine. And a little after: I am apt to believe that this Phenomenon has been formerly, and will hereafter appear always at that above-mentioned time of the Year. But the Cause and Nature of it, I can't so much as guess at; and therefore leave it to the enquiry of Posterity.*

PROPOSITION IX.

TO explain what is meant by the Poetic Rising and Setting of the Stars, and the kinds of it; namely, the Cosmical, Achronical and Heliacal

In the foregoing Propositions we have treated of the true Rising and Setting of the Stars; namely of their ascent above the Horizon of a certain Place, and descent below it, or rather of a depression of a given Horizon below a Star, and elevation of the same above it; and that without any consideration of the Sun. But now the Rising and Setting of the Stars are to be compared with the motion of the Sun along the Ecliptic, and consequently with the Daylight and Seasons of the Year. For the ancient Husbandmen, and from their traditions the Writers of Husbandry, as also Physicians, Poets and Historians, have made use of these marks to express the Seasons of the Year by, and consequently none of them can be understood without an explication of these Terms.

There are usually reckoned three kinds of the Poetic rising and setting of a Star, The *Cosmical*, *Achronical* and *Heliacal*. A Star is said to rise *Cosmically*, when it rises together with the Sun, but to set *Cosmically*, when it sets when the Sun rises; so that the *Cosmical* rising and setting is all one with the Morning rising or setting, as if the beginning of the Artificial Day, or the Rising of the Sun, were the same with that of the World. A Star is said to rise or set *Achronically*, that rises or sets when the Sun sets; and consequently the *Achronical* rising or setting is all one with the Evening ones. *Kepler* maintains that these words are to be taken in another sense; namely, so as that to rise and set *Cosmically*, signifies the same as to ascend above

above or descend below the Horizon; but that to rise or set *Achronically*, is the same as to rise or set in the Sun's opposite, or in the other *Achron* or extreme of the Night: In which sense *Ptolemy*, and to this day most Astronomers say, a Planet is *Achronical*, when it is opposite to the Sun and bright all the Night: So that to rise *Achronically* is, as usual, the same as to rise when the Sun is setting; but to set *Achronically*, is to set when the Sun is rising; which is commonly called the *Cosmical* setting.

A Star rises *Heliacally*, when it before lay hid under the Rays of the Sun, so as that it did rise and set together with the Sun, but now gets so much out of the Rays of the Sun, as when the Sun is below the Horizon the Star becomes visible, being to set soon after under the Horizon, or to disappear by the supervention of the Daylight. That Star is said to set *Heliacally*, which was lately seen above the Horizon when the Sun was not much below the Horizon, but now has hid itself so much among the Rays of the Sun, that the next Day being risen above the Horizon, or just setting below it, it can no more be seen. If there were no Twilight, this *Heliacal* rising and setting of the same Star (or rather Apparition and Occultation,) would not be above one Day distant from one another at most. For in that case a Star would be seen either before Sun-rise, if its *Oblique Ascension* in a given Horizon were never so little less than that of the Sun, or it would be visible after Sun-set, if but never so little greater. Nay, if there were no Atmosphere at all, the smallest Stars would be visible even in the Daytime when the Sun shines. For the reason why they don't appear is this: The particles of the Atmosphere enlightened strongly by the Sun, affect the Eye
of

of the Spectator with so vivid a light, that the Retina (or whatever it be that is the Sensor of Sight) is scarce moved by the very weak image of the Star, and consequently can't take notice of it or see. But if there were no Atmosphere spread about the Earth, nor any surrounding Terrestrial Bodies that reflect Light; if the direct Rays of the Sun were turn'd off from the Eye, the Eye thus free from the Rays of a Body either very bright in itself, or strongly enlightened, would see clearly (as it does now in the Night) even the smallest Stars, unless they are hid under the Atmosphere of the Sun: For this together with the Body of the Sun, make up one lucid Body. Since proportionable to the lesser brightness of a Star, there is need of a greater depression of the Sun below the Horizon, to make it visible; 'tis evident that the Heliacal setting of the Sun is sooner, and the rising later. That the least Stars may be seen, the Twi-light must be quite over, or the Sun must be full 18 Deg. below the Horizon: For the seeing of a Star of the sixth Magnitude, the Sun must be 17°; and soon, till you come to the Stars of the first Magnitude, which are to be seen in that part of the Horizon which is towards the Sun, when the Sun is 12°. The Planets shine with a brighter and fuller light, and therefore have no need of so great a depression of the Sun below the Horizon, so that 11 Degrees are enough for the seeing of Mars and Saturn; about 10 for Jupiter and Mercury; and 5 deg. are commonly look'd upon as necessary for Venus, tho' it has been frequently seen while the Sun shone, and not at an Heliacal Rising or Setting. But these things in the Planets depend upon their various distance from the Earth, and the more or less fullness of the Orb of the Inferior.

All

All the Fix'd Stars under the Zodiac, and the superior Planets, Saturn, Jupiter and Mars, the Sun in its Annual Motion towards the East getting before them, rise Heliacally in the morning, a little before Sun-rise; that is, a few days after their Cosmical rising: They set Heliacally in the evening; namely, a little before their Achronical setting. But the Moon, which always gets before the Sun, rises Heliacally in the Evening; when its very old and hastens to a Conjunction with the Sun. The inferior Planets Venus and Mars, sometimes getting before the Sun, and at other times being left behind by him, (as has been shewn at large in B. I.) sometimes rise Heliacally in the morning, namely, when they are Retrograde; sometimes in the evening, when they are Direct; which is to be understood in like manner of the Heliacal setting of them. But the Fix'd Stars, plac'd a great way from the Zodiac towards the elevated Pole, may in the same Day both rise and set Heliacally together, and undergo other changes in regard of this kind of Rising or Setting; as will appear to any Person that shall consider it.

The *Grecians* and *Romans* anciently used a Year not exactly fitted to the motion of the Sun; and by that means sometimes they were before the Sun, and sometimes behind him. But the Seasons of the Year not returning with their erroneous Calendar, but with the Sun and the Solstices; that their Country, Domestic, or Military Affairs might be dispatched in their proper Seasons, the Ancients propos'd the rising and setting of the Stars instead of a Calendar. For the Fix'd Stars moving from the Equinoctial Points but very slowly, and by a motion not then taken notice of, they made

no question, but that the Sun returning to the same place in respect of the same Fix'd Star, did alike come again to the same Station in respect of the Equinoxes or Solstices; that is, when the same Star rises or sets Cosmically, Achronically, or Heliacally, (which they chiefly regarded,) that the same season of the Year was restored to the Globe, and consequently that the same labours or affairs were to be returned to; from hence they called the stated Seasons of the Year *Sydera*, or the Stars. But it is found that the Fixed Stars, after a long series of Years, remove out of their places; and therefore, if the Day of the Rising or Setting of the Fix'd Star given among the Ancients is to be reduced to our Calendar, an account is to be taken of the motion of the Fix'd Stars in the intermediate time. And now-a-days Calendars may easily be had, and the *Roman* Calendar (which we use,) comes nearer to the Motion of the Sun, upon which the Seasons of the Years depend and are ordered, than the Rising and Setting of the Fix'd Stars; the observing their Rising and Setting came to be neglected by degrees, and is not used by any but the Poets, who are wont to describe and paint out as it were the several Seasons by the circumstances of the various Rising and Setting of so many Stars, than which nothing is more beautiful and affecting; tho' it be generally done by them very erroneously, because they describe at this time the Day of our Calendar by that Rising of the same Star by which it would have been justly described in the time of *Cæsar*; whereas the difference between these two times is nearly 14 Days.

SECTION II.

Of the Division of Time, and several other things depending thereon.

PROPOSITION X.

TO explain the Division of Time into Hours, Days and Weeks, and the Terms used in Astronomy depending thereon.

We have thus far been explaining the generation of the Circles of the Sphere in the apparent System of the World, and the more usual Terms in Astronomy depending thereon: But seeing there are no terms in Astronomy that occur oftener than those, whereby Time or any space of it is expressed; 'tis necessary to treat a little concerning them, and to shew, by the way, the Civil disposition of Time, so far as it is used in an Astronomical Calculation.

The most considerable parts of Time are, a *Day*, an *Hour*, *Month* and *Year*. The first of them is a *Day*, (because a space of Time most obvious and known by us;) and it is either Natural or Artificial. A *Natural Day* the duration of an intire apparent revolution of the Sun about the Earth. The *Artificial* is that part of the Natural, during which the Sun is above the Horizon; and it is opposed to the *Night*, which is taken for the Sun's continuance below the same: But the Natural Day takes in both. The Natural Day is either the Astronomical or Civil; and they differ from one another only as to their beginning, according to the custom of the People, and settlement of the Astronomers. The Astronomical Day is the space of time which flows
between

between the Sun's leaving a given Celestial Meridian, and next return to the same; that is, the space wherein a revolution of the whole Celestial Equator is performed, and of that part of it besides which answers to a portion of the Ecliptic, which the Sun describes in the mean while by its Annual motion towards the East. Because this portion of the Equator to be added to the intire Equator is not every where equal, (tho' its mean Quantity is nearly that of one Degree,) as well on the account of the Obliquity of the Ecliptic, as that the apparent Annual motion of the Sun about the Earth is not equable, the Natural or Astronomical Days are not precisely equal: But of this inequality of the Natural Days, I shall treat in its proper place afterwards; taking no notice at present of this nicety. Most Astronomers begin the Day at noon: But *Copernicus* following *Hipparchus* begins it at Night, and this beginning is retained in the *Prutenic* Tables. Differing Nations make different beginnings of the Day. The *Babylonians* begin their Day with the Rising of the Sun; The *Jews* and *Athenians* with the Setting of the Sun, as the *Italians* do still. The *Egyptians* at Midnight; which is the custom of the *Britains*, *French*, *Germans*, and several other *European* nations. And anciently the same was done by the *Jews*. For we naturally refer the Night, which we spend in sleep and silence, partly to the preceding, and partly to the following Day; that is, we begin the *Nox ἡμέρας* at Midnight. The *Umbri* anciently made the Noon to be the beginning of their Day, and this the *Arabians* observe at this time.

Tho' the Artificial Day was hardly divided anciently any other way, than into the Morning and the Evening, and the Night into

Watches;

Watches; yet afterwards the Natural Day was divided more accurately into 24 parts, called *Hours*. Hours are either equal or unequal. An *Equal Hour* is the four and twentieth part of a Natural Day. This sort of Hour was always used by Astronomers, and is now in use among almost all Nations. The *Italians* reckon the Hours from One to 24, from the beginning of the Night; (that being the beginning of the Natural Day,) and their Clocks are made after that manner. But We, the *French*, &c. reckon not 24, but twice 12 Hours; perhaps that the number of the Strokes struck by the Clock might not be tedious. So that we divide the Day into the Hours before Noon and the Hours after Noon. An Hour is commonly divided only into four Quarters (which are called Points by some,) a more minute division being in most busineses unnecessary: But the Astronomers (and now all the Politer sort of People) subdivide it into 60 Minutes, and each Minute into 60 Seconds. An *unequal Hour* is the twelfth part of an Artificial Day, as also the twelfth part of the Night. 'Tis call'd Temporary, because of a various length in different Times or Seasons of the Year: Thus the Summer Diurnal hour is longer than the Winter, and the Nocturnal shorter; and the Diurnal hour in Summer is longer than the Nocturnal, and in Winter shorter: At the time of either Equinox, the Diurnal hours are equal to the Nocturnal, and then these unequal hours become the same with the equal ones described above; upon which account these equal hours are called by Authors *Equinoctial Hours*. The *Jews*, *Grecians*, and *Romans* made use of these Temporary hours. Such as begin the Civil Day, at the Morning or Evening, do it because the Sun

is then in the Horizon, the most sensible of all the Circles of the Sphere. They have this advantage, that by the number of their equal Hours they know how much of the natural Day, that is, how much time is gone since Sun rise; but they have this disadvantage, that the times of the Sun-set, Noon and Midnight, in different Seasons of the Year, are marked with different numbers of the Hours, and they can't be known But by Calculation: They likewise have this advantage, they immediately know how much time remains till Sun-set, and so know how to accommodate their Labours or Travail to it; and this disadvantage again, that they can't reckon the Hour of Sun rise, Noon, or Midnight, without computing. Such as begin the Civil Day from Noon or Midnight, do it, because the Sun is then at his greatest Elevation or Depression; but the name of the Hour of Sun rise or Sun set, they get only by Computing. Lastly, such as shou'd use the unequal Hours, just now described, could always tell Sun rise or Sun set by the twelfth hour; Noon and Midnight always by their sixth; but could know the Quantity of the Hour only by Calculation.

A *Week* is the ancientest collection of Days as ever was; as is evident from the Sacred Writings. The *Jews* used this collection of old, and since the reception of Christianity, other Nations too: Every Day of it has a name given to it from some of the Planets, after the following manner: The Hours of the natural Day are four and twenty, the Planets but seven, and in this order in the common System, Saturn, Jupiter, Mars, Sol, Venus, Mercury, the Moon; they began at the first Day of the *Jewish* Week; ascribing to the Sun (the Author of the Day) the

the first Hour of it, to Venus the second, to Mercury the third, to the Moon the fourth; then beginning from Saturn, they assign'd the fifth Hour to him, and so on; by this means the first Hour of the following Day happens to the Moon, and she therefore bestows a name upon that Day of the Week from her own; which method is continued in the following Days to the end of the Week.

PROPOSITION XI.

TO explain the division of Time into Months, Years, and the various sorts and collections of them.

Tho' a Month be properly that space of time wherein the Moon goes thro' the Zodiac; yet since there are about twelve Months that pass while the Sun runs once thro' the Ecliptic, that space of time also wherein the Sun runs thro' one Sign of the Zodiac, is call'd a *Solar Month*; which is about $30\frac{1}{2}$ Days. Again, the *Lunar Month* is either *Periodic*, in which the Moon having departed from any Point of the Zodiac returns to it again, and is something less than 27 Days; or *Synodic*, which is something more than $29\frac{1}{2}$ Days, wherein the Moon runs thro' all its several Phases, and on that account is of principal use in the marking of Time. Both the Solar and Synodic Lunar Month, is either *Astronomical* (of which we have spoken already) or *Civil*, according to the different custom and institution of each Nation. For some, as the *Egyptians*, liked the Solar Months, and made each of them to consist of 30 Days; and to compleat the Year, at the end of twelve such Months they annexed five Days, made up of those twelve times ten Hours, that each Solar Month exceeds 30 Days by.

But most of the Ancients fancied the Synodic Lunar Month most, such as the *Jews* anciently, the *Grecians* and *Romans*, to *Julius Cæsar's* time, and the *Mahometans* at this time. But that these sort of Lunar Months, that do not consist of whole days, might be fitted to Civil use (where pieces of days can't be taken notice of, and considered,) they made their Civil Months alternately of 30 and 29 days, (calling the former *Full*, and this *Cave* Month;) so that two such Months are equal to two Lunar Months of $29\frac{1}{2}$ days a piece; and the New Moon would not sensibly remove from the first day of the Civil Month in the course of some Years.

A Year is sometimes taken for the time of the revolution of a Planet through the Zodiac; in which sense a Month is sometimes called a Year: and sometimes for the time of an intire apparent revolution of the Fix'd Stars thro' the Zodiac, which they call the *Annus magnus*, or *Great Year*. But a Year is properly that time which the Sun takes to run thro' the Zodiac in. And it is of two sorts, the Astronomic and the Civil: The Astronomic is also twofold, according to the two different bounds of the Sun's revolution; namely, the Sydereal and Tropical. The *Syderial* Year is the space of time, that the Sun having departed from a Fix'd Star returns to the same in; and it is 365 Days, 6 Hours, and 10 Minutes nearly. The *Tropical* Year is that wherein the Sun departing from one of the Cardinal points, the Equinoctial or Solstitial, returns to it again; and is something less than the Sydereal, because the Cardinal points of the Ecliptic themselves go backwards, and as it were meeting the Sun, make the Sun to return to the same point of the Ecliptic something sooner than to the same Fix'd Star, where
that

that point of the Ecliptic was, when the Sun was before in the said Point. This Tropical Year is 365 Days, 5 Hours, $49\frac{1}{2}$ Minutes very near; and wants about 21 Minutes of the Sydereal.

The *Civil Year* is the space of time, that the motion of the Sun or Moon, or both, points at, received by the settled Custom of any Nation. Hence 'tis evident, that there are three forms of the Civil Year; namely, either purely *Lunar*, or purely *Solar*, or *Luni-Solar*, which is made up of both. The *Lunar Year* consists of twelve Lunations or Synodic Months that are finished in 354 Days, at the end of which the Year begins again. And since this Year wants near 11 Days of the Tropical Year, that brings the Seasons over again; it comes to pass, that the beginning of this Year falling upon the Spring now, in 8 Years time would fall upon Winter, and after as many more upon Autumn, then upon Summer, and lastly, at the end of 33 Years return to the Spring again. And this is called the *Wandering Lunar Year*; because its beginning wanders thro' all the Seasons of the Year, and that in the memory of one Man. 'Tis called also *Annus Solutus*, because free and disengaged from the motion of the Sun, which is not at all considered in the ordering of it. This sort of Year is in use among the *Turks*; and seems to be found out at first in those Countries, where the differences of Summer and Winter are not very evident and sensible; and where, for want of Astronomy, the return of the Sun to the Cardinal points of the Zodiac, that defines the Solar Year, is not easily known: On which account taking the Signs of the Seasons from the Moon, they took the beginnings of their Years from the beginnings of her Phases

or Appearances; now, there is no number of Lunations, that comes so near to the Solar Year (which they had some small knowledge of) as the Twelve. I am apt also to think, the reason of the beginning the Civil Day from the Sun-set is to be taken from the Moon: Because it seems necessary for them to begin the Civil Day with their Year and Month, that is, with the New Moon, which at first they knew no otherwise than by the help of their Eyes; which was, when they saw the New Moon in the Evening immediately after Sun set. And upon this account, the first, second, third, &c. Day of the Month, was called the first, second, third Moon, and the day of the Full Moon was called consequently, the fourteenth Moon.

Such of the Ancients as were willing to retain the Lunar Year, thought it notwithstanding proper to adapt it so to the Sun, that its beginning might be in a manner fix'd, and that it might keep the Cardinal points of the Year shewn by the Seasons, from receding much from their Months; and to this purpose they us'd Intercalary or Embolimean Months. And first, in the space of three Years, they made up an intire Month to be intercalated every third Year, out of the eleven odd Days in every Solar Year. But because this artifice was not exact enough, in every eight Years they intercalated three Months, every one of which consisted of 30 days, (namely at the end of the third, fifth and eighth Year, tho' some others otherwise,) and at last eight Months in 19 Years. Hence 'tis evident that some of these Years are simple, namely of 354, or 355 days, the number that there is in the Wandring Lunar Year; some embolimen or intercalary, of 384 or 385 days, when there are thirteen Lunations

tions in the Year. These Luni-solar or fix'd Lunar Years, are used by the *Jews* and the Clergy of the Roman Communion.

There are three sorts of Solar Years, or such as are fitted to the motion of the Sun alone and the vicissitudes of the Seasons depending thereon; the *Egyptian*, *Julian*, and *Gregorian*. The *Egyptian Year* consists of 365 Days, which they divided into twelve Months of 30 days apiece, and five other days (called ἐπαόμηναι) added at the end of them, as was said above. In the constitution of this Year there is no regard had to the motion of the Moon: But because this Year wants 5 Hours 49 Minutes, or almost 6 Hours of the Solar Tropical Year; in four Years time it gets near a whole day before the Solar Tropical one, and in 1460 Years its beginning wanders thro' all the Seasons of the Year; on which account it may in a manner be called a *Wandering Year*, altho' in the Age of one Man, or 60 Years, it does not anticipate above 15 Days.

Julius Cæsar finding the *Egyptian Year* to get before the Tropical, because the six Hours, whereby it wants of the Tropical, are intirely neglected in its constitution; did therefore add those 6 Hours to each *Julian Year*; so that a *Julian Year* consists of 365 Days, six Hours. And because this piece or quarter of a Day, can't be considered or taken notice of in Civil use, he added the Day made out of them in four Years to every fourth Year, between the 23d and 24th of *February*, (because the *Romans* while they used the Lunar Year before *Cæsar's* time, intercalated the Embolimean month;) therefore from that time they writ *bis sexto Kalendas Martii*, or the sixth of the Kalends of *March*

twice; from which custom the Year had the name of the *Bissextile*.

And this *Julian* Year consisting of 365 Days, and adding in four Years one Day, so that after three simple Years there may follow a fourth of 366 Days, is the best fitted for Astronomical Computations; because it is a mean between the Natural or Tropical Year of 365 Days, 5 Hours and 49 Minutes, and the Sydereal Year of 365 Days, 6 Hours and 10 Minutes, and does, as it were, give an ocular Demonstration of the natural regrefs of the Equinoxes. This form of the Year held common among all Polite Nations from *Augustus* (who restor'd it when it was almost lost) unto the Year 1582, when the *Julian Calendar* was reformed by *Gregory* the XIIIth. But it is still in use among the *Britains*, *Irish*, and others. Yet it must be confess'd, that the quantity of the *Julian* Year is too big, on which account the beginning of this Year also by little and little creeps forwards in regard of the Seasons, or (which is all one) the Times of the Equinoxes and Solstices creep backwards in regard of the Days of this Year. And since this regrefs is about $10\frac{4}{7}$ Minutes every Year, in about 133 Years it will be a matter of a Day, and consequently from the Year of Christ 325, wherein the *Council of Nice* was held, to the Year 1582, wherein the Pope reform'd the Calendar, namely 1260 Years, this regrefs was a matter of 10 Days. Hence it came to pass, that whereas in the time of the *Nicene Council* the Vernal Equinox happened near about the 21st day of *March*, in the Year 1582 it was found to have crept up to the 11th of *March*. That he might therefore bring the Equinox to its former place, ten days were suppressed in the Month of *October*, in the Year 1582, and the 5th day

day was call'd the fifteenth; by this means what would otherwise have been the 11th day of the *March* following, and the time of the Equinox, becomes the 21st of *March*; that is, the Equinox happens the same day, viz. 21st of *March*, as it did in the time of the *Council of Nice*, which settled the time of keeping *Easter*. Still if no other reformation should be made, but the taking away this ten days, the Cardinal Points would again remove after the former manner, in some Years time. The Pope therefore to avoid this, order'd that once in 133 Years (in which time the excess of a *Julian* Year above the Tropical rises to a whole Day) one day should be taken away from the Calendar; that is, that three days should be taken out of four hundred Years; and this he appointed to be done, by making every hundredth Year of the Christian Æra common, which according to the *Julian* Account is always *Bissextile*; but every four hundredth, which was to continue *Bissextile*, as in the *Julian*. The New form of the Year settled by the Authority of Pope Gregory XIIIth, was call'd from him the *Gregorian*, and is observed in *Italy*, *France*, *Spain*, *Germany*, wheresoever the authority of the Pope reaches; and in the close of the last Age, was admitted, as to the Civil Months, by several of the Reformed in *Germany*: As for the finding the *Paschal Moon*, they kept themselves still to *Kepler's Rudolphine Tables*, till something more certain could be fix'd upon and settled.

As the form of the Year among different Nations is various, so likewise is its beginning. The *Jews* begin their Ecclesiastical Year with the New Moon of that Month, whose Full Moon happens next after the Vernal Equinox: The Church of *Rome* begin their Year with

the Sunday that falls upon the said Full Moon, or that happens next after it; or with the *Fest* of the *Resurrection* of our Lord, which was the custom in *France* before the Year 1564. The *Jews* begin their Civil Year with the New Moon, that has its Full Moon happening next after the Autumnal Equinox. The *Grecians* began their Year with the New Moon that happens next after the Summer Solstice: And the *Romans* anciently began theirs with the New Moon next after the Winter Solstice. And this seems to be the reason why *Julius Cæsar* did not fix the beginning of his Year (tho' a Solar) in the Winter Solstice it self, but tarried for the following New Moon, where he might place the beginning of his Year or the Calends of *January* according to the custom then receiv'd. Tho' perhaps the ancient opinion was not yet entirely lost, that the Solstices were celebrated in the eighth parts of the Signs of Cancer and Capricorn, and not in the very beginning of the Signs: by which no more was meant among the Learned, than that the Sun was really in the eighth parts of the Signs, when he was commonly thought to be in the beginning; which is all one as to say that the Equinoxes and Solstices were anticipated for 6 or 7 Days. And if the Sun had been in the first point of Capricorn on the 24th of *December*, he would be exactly enough in the eighth Degree of it on the 1st of *January*. By this Method he brought it about that the Celebration of the Olympic Games should fall upon the first Day of the Month *Quintillis*, which he call'd by his own Name, *July*: And perhaps these Games themselves were by ancient Rite instituted on the account of the Summer Solstice.

Tho' the first of January is now look'd upon as the beginning both of the *Julian* and *Gregorian* Year almost thro' all *Europe*; yet others assign other beginnings to it both formerly and now-a-days. We have spoken of the moveable Feast of *Easter* already. The *Venetians*, *Florentines*, the Inhabitants of *Pisa* in *Italy*, and of *Treves* in *Germany*, make the Vernal Equinox to be the beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feast of the Annunciation of the Blessed Virgin, that is, of the Incarnation of Christ, to be the beginning of the Year: Which the *Church of England* still retains as the beginning of the Year. And the Civil Year begins here also, tho' the *Vulgar* begin it at the first of *January*, with the neighbouring Nations.

The more celebrated collections of Years or Times, measured by a repetition of Years, are an *Olympiad*, of 4 Years; a *Lustrum*, of sometimes 4, sometimes 5 Years; a *Jubilee*, of 49 or 50 Years; a *Seculum*, or an Age, of 100 Years; an *Annus Magnus*, or Great Year, standing for an intire apparent Revolution of the fix'd Stars, till after having departed from a fix'd Point, they return to the same again, which is 25000 or 26000 Years, or thereabouts.

PROPOSITION XII.

TO enumerate the chief and most noted Epochas, and to explain their relation to each other.

As the Astronomer in computing the motion of the Celestial Bodies, assume certain fix'd Points to begin at; so they do certain moments of Time, from which as Roots, their Computations may begin and proceed. These Roots, which they also call *Epochas* and *Eras*, commonly have a name from some famous Action
or

or Event, that happen'd near the beginning of each. The most considerable, and which we are best acquainted withal, is the *Epocha of Christ's Nativity*, or the first day of the *January* that we suppose to follow next to the Nativity of our Lord: And 45 years before is the *Epocha of Julius Cæsar*, or the beginning of the *Julian Year* at which time Cæsar rejecting the old Roman Year, that had been used from *Numa*, as they believed, ordered the *Julian Year* to be observed thro' the *Roman Empire*. 'Tis true, this Year was not observed always from the beginning, but after various and confused intercalations extraordinarily; in about the 10th Year after its beginning, that is, the 5th of the *Vulgar Æra*, or Birth of our Saviour, it was restored by *Augustus* in the same order as if it had been continued without any interruption from the beginning.

Tho' the *Epocha* of the Nativity of Christ above described be established by common use, and now almost universally received, yet *England* and *Ireland*, in affairs both of Church and State, use an *Epocha* a full year after it; and suppose Christ to be born, not on the 25th of *March*, near the ending of the forty fifth *Julian Year*, but of the following Year; namely the forty sixth *Julian Year*; and they suppose Christ to be conceived on the 25th of *March* of the forty sixth *Julian Year*; and number the Year not named from the Nativity, but from the *Incarnation* of Christ, that is, the Conception; and by consequence the same in number, for the greatest part of the Year, with the remaining round Number, that shews the Year from the Nativity. But in the Months almost between the first of *January* and the 25th of *March*, they reckon a different year from the rest, as they

they ought according to their *Æra*. That this difference between the *Æras* that take their name from Christ, may be better understood, we are to take notice, that *Dionysius* called *Exiguus*, was the Author of this *Æra* above five hundred Years after Christ, from which time they began to reckon from the Nativity or Incarnation of Christ, whereas before they used to number their years by the Consuls, and from the Building of *Rome*, in the *Roman* Empire; and out of it by the Years of each King's Reign. This *Æra*, sometimes call'd from its Author, the *Dionysian*, is by no means to be looked upon as exact and agreeable to History. For 'tis not yet settled, whether it was the first, second, third, fourth, or fifth Year, before the Vulgar *Æra* that Christ was born, as may be seen in *Kepler's* Book of the Year wherein Christ was born. But this supposed *Æra* is not too far distant from the time, which is sufficient for an *Æra*. And according to *Dionysius*, the Author of this *Æra*, Christ was conceived the 25th of *March* of the first Year current of this *Æra*, and born the following Winter, at the end of the 46th Year of *Julius Cæsar*. This computation of the Year was at first universally received, but is now left and retain'd only in *England*, together with the beginning of the Ecclesiastical and Civil Year, (the Feast of the Annunciation,) which obtained in those days: For in the Christian World beside this *Epocha* was tacitly departed from; for now the common opinion is, that Christ was born the Winter before the *Dionysian* Annunciation, namely at the end of the 45th of *Julius Cæsar*, and therefore they reckon the Years from the Nativity; and so make Christ one Year older than *Dionysius* the Author of the *Æra* did, and the *English* with him.

Since

Since most Astronomical Tables are now fitted to the more common Christian *Æra*, the other more celebrated ones are to be compared and reduced to it. And the first of them is the *Epocha of the Creation*, about which there are several controversies. Some say the World was created 3950 Years before the common *Æra* of Christ, and these says *Gassendus* come nearest the truth; which is likewise approved of by the *French Astronomers*. Others 3983 years; and with these *Petauius* agrees. *Kepler* 3993 years, in the middle of Summer, from Astrology. *Hewelius* in the 3963d Year current before Christ, the 24th of *October* of the *Julian Year* continued backwards, at 6 a Clock in the Evening, in the Meridian of *Eden*; because he found in his Tables of the Sun, that at that time the Sun was in the beginning of $\cap$, and its Apogæum in the beginning of $\vee$. Others make it much more ancient, and place it 5199 Years before Christ.

The *Epocha of the Olympiads*, of all Profane ones, is the most Ancient and Celebrated: It is placed in the Summer of 777 years before Christ, the first of *July*, the *Julian Year* being continued backwards. Not much after is the *Epocha of the Building of Rome*, which is generally fix'd in 752 years before Christ.

The *Æra of Nabonassar*, always famous among the Astronomers, began at the 26th day of *February* of a *Julian Year* produced backward, and in the Year 747 before Christ. And because this was the first of the *Egyptian Years*, *Ptolemy*, and after him the other Astronomers, and *Copernicus* himself, compute from thence the motion of the Stars by *Egyptian Years*. For the *Egyptian Years* consisting of 365 Days, tho' unfit for common Use, because its Cardinal Points wander thro' all the Seasons, yet is very fit

fit for Astronomical Computations, because it is disturbed with no intercalation.

The *Epocha of the Death of Alexander the Great* follows next in order, on the 12th day of November, in the year 324 before Christ. And this also being the first of the *Aegyptian Wandering Years* (for so the Founders of *Æras* will have it, tho' the Event happen'd before or after the beginning of the Year) *Theo, Albategnius*, and others that used this *Epocha*, computed the *Aegyptian Years* from hence. Between the *Æras* of *Nabonassar* and the *Death of Alexander the Great*, there are 424 years pecisely.

I pass over the *Æra of the Abyssinians*, call'd also the *Æra of Dioclesian* and the *Martyrs*: As also the *Æra of the Arabians*, that is, the *Hegira*, or *Flight of Mahomet*: And the *Epocha* of the *Persians*, or as it is call'd *Jesdegird*; and others, (being such as are of small use to us) which may be seen at large in Writers upon that subject; and in fine, the *Epocha* of the *Julian Period* of 7980 years, invented by *Scaliger*, beginning 4714 years before the Christian *Æra*, and consequently before the World was created.

SECTION III.

Of Spheres and other Instruments invented to represent the first Motion of the Heavens, and of their Use.

PROPOSITION XIII.

TO describe the common Instruments, by which the First or Diurnal motion is imitated, and demonstrated to the Sight, namely the Celestial Globe and Sphere.

For

For the better conception of the aforefaid Circles in the Heavens, Artists present us with the Image of the Starry Heaven upon the Celestial Globe, which is a Sphere made of Metal, Paper, or any other like substance, easy to be moved upon two Poles. Upon the Convex surface you have marked the Places of the chief Fix'd Stars, so disposed in regard of one another and the Poles of the Sphere, as the Fixed Stars themselves, which they represent, are disposed in respect to one another and to the Poles about which the Heavens seem to revolve in the diurnal motion. The Images also of the Asterisms are drawn upon it, that these Stars seem'd to represent to the Ancients, drawn to it either by the force of Fancy or Religion.

There are drawn also upon this Globe, the Equinoctial Circle exactly between the Poles, and the two Tropics parallel to it, on each side of it at $23\frac{1}{2}$ Degrees distance, and as many Polar Circles at as great a distance from both Poles. The Ecliptic also is drawn reaching to both the Tropics, and intersecting the Equinoctial upon the Surface of the Globe, in the points so related to the Places of the Stars, as the points of the Celestial Equinoctial (where the Sun appears in the time of the Equinoxes) are related to the Stars themselves, in the Age wherein the Globes are made: As well the Equinoctial as the Ecliptic is divided into Degrees, beginning at the point that represents the Vernal Equinox; the Equinoctial into 360, without any break, but the Ecliptic into 12 Signs, distinguished by their proper Marks, every one of which is subdivided into 30 Degrees: And the Secondaries of the Ecliptic drawn thro' the beginning of each Sign, and meeting in the Poles, divide the whole Globe it self into as many

many Signs. Only two Secondaries of the Equator, namely, the Colures, are drawn upon the Celestial Globe, one of which, namely, the Solstitial Colure, is the Secondary also of the Ecliptic. Such a Globe as this representing the Starry Firmament, is made movable upon the aforesaid Poles, so inserted into the Meridian, a Brazen immovable Circle placed without the Globe, as to fit that side of this Meridian, which is divided into Degrees; which side therefore is to be taken for the true Meridian, the thickness of the Circle only serving for strength.

Besides, the Globe encompassed with the Meridian (as was said) is let into a Wooden contrivance representing the Horizon, so as that half of it is above the upper Surface of this Wooden contrivance, which is therefore to be looked upon as the Horizon. And that the Globe may come near enough in every part to the Horizon, there are notches in the Horizon for the insertion of the Meridian, that is, prominent without the Globe. Upon the breadth of this artificial Horizon, which is large enough, besides the inmost Circle divided into its proper Degrees, reckon'd from convenient beginnings, there are the *Julian* and *Gregorian* Calendars, with the Degrees of the Signs of the Ecliptic, that the Sun is in nearly upon those Days in that Age: There is also a Circle of the Winds or Points of the Compass.

The Horizon is supported by four or more little Pillars fasten'd to a Foot or Base, upon which there is placed a Mariner's Compass and Needle, to set the Globe pretty nearly agreeable to the Quarters of the World. You have also a Quardant of Altitude that may be fasten'd to the vertical Point of a Meridian by the help
of

of a Screw, its Limb divided into Degrees, is to be taken for a true Quadrant : and 90 Degrees are reckon'd from its extremity that touches the Horizon upwards towards the Zenith. In the Globes made now a-days, there is also a Circle called the *Hour Circle*, fasten'd to the Meridian, having the Pole for its Center. This little Circle is divided into twice twelve hours, that it might be fitted to our way of reckoning the hours, beginning at the Meridian. And since the Pole carries the Index along with it, the Index will point out the hours according to the revolution of the Globe; if it be at twelve a Clock of the day, at the same time as the place where the Sun is at that time is under the Meridian.

What is commonly call'd a *Sphere* differs from a Globe in this, that its surface is not united and perfect, as 'tis in a Globe, but the more principal Circles made of Armillæ or Hoops (from whence it has the name of an *Armillary Sphere*) shadow out a Spherical surface. In such a Sphere, besides the Horizon and Meridian, (between which being immovable, it is revolved like a Globe,) there are four great Circles; namely the Equator and Ecliptic, or rather Zodiac, (for like a Belt or Swath, it is 20 deg. broad;) and the two Colures and four lesser Circles, viz. the two Tropics, and as many Polar Circles: And thus the number of the Circles of a Sphere came to be reckoned Ten. This Sphere is so excavated, that by its transparence as it were, it is a representation of the World, and presents us with the little Globe of the Earth in its Center at rest, and suspended by the Axe, which could not be done so well in a Globe, whose Superfices is intire and united. From what has been said, 'tis evident, that in

an Armillary Sphere, the above described places of the Fixed Stars can't be marked at all, excepting some few, that appear in the Zodiac, tho' they likewise are commonly taken no notice of; and therefore the Problems relating to the Fix'd Stars can't be solv'd by such a Sphere. Tho' since the said Circles of the Sphere are delineated upon the surface of the Globe, all the Problems that are done by the Armillary Sphere, may be performed very easily by the Globe: And therefore Spheres, tho' of great name formerly, when the study of Astronomy began to revive, are very seldom used now, but to help the imagination of the young.

To these Armillary Spheres there are sometimes annex'd the Planets made to be immovable in different Circles, in the order as they are reckoned by the Astronomers of the *Ptolemaic*, *Tychonic*, or *Copernican* Sect. And therefore in the *Ptolemaic* and *Tychonic* Sphere, the Planets are carried in different Circles about the Earth at rest, in the Plane of the Zodiac, within the hoops that represent the Starry Heaven. But since there is required a Spherical surface (shaded out by the hoops) concentric with the Earth, to represent the First or Diurnal motion, and this is movable in the *Copernican* Sphere; therefore the Makers do place in it the Planets without the hoops compassing the Earth, which is movable about the imaginary Sun. Sometimes Spheres of this kind and Globes are made to go by a Weight or Spring, so as to imitate the rising, setting, and all the Diurnal motion of the Fix'd stars. Sometimes also the Planets perform their evolutions in their proper time: But things of this nature don't belong to this place.

PROPOSITION XIV.

TO describe the Terrestrial Globe, as it is commonly made.

The Terrestrial Globe does not differ from the above described Celestial Globe but in this, that instead of the Stars and Constellations composed of them, Lands and Seas are delineated upon it. And the Tracts, Bays, Promontories, and other things belonging to the Earth, are painted upon the convex spherical Surface, in regard of one another and the Poles upon which the Globe is turned in the Meridian, just as the Places themselves are upon the Earth, which they represent, in regard of one another and the Poles upon which the Earth really revolves by the diurnal motion. These places of the Earth thus imitated will really have that situation, each of them be transferr'd upon this little Earth agreeable to their Longitude and Latitude by Observation. The Equator, and its Secondary or Meridians passing thro' every ten Degree are also painted upon the Terrestrial Globe; one of which, namely that which passes thro' the *Azores*, or *Fortunate Islands*, is the most considerable, and called the first Meridian; it being that Meridian from which the Longitude of every place is reckoned, as was said above; and by consequence from whence the numbering of the degrees of the Equator towards the East is continued. The Parallels also to the Equator, namely the Polar Circles and Tropics, and several others at ten deg. distance from one another together with the Rhumbs (in the spaces representing the large and vast Seas) shewing the Courses of Ships guided by the Mariner's Compass and Needle, are all drawn upon it. And that the Terrestrial Globe, besides its own uses, might

might supply the place of a Celestial Sphere (when such one is wanting,) the Makers have fitted an Ecliptic to it. But because this Circle is purely a Celestial one, and has no proper place upon the Earth, therefore they have placed its intersection with the Terrestrial Equator, (which intersection they might have placed any where) at the point where the first Meridian intersects the Equator.

PROPOSITION XV.

TO resolve several Problems relating to the Situation of places of the Earth to one another, and to the Diurnal motion of the Stars, by the Globes; that is, to shew the chief Use of both Globes.

Tho' the solution of the following Problems by the Globes is not accurate and exact, yet it serves very much to form and instruct the minds of Juniors; and we shall dispatch them in one Proposition.

And here the Celestial Globe is to be looked upon as the great Machine of the World itself; the Sun and all the Stars, and whatever we see in the concave Firmament, are to be conceived as upon the convex surface, view'd by an Eye plac'd without that surface: But there is no need of such a fiction in the Terrestrial Globe. And because the Globes are the Instruments we make use of in this Proposition, they are to be look'd upon as perfect; as for the manner of making them so, we are not now to enquire, but to apply them as they are already made to the solution of the following Problems and others like them.

I. To find the Longitude and Latitude of any given Place upon the Terrestrial Globe.

Bring the given Place to the Meridian, (always understand that side of it as is divided into

Degrees, which, as we said before, is the true Meridian) and the degree of the Equator, then found under the Meridian, shews the Longitude of the Place. For the beginning of reckoning those degrees is taken from the first Meridian as was said before. Besides, reckon from the Equator in the Meridian to the given Place, the degrees of Latitude: For the Meridian is divided into four times 90 degrees, two of which 90 begin to be numbered from the Equator towards both Poles; the reckoning of the two other 90 degrees begins from both Poles, and ends in the Equinoctial under the Horizon. The Latitude is North or South, according as the Place given is to the North or South of the Equator.

2. *The Longitude and Latitude being given, to find the Place itself upon the Terrestrial Globe.*

Look in the Equator for the degree of the given Longitude; 'tis evident that the Place sought is in the Meridian drawn thro' this point taken in the Equator. Apply this point therefore to the divided Meridian, and reckon the degrees of the Latitude given toward the North Pole, if it be northern, or towards the South Pole, if it be southern, and the point at the end of the reckoning will be the Place sought.

3. *To rectify both the Globes to a given Latitude together with the Quadrant of Altitude; and to dispose the Globes agreeable to the four Quarters of the World*

If the Latitude of the Place given be North let the North Pole be raised above the Horizon if South, the South Pole, till the Arc of the Meridian intercepted between the Pole and the Horizon, be of as many degrees as the Latitude of the place given is. For by Prop. 7. the

Eleva

Elevation of the Pole is an Arc similar to the Latitude of the place ; that is, containing as many degrees. From whence 'tis evident to the sight what Stars always appear ; namely, such as upon the turning of the Globe quite round, don't descend below the Horizon : As also what on the contrary, never appear. Then compute from the Equator in the Meridian upwards, the degrees of the Latitude given, and the point where the reckoning ends, shall be the Vertex of the given Place : (For the height of the Pole is equal to the distance of the Equator from the Vertex ; since the same Arc, namely the distance of the Pole from the Vertex is the complement of both to 90 deg. :) Fix the Quadrant of Altitude by means of the Screw, to the Vertex thus found, so as that its divided edge, (which serves instead of a Quadrant,) may proceed as it were from thence. Lastly, by the help of the Mariners Compass fix'd to the foot of the Globe, let the Right line that is the common Section of the planes of the Horizon and Meridian be placed in the Meridian line, so as that the elevated Pole of the Globe may look towards the elevated Pole of the World. For we suppose the Magnetic Needle to be directed along the Meridian line ; at least, that its declination (that hinders it) is known. By this means, the Meridian of the Globe will coincide with the Meridian of the Place ; and its other points with the correspondent points of the Place. But such an Horizontal plane must be pitched upon for the Globe to stand upon, as will make the Horizon of the Globe to be parallel to the Horizon of the Place, as it ought to be.

4. *To find the Degree in the Ecliptic that the Sun is in at any given time, by the help of the Calendar and Circle of Signs drawn upon the surface of the Horizon, and by that to assign its Place in the Ecliptic itself.*

Look into the Calendar for the Month and Day proposed, and opposite to it you will find the Sign and degree thereof, that the Sun is therein. Then in the Ecliptic drawn upon the Globe look for that Sign of the degree of it, and that will be the Place of the Sun sought. If you would be more exact, you must get the Place of the Sun, the help of an Ephemerides, and find that out, in the Ecliptic drawn upon the Globe. After the same manner may the Places of the Planets found by such Ephemerides, be marked upon the surface of the Globe; by seeking first the Longitude of the Planet in the Ecliptic, and by protraction its Latitude from the Ecliptic either way, by the help of a pair of Compasses. To find the degree in the Ecliptic the Sun is in pretty nearly at a given time, you need only to know what Day of each Month the Sun enters a Sign of the Ecliptic, and compute one degree for every Day from thence. For this purpose there are some memorial Verses composed, which may be seen in the Writers of the Doctrine of the Sphere.

5. *To find the right Ascension and Declination of the Sun, or Star, or any given Point; and consequently to find the Time of the Culmination of a given Star or Point.*

The right Ascension of the Sun or any point upon the Celestial Globe, is found after the same manner as the Longitude and Latitude of a place on the Terrestrial Globe; (by *Probl. 1.*) For the right Ascension and Declination in the Celestial, is all one with the Longitude and

Lati-

Latitude in the Terrestrial Globe. Besides, having brought the Place of the Sun to the Meridian, bring the Index to twelve a Clock, (we reckon it twelve a Clock when the Sun is in the Meridian,) and turn the Globe round till the Star or given Point comes to culminate, then the Index shews the hour, when that will happen. But because the Hour Circle is too small, the part of an hour can't be easily distinguish'd in it: To be more exact, you must do thus. Because the intire Equator (together with a certain portion of it, corresponding to the proper motion of the Sun during that time) passes the Meridian in the space of a natural day; 'tis evident that a 24th part of the Equator, or 15 deg. pass the Meridian in one hour, and one deg. in four Seconds of time, and so on. If therefore from the right Ascension of a Star, you take away the right Ascension of the Sun, you will have that portion of the Equator which passes the Meridian, in the time between one culmination of the Sun and the next following culmination of the Star: If instead of that, you take the time necessary for the passage of the said Arc of the Equator by the Meridian, (allowing an hour for fifteen degrees, and four minutes for one degree, as was said before; which Operation is called the *Converting an Arc of the Equator into Time*;) you will have the time of the Culmination of that Star from Noon. But if the right Ascension of the Star be lesser than the right Ascension of the Sun, so that the latter can't be Subtracted from the former, you must add to it a whole Circle or 360 degrees.

6. *The Place of the Sun or any Star being given, to find its Oblique Ascension or Descension, as also its Eastern and Western Amplitude.*

Bring the given Place of the Sun or Star to

the Eastern part of the Horizon, (having rectified the Globe for the Latitude of the Place by *Prob. 3.*) and mark the degree of the Equator that ascends with it: For the Arc of the Equator, intercepted between the beginning of Aries and the Point thus marked (expressed by the number annexed to the point) is the Oblique Ascension of the Sun or Star it self, And the Arc intercepted in the Horizon between the East (that is, the intersection of the Equator and Horizon) and the Place of the Sun or Star is the Eastern Amplitude of that heavenly Body. If you bring the same Place of the Sun or Star to the Western part of the Horizon, the degree of the Equator then descending, will be the Oblique Descension; and the Arc of the Horizon, intercepted between the Western point of the Horizon, and the setting Celestial Body will be its Western or setting Amplitude.

7. *The right and oblique Ascension of the Sun, or a Star being given, to find half the time of its continuance above or below the Horizon; and if it be the Sun, whose Ascensions are given, to determine the Length of the Day and Night, and the hour of the Rising and Setting of the Sun; but if it be a Star, to find the hour of its Rising or Setting, by the time of its Culmination being given: And on the contrary, the Place of the Sun and Length of the Day being given, to determine the Places on the Earth to whom they agree.*

If you fix the Index to twelve a Clock when the Sun's place is under the Meridian, and turn the Globe backwards till the Sun's place is at the Eastern part of the Horizon, the Index will tell you the Hour of Sun-rise; and if the Place of the Sun be brought to the Western part of the Horizon, the Index will point out the Hour of its setting; the same Index will likewise shew the

the Culminating, Rising or Setting of the given Star, when the Star marked on the Globe comes to the Meridian, Eastern or Western part of the Horizon: For in all these instances Nature itself is imitated. And the Hour of the rising or setting of the Sun or Star being known, the continuance of them above or below the Horizon is self-evident.

To find the same more exactly by computation without the Globe, the Ascensions being given, do thus: Let the Ascensional Difference be converted into time, and the addition of it to six hours, if the Celestial Body decline from the Equator towards the Pole that is elevated, or the subtraction of it from six hours, if it decline towards the Pole that is depressed will give half the continuance of that Celestial Body above the Horizon: And this added to the time of its Culminating will give the Hour of its Setting, subtracted from it will give the Hour of its Rising. And consequently its whole continuance above the Horizon, (which is the Length of the Day, if the Celestial Body be the Sun) will be given, which subtracted from 24 hours, gives its continuance below the Horizon.

If the Place be required where the Day is of a given Length, when the Sun is in a given point in the Ecliptic; bring the Place of the Sun to the Meridian, and the Index to twelve a Clock; turn the Globe till the Index show the Hour of the Rising or Setting of the Sun agreeable to the given Length of the Day; then elevating or depressing the Pole, let the Globe be brought to such a situation as the Sun's place may be in the Horizon when the Index is at the said hour; for in this case, the Globe is in that situation, wherein the Sun being in a given point of the Ecliptic, makes the Day of the given

given Length. Therefore all the Places of the Earth, whose Latitude is equal to the height of the Pole of the Globe in this position, will answer the question. And if the given Place of the Sun be in the Tropic towards the visible Pole; (that is, if the Place be required whose longest day is a given length,) this Problem will serve to distinguish the Parallels and Climates.

8. *The Latitude and Sun's place being given, and besides any one of these three, namely the Hour of the day, the Altitude of the Celestial Body, and the Azimuth of the same be given, to find the other two.*

Rectify the Globe for the Latitude of the Place given, by *Prob. 3*; bring the Sun's place to the Meridian, and the Index to twelve a Clock; then if the Hour be given, turn the Globe about, till the Index point at that Hour, or move it accurately, till as many fifteen degrees of the Equator pass the Meridian forwards or backwards, as there are Hours since or before Noon; and if the given Hours have minutes, for each four minutes, make one degree pass by the Meridian. Fixing the Globe in this position, let the Altitude and Azimuth of the Sun or Star, be measured by the Quadrant of Altitude first fixed to the Zenith: Namely the Altitude in the Quadrant, reckoning upwards from the Horizon; but the Azimuth in the Horizon from South or North point to the intersection of the Quadrant with the Horizon. If the Altitude of the Sun or Star be given, turn the Globe about till the Star arrive at the given Altitude, (which may easily be try'd by the Quadrant of Altitude,) and the foot of the Quadrant will shew the Azimuth, and the Index the Hour. When the Azimuth is given, bring

bring the foot of the Quadrant to the Azimuth given, and turn the Globe till the given Star come to the edge of the Quadrant, and immediately the Altitude is known, being reckoned upon the Quadrant, and the Hour by the Index: Unless it be better to find it by the number of degrees of the Equator, that have pass'd the Meridian, since the Sun was upon it. By Hours we here understand Equinoctial ones reckon'd from Noon to Midnight, tho they may be easily reduced to *Italic* or *Babylonic* Hours, by finding (by preced. *Prob.*) the Hour of the rising or the setting of the Sun, from whence they (being likewise Equinoctials) are reckoned: 'Tis an easy matter also to reduce any given Equinoctial hour, reckoned from any beginning, to an unequal or *Planetary* hour, by the length of the day. And if the places of the Planets (found for that time by the help of an Ephemerides) be mark'd upon the surface of the Globe, the Constellations and Planets may be easily found and distinguished in the Heavens by the Globe, if rectified to the hour of the Night, and in a clear serene night be compared with the face of the Heavens, and turn'd about gradually as the night passes, to keep the same situation with the Heavens.

9. *To find the time when a Fix'd Star rises or sets Cosmically or Achronically.*

Bring the given Star to the Eastern part of the Horizon, and mark the point of the Ecliptic that rises with it; the time when the Sun is in that point (which you may easily find upon the Horizon of the Globe by the Circle of Signs and the Calendar annex'd) is the time of the Cosmical rising of that Star: But if the time of the Cosmical setting were required, mark in the Globe the point of the Ecliptic rising when the given
Star

Star comes to the West part of the Horizon, and the time wherein the Sun enters that point, is the Time sought. To find the Achronical Rising, mark that point of the Ecliptic which sets when the Star rises, and the Time wherein the Sun passes that point, is the Time sought. And if that point of the Ecliptic be taken notice of, that comes to the Western part of the Horizon together with the Star, that day the Sun is in that Degree, the Star sets Achronically. And this is sufficient, if by the Cosmical and Achronical Rising and Setting be understood Morning and Evening rising and setting, as it commonly does. But if the Cosmical rising and setting (as *Kepler* would have it) signify the same as its ascent above the Horizon of a given place, and descent below it, then this happens daily (if at all) to any Fix'd Star, the time of which is found by *Prob. 7.* And the Achronical Rising of a Star (which according to the same Author, signifies a Star's rising when the Sun sets) will be the same, and found after the same way as the former: But the Achronical setting comes to the same thing with the Cosmical Rising in the common sense.

10. *To find the time, when a given Fix'd Star rises or sets Heliacally.*

Since all the Fix'd Stars rise Heliacally in a Morning, and set in an Evening, to find the time when a given Fix'd Star (for instance, of the first Magnitude) rises Heliacally, that point of the Ecliptic is to be found, which is 12° below the Eastern part of the Horizon when the Star rises: For when the Sun is in that point of the Ecliptic, he is depressed below the Horizon, just as much as is required, that such a Star may be visible when it is just come above the Horizon. To find this by the Globe, bring the

the Star to the Eastern part of the Horizon, and find with the Quadrant of Altitude that point of the Ecliptic which is elevated 12 degrees above the Western part of the Horizon: The point opposite to this is the point sought, *viz.* one depressed 12 deg. below the Eastern part of the Horizon: That day therefore wherein the Sun possesses that point of the Ecliptic (which may be easily found in the Calendar annex'd to the Circle of Signs in the Horizon of the Globe) the said Star of the first Magnitude rises Heliacally. If the Star be of the second magnitude, a point of the Ecliptic must be found, that is depressed thirteen degrees below the Horizon, when the Star is just at it: If of the third, 14 degrees, and so on; as is evident from *Prop. 9.* If the time of the Heliacal setting of a given Star of the first Magnitude be required, bring the Star to the Western part of the Horizon, find as before, that point of the Ecliptic which at that time is elevated 12 degrees above the Eastern part of the Horizon, and the point opposite to it (namely that which is depressed as many degrees below the Western part of the Horizon) is the Sun's place in the Ecliptic, when the said Star sets Heliacally. After the like manner you may find the Place of the Sun in the Ecliptic for any given time past, when a given Fix'd Star did at that time rise or set Heliacally; and consequently the times anciently described and marked out by the Writers of Husbandry and Medicine, may be compared with ours: For instead of a given Star, as it is painted upon the Globes, you are to make use of that place of it, in which it was in that Age; lessening the Longitude of the Star one degree and a third part for every Age since the given, but keeping the same Latitude.

11. *To determine the beginning of the Morning and end of the Evening Twilight for any time given.*

Rectify the Globe for the Latitude of the Place, (which is supposed in this and all the preceding Propositions to be given,) bring the Place of the Sun at the time given to the Meridian, and the Index to twelve a Clock; then turn the Globe about towards the East, till the point of the Ecliptic opposite to the Sun's place, be elevated 18 degrees above the Western part of the Horizon, in which case the Sun itself is depressed as many degrees below the Eastern part of the Horizon; and therefore (by *Prop. 8.*) the Morning Twilight is beginning: and the time of it shewn by the Index. But 'tis better to turn the Arc of the Equator, that passes the Meridian, (while the Globe is moved about into this situation from that which it had when the Sun's place was at the Meridian,) into time, to be subtracted from 12 hours, for finding the Time of the beginning of the Morning Twilight. After the same manner the end of the Evening Twilight is found, by turning the Globe about till the point opposite to the Sun's place be elevated 18 degrees above the Eastern part of the Horizon, &c.

12. *To find the Distance of any two Places on the Earth, that are given, as also the Angle of Position; or that which a great Circle passing thro' the given Places, makes with the Meridian of either of them.*

The nearest distance of any two Places, which is always to be computed in a great Circle, is found by taking their distance as they are drawn upon the Globe; (by a pair of Compasses form'd for that purpose,) and applying that interval to the Equinoctial or first Meridian, to know the

number of degrees between the two Places; and reckoning for every Degree, 60 *English* or 15 *German* Miles. For the magnitude of the Earth is such, that a degree of a great Circle described upon its surface, is nearly 60 *English* or 15 *German* Miles.

To find the Angle of Position, the Globe must be turn'd about till the first of the two Places becomes Vertical, (which it will be, when it arrives at the Meridian of the Globe, being rectified for the Latitude of that Place;) then move the Quadrant of Altitude fasten'd to the Vertex, and consequently to the first Place, till it touch the second Place; and the Arc of the Horizon intercepted between the Meridian and Quadrant of Altitude, will be the measure of the Angle of Position between the Meridian of the first Place and the Great circle (namely the said Quadrant) passing through the two Places, because the Zenith is the Pole of the Horizon. The Arc of the Quadrant intercepted between the two given Places converted into miles, (without the help of the Compasses used above) will give the distance of the Places. But if, repeating the former Praxis, the Angle be found, that the same Circle passing thro' the two Places makes with any number of intermediate Meridians; (passing thro' for instance, the several degrees of the Equator;) the *Rhumbs* will be determined, along which you are to sail to the intermediate Places situated in the said Meridians, if you would sail nearly in an Arc of a great Circle (that is, the shortest way) thro' given Places.

13. Given the Hour of the day according to the reckoning of one given Place, to find the Hour at that time in another given Place; and to find the Place where a given Hour is reckon'd.

Bring

Bring the Place to the Meridian, and the Index to the given Hour according to the reckoning of that Place, then turn the Globe about till the other given Place comes to the Meridian, and the Index will point at the Hour sought. For the second case, bring the Meridian and the Index to the given Hour, as before; then turn the Globe till the Index point at the other given Hour; and all the Places then under the Meridian will reckon or have the given Hour of the day.

If you are for calculating it, you will find it more accurately. If the two given Places are under the same Meridian, or have the same Longitude, they will also reckon the same Hour from Noon or Midnight; But if their Longitudes differ, the difference of their Longitudes found by the Globe and turn'd into Time, gives the distance of the Hours in those Places reckoned at the same moment, so as that the Place which is towards the East reckons the greater of those two Hours. In the latter case, where the difference of time is given, let it be converted into an Arc of the Equator: And this Arc will be the difference of the Longitude, which being added to the Longitude of the given Place, if it reckons the former Hour, or subducted from it, if the latter, will give the Longitude sought; and all the Places of the Earth under the Meridian having that Longitude, will answer the Question. What is here said of our Hours reckoned from Noon or Midnight, may easily be applied to *Italic* or *Babylonic*, or even *Planetary* Hours, by reduceing them; as was shewn in *Prob. 8.*

14. *To find the Place where the Sun is vertical at a given time; and all the Places where the Sun at that time appears in their Horizon rising or setting.*

Having rectified the Terrestrial Globe for the Latitude of your Place and the Cardinal points of the World, turn the Globe about till that Place become vertical; and let the Globe in this position be exposed to the Beams of the Sun, (by removing the Horizon, that hinders the Rays of the Sun,) and at one view it will appear in what places of the Earth it is Day, and where it is Night; where the Sun is rising and where it is setting. For since the Globe and Earth it self are similar in Figure and similarly posited, they will be similarly enlightened by the Sun, whose immense distance may be looked upon as infinite. That Place, which is the Pole of the Circle, marked out by the confines of Light and Shade, is the Place where the Sun is vertical, and may be determin'd mechanically, by seeking the place, where the Shadow of a style erected perpendicularly upon the Surface of the Globe vanishes, or falls upon the Foot of the Style it self. If the same things be required when the Sun does not shine, find (by preced. Prob.) the Meridian of the Places where it is then Noon, and in this Meridian take that place of the Earth, whose Latitude towards the same Pole is the same with the Sun's declination at that time; and that is the place where the Sun is vertical, and if it be made vertical, the Horizon will shew the confines of Light and Shade, or all the places where the Sun appears in the Horizon; and then 'tis easy to distinguish the Places where he is rising, from the places where he is setting.

15. *To make an Horizontal Dial for a given Place by the help of a Globe.*

Rectify the Globe (as before) for the Latitude of the Place and the Cardinal points of the World: If besides the Meridian, there were eleven other Hour Circles (that is, 24 Hour Semicircles) immovable, standing without the Globe, and the whole Globe, besides the Arc about which it revolves, were pellucid; 'tis evident that the Sun placed in the Meridian would cast the shadow of the Axe upon the other half of the Meridian, and placed in the One a Clock Hour Circle after Noon, would cast the shadow of the opake Axis upon the One a Clock at Night Hour Circle, or the other half of the same Meridian beyond the Poles, and so on in the other Hour Circles. If besides all this, you imagine an Horizontal Plane shewing the shadow of the Axe cast upon it by Reflexion, when the Sun is in the Meridian, the shadow of the Axis projected upon the horizontal Plane, will be in that part of the common section of the Meridian and Horizon, which is beyond the Center; that is, in a Right line connecting the Center with the intersection of the Meridian and Horizon: When the Sun is come to the One a Clock Circle, the shadow of the opake Axis projected upon the horizontal Plane, will be that part of the common Section of the One a Clock Hour Circle with the Horizontal Plane, that lies beyond the Center, or which connects the Center with the common intersection of the One a Clock at Night Hour Circle; and which does therefore contain with the Meridian or Twelve a Clock line, an Angle, that is measured by the Arc of the Horizon, which is intercepted between the Meridian and One a Clock Circle

after

fter Noon or Midnight. And after the same manner the Sun, when it is at the Two a Clock Circle after Noon, cast the shadow of the Axe upon the Horizontal Plane in a Right line connecting the Center and intersection of the Two a Clock at Night Hour Circle and Horizon; which therefore makes with the Meridian line an Angle, that is measured by an arc of the Horizon intercepted between the Meridian and the two a clock Circle: and after the same manner in the other Hour lines upon the Horizontal plane, every one of them makes an Angle with the Meridian, which is measured by an Arc of the Horizon intercepted between the Meridian and Hour Circle it elongs to. That a Sun-Dial therefore may be drawn upon an Horizontal Plane, upon any point as a Center, let a Circle be described representing the Horizon of the Globe, (for the center of it represents the Center of the Earth, or in the present case, the Center of the World:) thro' the Center draw a Meridian line; that is, one that being produced would reach to the Cardinal Points of North and South, and upon the Center erect an opaque Right line representing the Axe of the World, tending to the Pole; that is, so as in the plane of the Meridian to make an Angle with the Meridian line equal to the Angle of the Elevation of the Pole above the Horizon of the place. Then let the one a Clock line after Noon be drawn from the Center of the Circle on the Eastern side of the Meridian line making an Angle with it, measured by the Arc of the Horizon intercepted between the Meridian and One a Clock Hour Circle. But because the Hour Circles are not to be found in Globes, let some certain Secondary (for instance, one of the Colures) be pitched upon, and

let the Globe be so placed, that this assumed Secondary may be instead of the first Hour Circle; that is, that 15 degrees may be intercepted between the fix'd Meridian and this assumed one; then let the degrees intercepted between this assumed one and the fix'd Meridian, be reckoned in the Horizon. Turn the Globe then, till the assumed Secondary of the Equator come to be in the place of the Two o'Clock Circle in the Afternoon; that is, till 30 degrees of the Equator be between it and the fix'd Meridian, and the degrees of the Horizon intercepted between the Meridian and the Circle assumed thus posited, being numbered; let lines be drawn to the Eastern side of it from the Center of the Sundial, making Angles with the Meridian line measured by the same number of degrees, and they will be the Hour lines of One and Two a Clock. And proceeding, after the same manner must the Lines of the following Hours be drawn, till you come to that Hour that the Sun sets at in the Longest day. Where, 'tis evident, that the Hour line of six a Clock is perpendicular to the Meridian line since in any Latitude whatever, there are 90 degrees intercepted between the Meridian and the six a clock Hour line. After the same manner are the Angles to be determined, that the line of the half Hours or any other parts of Hour contain with the Meridian line upon the Horizontal Plane.. Having done drawing the Hour lines upon one side of the Meridian, for instance for the Hours after Noon, the Morning Hour lines are to be drawn so, as that the lines representing the equally distant Hours from Noon may make equal Angles with the Meridian but lying on the other side. After the like manner (by the help of the 13th Prob.) may a Sun di

ial be drawn upon any Plane: For any Plane the Horizontal Plane of some place.

SECTION IV.

about the determining by Observation, the situation and respect, that the Circles of the Sphere have to one another.

PROPOSITION XVI.

TO determine by Observation, the Plane of the Meridian Circle in any Place.

The Place being given, the Horizon is also given: for that is determined by the Sight. And this being the only one that comes under the notice of our Senses, the others must be determined by the respect they have to this; and first the Meridian. Let two planes of Vertical circles be found by Observation, that a Celestial Body, which does not change its Declination, is upon, when it is at the same Altitude before and after its greatest Elevation above the Horizon; the Vertical Plane bisecting the Angle contained by these two Planes, will be the Plane of the Meridian, and produced unto the Heavens by the sight will mark out the Meridian right. Since (by the supposition) the Celestial Body does not change its declination, it will describe by its Diurnal motion a Circle parallel to the Equator: The highest point therefore of this Circle (as also of the Equator itself) in the Meridian Circle, and the points that are at equal Heights on either side, will be equidistant from the Meridian. Wherefore conversly, that Circle from which the Verticals, in which the Celestial Body was at the same

Altitude, are equidistant on both sides, is the Meridian; that is, the Circle, whose Plane bisects the Angle contained by the abovementioned Verticals is the Meridian itself. *Q. E. I.*

As for the Practice, this is a good way enough of solving the Problem: Upon a Plane parallel to the Horizon, that is, one to which a Plumbet is perpendicular (since the Zenith and Nadir, upon which the Plumbet falls when produced, are the Poles of the Horizon,) erect a Style, on whose lower extremity, as a Center, let a Circle be described upon the Horizontal Plane, and let a point be marked in the Circumference of it, upon which the shadow of the top of the Style falls before Noon; and again that point of it upon which the shadow of the same falls in the Afternoon: A Right line connecting the center of the Circle with that point which bisects the Arc lying between the two points thus marked, is the common Section of the plane of the Meridian with the Horizontal Plane, which is also called the *Meridian line*; because it shews the North and South Cardinal points by its direction. A Plane therefore erected upon this line, perpendicular to the Horizontal plane, is the Plane of the Meridian sought. When the Celestial Body made use of in the solution of this Problem is the Sun, 'tis evident that it must be done, when the Sun does not sensibly change his declination; (for this is a necessary qualification in the Celestial Body pitched upon for this purpose;) that is, in one of the Solstices. Because these Observations are to be made before and after Noon, the Summer Solstice is to be preferred, seeing that in the Winter Solstice, by reason of the Sun's nearness to the Horizon, the

uncer-

uncertain Refractions will render the operation doubtful; and besides, then the Sun ascends and descends too obliquely, altering its Vertical much more than its Altitude; which is inconvenient in this operation, where the Vertical is to be determined by the Altitude. Therefore when the Sun is in the Summer Solstice, a Circle is to be drawn upon the Horizontal plane, so as that the extremity of the shadow of the Style or Gnomon may fall upon it, when the Sun alters its Altitude pretty much, and his Azimuth but little; and therefore several concentric Circles are to be drawn, that the most commodious may be chosen.

To determine the Meridian line drawn for the fixing your more accurate and perfect Instruments, this operation should be repeated, and others made use of, to be seen amongst several others, or easily thought upon by any one: And this is to be understood of any other Observations, that are fundamental, and upon which a long train of others are built. The longest Styles or Gnomons are the fittest for the making this Observation: And instead of an opaque extremity or top, in a large Building, a hole letting the Rays of the Sun pass thro' it into the Ædifice, would be more proper; as *Cassini* did in the Church of St. Petronius at *Bononia*.

PROPOSITION XVII.

TO determine by Observation the Height of the Pole, and Situation of the Equinoctial, in regard of the Horizon.

Where one of the Poles has a considerable Elevation, [Fig. 5.] let the greatest and least Altitudes OS and $O\sigma$ above the Horizon HO of any Fix'd Star that never sets, (both of which happens in the Meridian,) be observed by an

Instrument fix'd in the plane of the Meridian, (determined by the preced. *Prop.*) And PS or $P\sigma$ half the difference $S\sigma$ of these two Altitudes, is the distance of the Star from the Pole: This distance $P\sigma$ therefore, or PS added to the lesser Altitude $O\sigma$, or subtracted from the greatest OS , gives OP the Height of the Pole sought. If the Star, when it is at its greatest Height is situated upon the other side of the Zenith Z , between Z and H ; then instead of the greatest Altitude of the Star, you are to make use of its complement to a Semicircle, in the foregoing Calculation.

Great care and diligence is to be used in making this Observation: For upon this are all the other observations of the Sun and Fixed Stars, and consequently all Astronomy is founded. Such a Fix'd Star therefore is to be chosen as is least liable to Refraction in its least Altitude; that is, the nearest to the Pole, which does not approach the Horizon too much: For the Refraction of Stars are liable to make their places observed near the Horizon uncertain; as shall be shewn hereafter.

If neither of the Poles be elevated enough above the Horizon, then the thing is to be done by the Meridian Altitude of a Star, whose Declination is known; as shall be shewn in the *Corollary* to the following *Proposition*. And in most places, where but few Observations are design'd to be made, this method is usually taken. But since the Observation of the Declination of the Stars, depends upon the knowledge of the Height of the Pole, 'tis more proper to lay down such a manner of observing the latter as does not depend upon the former; which also must necessarily be used by all cautious Observers, that design to determine the places of the
Stars,

Stars, and without trusting the Observations of others at all, since they may repeat them.

The Height of the Pole being found, the Altitude of the Equator, or HE the Arc of the Meridian intercepted between the Horizon HO and the Equator EQ is determined; For it is the Complement of the former to a quarter of a Circle: For because HZO is a Semicircle, and PE a Quadrant, PO and HE together will be equal to a Quadrant. The situation therefore of the Equator in regard of the given Horizon HO , is determined: For it intersects it in the points of the true East and West, which are marked out by a Right line perpendicular to the Meridian (found by the preced. *Prop.*) and the Plane of it is inclined to the Plane of the Horizon by an Angle, whose measure is the Arc HE lately found.

SCHOLIUM.

By the help of these two Propositions the magnitude of the Earth may be known. If a journey made along the Meridian (drawn by *Prop.* 16.) be exactly measured, and by an Observation of the Altitude of the Pole made at the beginning and end of the journey, the ratio of the Arc of the Terrestrial Meridian intercepted, to the intire Meridian be determined; the Circumference of the intire Terrestrial Meridian will not be unknown: And consequently, the Earth's Diameter, and from thence (by Geometry,) its Surface and Solidity will be known.

Besides, because in practise, a journey taken under any Meridian, or the length of the Way, is every where very nearly proportional to the change of the Elevation of the Pole by Observation; 'tis evident that the common Section of the surface of the Earth, and of the plane of any Meridian is a Circle, and consequently that the figure

figure of the Earth is Spherical. We abstract now from the little elevation at the Equator (concerning which in *Prop. 31. B. 1.*) which in the before-mentioned operation will be insensible.

PROPOSITION XVIII.

TO observe the Declination of any Celestial Body.

Let the Meridian Altitude of the Star proposed be observed. If the Star be in the quadrant ZH of the Meridian [*fig. 6.*] that the Equator EQ intersects the Meridian in, as at σ , the difference of the Altitude of the Equator HE and of the Altitude of the Star $H\sigma$ (namely $E\sigma$) is the Declination of the Star sought. But if the Star be in the quadrant ZO of the Meridian, in which the Pole P is, as in *S*, then the difference of the Altitude of the Pole P , and of the Altitude of the Star OS , (namely, PS ,) or the distance of the Star from the Pole, is the complement of the Declination sought. *Q. E. I.*

We add nothing at all here about the actual making the Observations and the construction of the Instruments, in which the chief difficulty of these things consists. For we suppose the Maker very well versed in *Practical Geometry*, *Mechanics* and *Optics*. And it is evident that it will be most commodious for any one that is about to determine the Places of the Fix'd Stars, to have a Quadrant or rather a Semicircle entirely immovable in the Plane of the Meridian, and divided into Degrees, and their lesser parts according to art, and an Index moving upon its Center, furnished with Telescopic Sights, for the ready measuring the Meridian Altitude of any Star, when it comes to the Plane of the Meri-

Meridian: For this Observation of the Declination of the Stars is of great use in correcting their Places.

COROLLARY.

As the Declination of an Heavenly Body is found by the Elevation of the Pole or Equator, and the Meridian Altitude of the Celestial Body being given; so, on the contrary, the Elevation of the Pole or of the Equator may be had, by the Meridian Altitude and Declination of the Star being given. For in the quadrant ZEH , the sum or difference of the Lines $H\sigma$, $E\sigma$ gives the Elevation of the Equator HE . But in the Quadrant ZPO , the sum or difference of the Lines OS and PS gives the Altitude of the Pole.

PROPOSITION XIX.

TO observe the Inclination of the Ecliptic to the Equator or the Obliquity of the Ecliptic, and to determine its Situation and consequently that of the Colures, Tropics and Polars, in respect of the Equator.

Let the Sun's Declination be daily observed about both Solstices: (by the preced.) This, where it is greatest, is the Obliquity of the Ecliptic sought. For since the Ecliptic is the way of the Sun, as big as the greatest Declination of the Sun is, so big is that of the Ecliptic it self; consequently the one being observed, the other is also observed. *Q. E. F.*

But since from the preceding Praxis, the Declination of a Celestial Body can't be observed, excepting when that Star is in the Meridian; if the Solstice should happen to be celebrated when the Sun is not in the Meridian of the Observer, (which will generally happen) there will be some error; but so small as to be not worth regarding. For we gave instructions that

that the Declination should be taken any Noon; and consequently the greatest error will be when the Solstice is celebrated at Midnight; that is, at that point of time which is most remote from Noon. The error therefore, when it is greatest, will be equal to what the Sun's Declination 12 hours before or after the Solstice wants of its greatest Declination: But this does not arise to above four Seconds; tho' the Obliquity it self is not determined to a minute, or even two, by Artists themselves. Of the Solstices that is to be chosen, in which the Sun may be best observed without any danger of error, and in this regions near the North Pole, the Summer Solstice; because the Sun in the Winter Solstice, even at Noon it self, is not out of all danger of Refraction; which is a matter of very great moment in this kind of Observation. There are some places, where to find the Obliquity of the Ecliptic, it may be advisable to subtract the Meridian Altitude of the Sun, when in one of the Solstices, from its Meridian Altitude in the other, if the Sun in both cases be on the same side of the Zenith; but if on different sides, from the Complement to a Semicircle of the latter Meridian Altitude, to have the distance of the Tropics remaining, half of which is the Obliquity sought.

To get the Obliquity of the Zodiac with great exactness, Observations continued for several Years, says the skilful and industrious *Hévelius*, and such as are compleat on all accounts, taken by your larger Instruments, and such as may be depended upon, are necessary. And upon this account it is that now-a-days, when Instruments are both great and accurately divided and furnished with Telescopic Sights, there is a sensible difference observable between the

the Obliquities of the Zodiac set down by different Artists. For *Hévelius* makes it $23^{\text{deg.}} 30' . 20''$. The same does *Ricciolus* in his *Reformed Astronomy*. *Mouton*, who has writ almost an entire Book upon this Argument, choosing, after several repeated Observations, the middle between the extremes, fixes upon $23^{\text{gr.}} 30'$, as *Ricciolus* in his *Almagest* does from some select Observations: This is what Mr. *Street* also, in his *Astronomia Carolina*, sets down, and most Astronomers now make use of. Notwithstanding Mr. *De la Hire*, in his *Astronom. Tables, Part first, of the Motions of the Sun and Moon*, makes it an entire minute less; namely, only $23^{\circ} . 29'$, and that from Observations near the Zenith, and out of all danger from Refraction.

And were the judgment of the Ancients about this matter consulted, so great a difference would be found between the Obliquity observed by them and that taken from the Heavens now, that some Authors have thought it to be changable, besides the change made in it twice a Year; which we don't here dispute about, because almost insensible: Since from the time of *Aristarchus* the *Samian*, who flourished near 300 Years before Christ, it was reckon'd to be about 24° : But in *Ptolemy's* time, about 140 Years after Christ, not above $23^{\circ} . 51'$. And again, in *Albategnius's* time, near 900 Years after Christ, only $23^{\text{gr.}} 35'$. And *Tycho* by his Observations makes it to be but $23^{\circ} . 31' . 30''$. Yet the most industrious observers of the present Age, will have it, notwithstanding, to be constant; namely, *Gassendus*, *Ricciolus*, *Horrox* and *Hévelius*, who does not think, in his *Prodromus*, that we ought to depend so much upon their Observations, since they did not make use of the best Instruments, nor proceeded in the shortest method.

Nay,

Nay, some of the Ancients, and *Ptolemy* himself, in his *Almagest*, think so too ; *Albategnius* for the same reason with *Hevelius*, prefers his own Observations to those of the Ancients. And *Gassendus* in the *Life of Peirescius*, shews, by some Observations of his own making, that the Obliquity of the Zodiac is the same now-a-days, as it was in the time of *Alexander the Great* ; because he found by an Observation made at *Marseilles* by himself, that the ratio of the Gnomon to its shadow at Noon in the Summer Solstice, was the same with that observ'd in the same City at the same time of the Year by *Pytheas Massiliensis*.

Besides, the points in the Heavens, where the Ecliptic intersects the Equator are they, in which the Sun appears when the days are equal to the nights to an Observer, to whom either of the Poles is elevated above the Horizon. This therefore is the situation of the Ecliptic, that it may cut the Equator in the said points, and be inclined to the Equator in the Angle above determined, half of it tending to the North Pole, in which the Sun is, when the Days are longer than the Nights to the Inhabitants of the Northern Hemisphere of the Earth. The situation of the Ecliptic being given, the situation of the Tropics is immediately given ; because they are parallel to the Equator, and touch the Ecliptic, and are as far from the Equator, as the Obliquity of the Ecliptic is great, and the Polar Circles are as far distant from the nearest Poles. The situation of the Colures is deduced from hence, for they pass thro' the Equinoctial and Solstitial points found before, and intersect one another at right angles in the Poles already determined by Observation.

PROPOSITION XX.

THE Obliquity of the Ecliptic being given, to find by calculation, the Right Ascension and Declination of a given point in it, as also the Angle that the Ecliptic makes with the Meridian at the said point; and conversely, to define the point of the Ecliptic, to which one of these agrees, and consequently to determine the place of the Sun in the Ecliptic, by an Observation made in Prop. 18.

Let $E \odot$ [Fig. 7.] represent the Equator, whose Pole is P ; EL the Ecliptic; E the point of one of the Equinoxes; L a point given in the Ecliptic, thro' which let a Circle of Declination, as $PL \odot$, be supposed to be drawn. In the Spherical Triangle $EL \odot$, there is given the Side EL , namely the distance of the point given from L , and the Angle $\odot EL$, the Obliquity of the Ecliptic; the Angle $\odot$ also is given, namely a right one, because P is the Pole of the Circle $E \odot$: The side $\odot L$ therefore will be found (by Spherical Trigonometry,) which is the Declination of the point L sought, and is North or South, according as the adjacent Pole is: In like manner the Side $E \odot$ will be found, which (according to the situation of the point L in this or that Quadrant of the Ecliptic, and the nature of the point E) will be either the Right Ascension sought, or its complement to a quarter, or half, or three quarters, or an intire Circle. For the Angle $\odot LE$ is given, that the Ecliptic makes in the said point with the Circle of Declination or Meridian. The converse of this is done after the like manner: Yet we are to take notice, that there are two points of the Ecliptic, whose Declination towards the given Pole is the same; and therefore some sort of a determination is necessary.

SECTION

SECTION V.

Of the Fix'd Stars, and the determining their Places by observation, and some other things that belong hereto.

PROPOSITION XXI.

TO explain the several Orders of the Fix'd Stars, into which they are divided on account of their different apparent Magnitude; and to give an account of this difference.

In some of the preceding Propositions we have laid down the method, whereby the position of the chief Circles of the Sphere are determined in respect to the sensible Horizon; namely, of the Meridian, Equator, Zodiac, both Colures, Tropics and Polar Circles: We should now shew the way, how the Places of the Fix'd Stars in respect of these Circles are to be found. But before that, we must say something about the Fix'd Stars themselves, their differences and order.

And first, there appears a vast difference among the Fixed Stars as to their apparent Magnitude and efficacy of their Rays, to any one that shall but look up to the Heavens in a clear night when the Moon does not shine. On this account Astronomers have made six Classes, or Orders of them, the greatest of which they call *Stars of the first Magnitude*; the least, *Stars of the sixth*, and the intermediate in the same order. Some make the difference to lie in the real magnitude of the Stars, asserting some of them to be really bigger than others, while they are all equally distant; because placed according to
them

hem in the same Spherical surface: Others following the Ancients, make their different distance to be the cause of their different Magnitude.

And this opinion is very much favoured by the number of the Fix'd Stars of the first and second Magnitude. For if every Fix'd Star did the office of a Sun, to a portion of the Mundane space nearly equal to this that our Sun commands, there will be as many Fix'd Stars of the first Magnitude, as there can be Systems of this sort touching and surrounding ours; that is, as many equal Spheres as can touch an equal one in the middle of them. Now, it is certain from Geometry, that thirteen spheres can touch and surround one in the middle equal to them, (for *Kepler* is wrong in asserting, in *B. 1* of the *Epit.* that there may be twelve such, according to the number of the angles of an *Icosaedrum*,) and just so many uncontroverted Stars of the first Magnitude are taken notice of by Observation. For Astronomers are not as yet agreed upon their number: *Hewelius* reckons the *bright Star in the Eagle's Shoulder* of the second magnitude, whereas *Tycho* made it of the first; and on the contrary *Hewelius* makes the *little Dog* and the *right Shoulder of Orion* of the first magnitude, and *Tycho* of the second. And there are others that *Hewelius* himself doubts of.

Again, if it be asked how many Spheres equal to the former can touch the first Order of Spheres, surrounding the Sphere placed in the beginning, (or rather a Sphere comprehending those former thirteen together with a fourteenth in the Center;) the number of these will be found to be 52, or 4×13 . For the centers of these Spheres of the second Order are in a Spherical surface, which is quadruple of that in which

the centers of the Spheres of the first Order touching the inmost are, since the Diameter of the one is double that of the other; and nearly as many Fix'd Stars of the second magnitude have been taken notice of; *Herveli* making them to be about 50; *Kepler* following *Tycho* 58; and *Ptolemy* not above 45; now we assume the middle between them: For there is a greater difference among Astronomers about Stars of the second magnitude than about Stars of the first. Nor will there be any great difference from Observation in determining the number of the Fix'd Stars of the other magnitudes according to this method. For the farther consideration of this matter, it only remains that we shew, that there is the same Order observed among the Fix'd Stars of the first rank, as there is between the central Bodies of the Spheres touching and surrounding the inmost Sphere (near whose Center the Eye is placed;) and the same between the Fix'd Stars of the second magnitude, as there is between the central Bodies of the Spheres second in order from these; and so on. And here indeed the matter does not go on so well, (which made *Kepler* of another opinion,) as is evident even from hence, that upon the first cast of our Eye upon the Heavens, some tracts of the Firmament appear fill'd with innumerable Fix'd Stars whereas others are found to be almost empty and void of any.

But there is no great error committed in the order of the Stars of the first and second magnitude, as will appear to any one that makes a comparison: For there are six Stars of the first magnitude in the Zodiac, three to the North, and four to the South, nearly, as it ought to be according to this Theory.

PROPOSITION XXII.

TO enumerate the Constellations, into which the Fix'd Stars are distributed by Astronomers.

The Ancients either following the Shape and Figure of the disposition the Stars appeared in, or led by Religion, have distributed all the Fix'd Stars visible in our Temperate Zone, into 48 Images: Twelve of which are situated along the length of the Zodiac, and give names to its twelve catemories: And there are in the Northern half, *Aries, Taurus, Gemini, Cancer, Leo and Virgo*; in the Southern, *Libra, Scorpio, Sagittarius, Capricornus, Aquarius and Pisces*. The other Images are placed in the Hemispheres separated from each another by the Zodiac: in the Northern 22, namely the *little Bear, the great Bear, Draco, Cetus, Bootes, the Northern Crown, Hercules, Ursa, Cygnus, Cassiopeia, Perseus, Andromeda, the Triangle, Auriga, Pegasus, Equuleus, Dolphin, Sagitta, Aquila, Serpentarius, Serpens*. To these were afterwards added the Constellations of *Antinous* made out of the unformed ones near *Aquila*, between it and *Capricorn* and *Sagittary*, and of *Coa Berenices* out of the unformed ones near *Lyon's Tail*. *Ptolemy* makes *Antinous* belong to *Aquila*, and *Equuleus* to *Pegasus*. On the Southern part of the Zodiac there are 15 Constellations known to the Ancients, namely *Cetus, Eriantus, Lepus, Orion, the Great Dog, the Little Dog, the Ship Argo, Hydra, the Cup, Corvus, the Centaur, Lupus, Ara, the Southern Crown, and Pisces Austrinus*.

To these are lately added 12 Constellations, which are invisible to us, being near the South Pole, namely the *Phoenix, Grus, Pavo, Indus, Apus, South Triangle, Musca, Chamæleon, Piscis volans, Toucan, or insæ Americanus, Hydrus, Xiphias or Dorado*.

There are some other Stars, which are not within the compass of the Images described, or reducible to them, these therefore they call *Unformed*; and out of these, some of the more considerable Observers of the Fix'd Stars, make now and then some new Constellations, and give them proper names. Thus the Accurate Mr. *Edmund Halley*, who first of all observed with exactness the Stars near the South Pole, while he was at *St. Helena*, has taken this liberty, in forming a new Constellation into the Image of an Oak, out of the unformed Stars that lie between the Ship *Argo* and the *Centaur*, and consecrated it under the name of *Robur Carolinum*, or the *Royal Oak*, translated into the Heavens in perpetual memory of the Preservation of King *Charles II.* in the hollow thereof. Thus *Bartschius* in his Globe of four foot diameter, has two other new Constellations, the *Camelopard*, and the *Monoceros*, that *Hevelius* retains.

But *Hevelius* himself has gone farther in this matter, and has made up more new Constellations out of the unformed ones, than any of the Moderns besides, as is evident from his *Firmament*; where he has plac'd *Leo minor* between *Leo* and *Ursa major*, and *Lynx* between *Ursa major* and *Auriga* above *Gemini*; the *Canes Venatici* after *Ursa major* and under its Tail; so that what to *Tycho* was an unform'd Star between the Tails of *Helice* and *Leo* (afterwards call'd *Caroli* by the English,) by *Hevelius* is said to be in *Annulo Armillæ Charæ*, or Collar of *Chara*; the *Lacerta* or *Stillio* between *Andromeda* and *Cygnus*; the *Sextans Uraniæ* between *Leo* and *Hydra*; *Scutum Sobiesciannum* between *Aquila* and *Serpentarius*; *Vulpecula cum Ansere*, between *Aquila* and *Lyra* under *Cygnus*; *Triangulum minus* between *Triangulum Boreale* and the *Heads*

of Aries. He has altered some of the ancient Constellations a little, and added others to them; as he has armed Antinous with a *Bow and Arrow*, who was before unarmed, and placed *Cerberus* above the bending of *Hercules's* left Arm, and *Mons Mænalus* under the feet of Bootes. The Constellations are set in the places described, as a continuation of the ancient Fables of the adjacent Constellations, lest he should break too much in upon or disorder the Rules and Judgments of the Astrologers, or to perpetuate the memory of some noble Action.

Others again are for giving the Stars new Names, such as are *Christian* ones; a specimen of this we have in the Venerable *Bede* in the signs of the Zodiac; and *Julius Schillerus* has continu'd in his *Cælum Stellatum*, or Starry Heaven, published in the Year 1627. This design has been very justly disapproved of by *Copernicus*, *Tycho*, and *Hevelius* in his *Firmament*, and other Astronomers, who think the ancient names ought to be retained, for fear of confusion, and by degrees losing the Ancient Astronomy. or by this method, which has been conveyed down to us from the original of Astronomy, we obtain what was of the greatest importance in this matter; namely (as *Copernicus* says) that *so vast a multitude of Stars might be distinguished into parts, and that these may have some names to express them by.*

The *Via Lactea*, or *Milky Way*, comes under this head of Constellations; for the present Age has found it to be a collection of innumerable little fix'd Stars, as some of the Ancients believed, as *Manilius* informs us: 'Tis a broad Circle white like Milk, compassing the whole Heavens; sometimes in a double, but generally a single path, passing thro' *Cassiopeia*, *Perseus*, *Auriga*, the

Feet of Gemini, Orion's Club; the fore parts of Monoceros, the Tail of Canis Major, Argus, Navis, Robur Carolinum, Crux and feet of the Centaur; after which, it is divided into two parts, over against Ara; its Eastern part passes thro' Ara, the extremity of the Tail of Scorpio, the Eastern foot of Serpentarius, the Bow of Sagittarius, Scutum Sobiescianum, the feet of Antinous and Cygnus, where its greater part joins the other: But the Western part passes thro' the former part of the Tail of Scorpio, the Right of Serpentarius and Cygnus; and ends its course in Cassiopeia, where *Manilius* begins the description of it. There are other lesser divisions of the *Galaxy*, the which may be seen in *Hewelius's Firmament*. There are also two *Nubecula* somewhat resembling the Milk Way, situated near the South Pole, invisible in *Europe*, and called by Sailors, the *Magellan Clouds*, are exactly like the whiteness of the *Galaxy*, as *Mr. Halley* assures us; and being viewed by a Telescope, they present us here and there with little *Nebulae*, and small Stars. The greater of these two is situated between Hydrus and Dorado, the less between Anser Americanus or Toucan and Hydrus.

PROPOSITION XXIII.

THE Theory of the Sun and a Pendulum Clock being given, to determine the Right Ascension of the Fix'd Stars by observation.

Having explained the difference of the Fix'd Stars, as well on the account of their apparent magnitude, as of the figure of the Constellations, into which they are commonly divided; what remains is to shew the method of determining the Place of any of them, in regard of the Circles of the Sphere, whose situation has been

been determined before, and first in regard of the Equator; for which purpose, we have shewn how to observe the Declination of any one of them in *Prop. 18*; it remains therefore that we find the Right Ascension: But, because the beginning of the Equator, from whence the Right Ascension of the Stars is reckoned, is its intersection with the Ecliptic; in which, though the Sun be found once a Year, *viz.* in the beginning of our Spring, yet it presently departs from thence, and leaves that point obscure or marked out by no appearance that may come under the cognizance of our Senses; and lastly, such an one as that the Ascensions of the Stars can't be immediately deduced from thence by Observation; some Phænomenon therefore is to be pitched upon, whose Right Ascension is given every moment, that the Right Ascensions of the others may be observed from this. And of all others, the fittest for this purpose is the Sun, because its motion is the most simple, and its Right Ascension is quickly found, by *Prop. 20*, its place in the Ecliptic being given. If its Theory therefore be given, that is, if its place in the Ecliptic at a given time be given, and a Clock also whose Index moves round equably, the Right Ascensions of the Fix'd Stars may be thus observed.

First, let the Clock be order'd so, as that the Index may go over the 24 Hours, during the time that any Fix'd Star, departing from the Meridian, returns to it again, which is a little less than a natural Day. Let the Index of a Clock thus order'd, be set to twelve a Clock, when the Sun is in the Meridian, and let the hour the Index points at, be taken notice of, that the fix'd Star whose Right Ascension is sought, comes to the Meridian at. Let these hours and

parts be turn'd into degrees of the Equator and their parts, as was shewn before: And you will have the difference between the Right Ascensions of the Fix'd Star and the Sun; this added to the Right Ascension of the Sun, gives the Right Ascension of the Fixed Star sought.

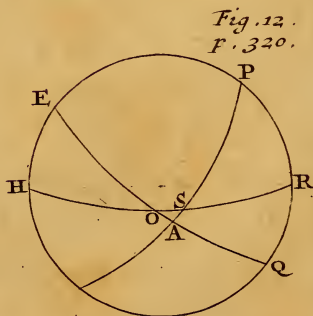
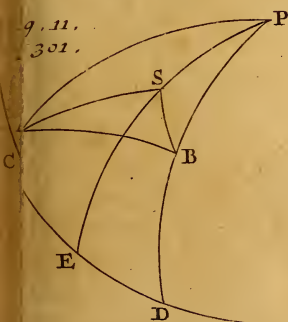
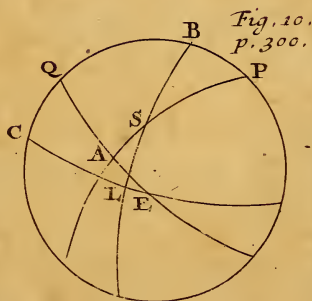
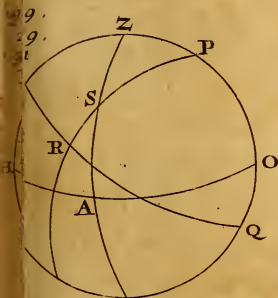
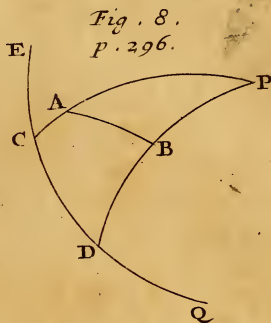
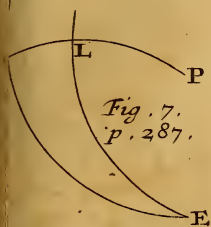
If in the Clock fitted to the diurnal Motion of a Fix'd Star, instead of the 24 Hours, there are placed 360 Degrees, and their Sexageffimal parts, and if while the Sun is in the Meridian, the Index be placed to the number of Degrees and Minutes, the Right Ascension of the Sun then consists of ; it will then point at the Right Ascension of the Fix'd Star, without any farther reduction, whenever it arrives at the Meridian.

PROPOSITION XXIV.

THE Declination and Right Ascension of one Star, and the Declination of another together with their Distance being given, to find the Right Ascension of the latter; and vice versa, to find the Distance of two Stars, whose Right Ascensions and Declinations are given.

Let *E Q* represent the Equator, [fig.8.] whose Pole is *P*; and the Stars *A* and *B*, whose Declinations *CA*, *DB* being given, their Complements also are given, or the distances from the Pole, namely *PA*, *PB*, and by supposition, the Distance *AB* of the Stars is given. Wherefore in the Triangle *APB*, the Sides being given, the Angle *APB* will be found, and consequently its measure the Arc *CD*; namely the difference of the Right Ascensions of the Stars *A* and *B*, which therefore added to, or subtracted from the Right Ascension given of one of them; (according as the Circle of Declination passing thro' this, is to the East or the West of the

Plate 2. Book 2.





the Circle of Declination passing thro' that,) gives the Right Ascension sought of the other.

And on the contrary, the Right Ascensions of the Stars A and B being given, the difference of them CD is also given; that is, the Angle CPB , whose measure that Arc is: And in the Triangle APB , the Sides PA , PB , the Complements of the given Declinations, together with the Angle APB contained are given; the other Angles will consequently be found, and the Side AB the distance of the Stars sought.

After the like manner you may find the Distance computed in a great Circle of two places in the Surface of the Earth, whose Longitudes and Latitudes are given, and the Angles which that great Circle (which answers the common Celestial Azimuth) makes with both Meridians.

PROPOSITION XXV.

TO determine by Observation the Right Ascension of a given Star, by a Phænomenon visible by day and night.

Let the Declination of the Phænomenon visible by day and night, be observ'd (by Prop. 18. and the distance as well from the Sun as from a Star; the latter in the night, the former in the day. From the Declination of the Phænomenon, the Distance from the Sun, and the Sun's Place, the Right Ascension may be deduced, by the preceding; from that and the Declination of the same being observed, together with the Declination of the Star and Distance from the Phænomenon being observed, the Right Ascension of the Star sought may after the same manner be found. Q. E. F.

S C H O L I U M.

The Ancients, who had not an accurate measure of Time, (such as *Prop. 23.* requires, their Hourglasses, the instruments they made use of, being too fallacious,) sought after the places of the Fix'd Stars by the Moon, taking the distance of the Moon from the Sun, computed according to the Equator or Ecliptic a days, and the distance of a Star from the Moon a nights, from whence they concluded upon the distance between the Star and the Sun, and consequently (by the Sun's Theory) the distance of the Star from the beginning of the Ecliptic.

The Moderns, instead of the Moon use Venus, because its Diameter being less will make the error less in observing the distance; and because the Moon's place comes often to be observed by the places of the Fix'd Stars first corrected, that its Theory may be corrected, which is not as yet accurate, especially about the Quadratures, which was the times that the Ancients made these Observations in; and also because the Motion of Venus is neither so swift, nor so uncertain. Therefore it will be best to observe Venus in its greatest Elongation from the Sun, when its Motion is nearly equal to that of the Sun; and when by reason of their Elevation the Sun and Venus are least obnoxious to Refraction. In making these Observations special care must be taken to repeat them, as well when Venus is to the East of the Sun as when it is to the West, so that by these different Observations conspiring to determine the same place of the Fix'd Star, the true place may be at last certainly had. *Ricciolus* is for using *Sirius* instead of Venus, which, as he says, shines so bright as about the Equinoxes is visible above eight minutes before Sun-set in *March*, and
after

after Sun-rise in *September*. But both *Regulus* and *Arcturus* have been observed by the naked Eye in the Meridian, while the Sun shone, by the Astronomers of the *French* Royal Academy of Sciences, even when the Sun was elevated above the uncertain Limits of Refraction: Which may be done every day in any of the brighter Fix'd Stars by the help of the Telescope.

PROPOSITION XXVI.

THE Height of the Pole, Hour from Noon, and Place of the Sun being given, if besides, the Altitude and Azimuth of a Star be known by Observation, to find the Right Ascension and Declination of that Star.

Let $PZHO$ represent the Meridian; [fig. 9.] HAO the Horizon, whose Pole is the Zenith Z ; ERQ the Equator, whose Pole is P ; S the Star, thro' which we are to imagine the Vertical Circle ZSA , and Circle of Declination PSR to be drawn. In the Triangle SZP , there are given the Side ZP , the Complement of the Elevation of the Pole OP , the Side ZS , the Complement of the Altitude of the Star SA ; and the Angle SZP , that the Arc AO , given by the Azimuth, measures; therefore the Side PS , the Complement of the Arc RS , of the Declination of the Star sought is found; as also the Angle ZPS , and consequently its Measure, namely the Arc ER . But the Hour of the Day being given, the Arc of the Equator intercepted between the Meridian and Circle of Declination, drawn thro' the Sun, is also given, consequently the sum and difference of the given Arcs are given. But this sum, if the Sun and Star be on different sides of the Meridian; or difference, if on the same, is the difference of the Right Ascensions of the Sun and Star.

And

And the Sun's Place being given, its Right Ascension (by *Prop.* 20.) is given; consequently the Right Ascension of the Star sought is also become known.

PROPOSITION XXVII.

TO find the Longitude and Latitude of a given Star.

In Figure 10, let $E \odot$ represent the Equator, whose Pole is P ; EC the Ecliptic, whose Pole is B ; the Point E the common intersection of these Circles, the beginning of γ or α ; S the given Star, thro' which the Secondaries BSL , PSA , of the Ecliptic and Equator are to be imagined to pass. And the great Circle $PB \odot C$ connecting the Poles is the Colure of the Solstices. Let AS the Declination of the Star be found (by *Prop.* 18,) then you will have its Complement PS ; and (by *Prop.* 23 or 25) EA the Right Ascension of the Star (at least from the given Ascension, the Right Ascension EA will be given,) from whence you will have its Complement $\odot A$, and the Angle $\odot PA$, whose measure it is. Therefore in the Triangle BPS , there are given the Side PS and the Side PB , namely the distance of the Poles of the Equator and Ecliptic equal to the Obliquity of the latter, together with BPS , the Angle contained by them; the Side BS therefore is known, and consequently its Complement SL , the Latitude of the Star sought. The Angle PBS also will be found, or CBL the next to it, and its measure the Arc CL , and therefore its Complement LE . But this Arc either is the Longitude itself, which is sought, or its Complement to one, two, three or four Quadrants.

In like manner, by the situation of a Star in respect of the Ecliptic being given, its situation,

tion, in regard of the Equator will be determined:

PROPOSITION XXVIII.

THE Longitude and Latitude of two Stars, and the Distance of the third from both being given, to find the Place of that third.

Let CED (in *fig. 11.*) represent the Ecliptic, whose Pole is P ; A and B the Stars, whose Longitude and Latitude are given, thro' which let the Circles of Latitude PAC , PBD be imagined to be drawn: The Arc of the Ecliptic CD is the measure of the Angle CPD , contained under them, the difference of the given Longitudes, and consequently is given. In the Triangle APB , the Sides PA , PB , the Complements of the Latitudes CA , DB , and the Angle APB , being given, the Side AB , and Angle PBA will be found. Again, in the Triangles ASB , all the Sides being given, namely SA , SB , the distances of the Star S from the two Stars A and B being given, and AB being first found, the Angle ABS will be found, and consequently SBP , the difference of this and PBA found before: And lastly, in the Triangle SBP , two Sides BP , BS , with the Angle SBP contained between them, being given, PS will be found, and consequently its Complement ES , namely the Latitude of the Star S ; and the Angle BPS , and its supplement DE , namely the difference of the Longitudes of the Stars B and S , to be added to the Longitude of the Star B , that the Longitude of the Star S may be had, if PSE be more to the East than PBD ; but to be subtracted, if more to the West. There are other cases of this Problem, according to the various

ous situation of the points *A* and *S*, but they are solved after the same manner.

And after the same manner, if the situation of the Stars *A* and *B* in respect of any other Circle, for instance the Equator, be given; that is, if their Right Ascensions and Declinations, and Distance of the third from both be given; the situation of it, in regard of that Circle, will also be given. And these things are done on the Earth after the like manner.

PROPOSITION XXIX.

TO describe the method of making a Catalogue of the Fix'd Stars; and to give an account of the chief Persons who have done so, and the methods they used; and to make a Celestial Globe.

Let every single Star of a Constellation be described in order, according to the place it has in the Constellation; and over against each set down the Longitude and Latitude thereof, found by *Prop.* 27, and then annex its Magnitude. If there be several Stars in the same part of the Constellation, (for instance, in the head or the hand) let them in the description be distinguished into Northern and Southern, and into such as precede, are in the middle or follow; where such as are situated toward the West are said, by such as follow *Ptolemy*, to precede; or such as possess the preceding parts of the Zodiac, *i. e.* nearer to the beginning of Aries; that is, such as precede in the Diurnal motion: They are also distinguished by some by the Letters of the Alphabet annex'd. If there be any remarkable Star in the Constellation, which has a peculiar name, this name also is put down; as *Arcturus* in Bootes, *Regulus* in Leo, *Syrus* in the Great Dog, &c. After the Stars proper to each Constellation, the unformed Stars about
it

it are annexed, described after the like manner by the nearness of this or that part (about which they stand,) together with their Longitude, Latitude and Magnitude.

The first that attempted to reduce all the Fixed Stars into a Catalogue, and determine their Places, was *Hipparchus of Rhodes*, about 120 Years before Christ, who, in *Pliny's* judgment, dared to do a thing that God himself did not approve of, to tell the number of the Stars for Posterity, and reduce them to a Standard. Tho' 'tis certain from *Ptolemy* that *Tymocharis* and *Aristyllus* left several observations about this matter, made 180 Years before *Hipparchus*. *Menelaus* also the Geometrician, in the first Year of *Trajan*, observed some of the places of the Fix'd Stars at Rome. After these, *Claudius Ptolemy*, about the 140th Year of Christ, began to observe and commit to Writing, some things about the progress of the Fixed Stars in *consequentia*; the Catalogue of *Hipparchus*, which rises to 1026 Fix'd being still retained in regard of the place as to Longitude and Latitude; nor have we any Observations of the Ancients besides these, which are delivered to us by *Ptolemy*. *Albategnius* the Syrian has done the same about the 880th Year after Christ: For in the Book that he has given the title of the *Science of the Stars*, Chap. 52. Pag. 202. he says that he has added 11 degrees and half and a third part to the places in which he found them to be described in *Ptolemy's* Book. For neither the *Persian* nor *Alphonsine* Astronomers, nor *Copernicus*, who is later than any of them, observed all the Fix'd Stars, but rested satisfied with the Catalogue of *Hipparchus*, only adding in the respective Catalogues the portion of the Motion, which was made in the mean while from them all; of which we shall speak here-

hereafter. *Copernicus* reckons the Longitudes of all the Stars from the preceding of the two in the Ram's Horn, which he calls the *First of all*, and which, by consequence, are the same in all Ages, tho the Longitude of this first Star from the Equinoctial point continually increases.

All these Astronomers in the determining the Places of the Fix'd Stars, for the most part, made use of an Armillary Sphere, or a part of it, (which they called an *Astrolabe*.) Its description may be seen in Chap. 1. Book 5. of *the Great Construction*. This Instrument being fitted to the Latitude of the Place and the Cardinal points of the World, they placed the Armilla or Hoop representing the Ecliptic, in the day time, by the help of the Sun's Rays, (whose place they knew by the Theory of it,) in such a situation as the Ecliptic had then, in regard of that Horizon; then they observed the place of the Moon in the Ecliptic, by the help of a movable Circle of Latitude in the Astrolabe. The next Night, by the help of the Moon, (whose place found before, they corrected according to its motion performed in the mean while,) they placed this Zodiacal Armilla in such a situation as was agreeable to the present moment of time, (as before they had done by the mediation of the Sun;) thus they observed the Longitude of each Star reckon'd according to the Ecliptic, as they had before observed in the day time the Longitude of the Moon (looking along the moveable Circle of Latitude,) and its Latitude by the help of a moveable Sight in the said Circle. You may see this method of Observing, fully described in Chap. 4. Book 7. of *the Great Construction of Ptolemy*, and in Chap. 14. B. 2. of *Copernic. of the Revolut.*

Tho' the Ancients thought the Moon was absolutely necessary to determine the places of the Stars in respect of the Equinoctial and Solstitial points, and together with these the most sagacious Copernicus, Chap. 14. B. 2. says thus: *For without her there would be no way to comprehend the places of the Stars, because she alone of them all partakes of Day and Night: But Cardan, in a Book, which he entitles Supplementum Almanack,* (after he had told us that the Fix'd Stars were to be compared with the Moon during a Lunar Eclipse, because the Moon's Place in respect of the Sun, is at such a time best known, because opposite to the Sun's,) very happily substitutes Venus instead of the Moon in this affair, because she is both a day and a night Star, and has some other advantages, mention'd in the *Schol. of Prop. 25.*

The next after Hipparchus, that observed the Fixed Stars themselves a-new, was Ulugh Beigh, the Grandson of Tamerlane the Great. He says in the Preface, that he observed all that could be observed, besides 27 in the South. This Prince died about 1449, and set down in his Catalogue (translated into Latin by Mr. Tho. Hyde) the stars places for the Year of Christ 1437.

At length in the 16th Century appear'd the Atlas of that Age, the immortal Tycho Brahe, who by indefatigable labour and pains observed a-fresh all the Fix'd Stars that are easily seen above the Horizon of Uraniburgh. And at the same time William the most illustrious Landgrave of Hesse, attempted the same great Work at Cassel, and with his Mathematicians Rothman and Byrgius, continued in it for above 30 Years. He sought for the Longitude and Latitude of the Fix'd stars by Calculation, from the Right Ascension and Declination first observed by the method in

Prop. 27 ; as *Tycho* assures us in *Chap. 2. Part 1. Progymn. pag. 147* ; tho' he does not intirely approve of it, because of the difficulty of reckoning the time exactly from Noon, which tho' often times tried by himself, by making Clocks and other ways described in the before quoted *Ch. of the Progymnasmatata*, yet never succeeded according to his wishes. But tho' the Observations of the Prince don't extend to above 400 Fix'd Stars, and were made by smaller Instruments, yet *Hewelius* in *Prod. Astro.* prefers then to the labours of *Tycho* himself, because in his opinion, the Prince and his Observers used more diligence than *Tycho* and his fellow Observers. The Prince afterwards performed this business by the help of Venus ; but *Tycho*, tho' in *pag. 146. B. 1. Progymnasmatata* he charges *Cardanus* with an error in settling the place of the most Southern and Bright Star of the Scales of Libra by Venus, greater than it is in the *Alphonsine Tables* ; nevertheless confesses, that it arose from hence, that the situation of Venus was not at that time well determined by Numbers. And he himself uses Venus for the same purpose after the manner described in *Prop. 25*. Instead of a Zodiacal Armilla used by the Ancients in observing the different Longitudes of the Fix'd Stars and their Latitudes, *Tycho* for very good reasons, substituted the Æquatorial Armilla by which he might observe the differences of the Right Ascensions and the declinations of the Meridian, the Meridian Altitude being always made use of to confirm the others. All the other cautions which this most diligent Observer took in this Affair, may be seen in *Chap. B. 2. Progymn.* The Places of some of the Fix'd Stars being once laid down, found by the methods already delivered, he determined the Pl.

ces of the other Stars which were near the Ecliptic or Equator, after the manner related in *Prop. 24*. He sought the Right Ascension of any unknown Star by two, whose places were already verified, and if the Right Ascensions of the same Star thus found, were a little different from one another, he always took the mean for the true one without any hesitation: Afterwards from the Right Ascension and Declination of a Star he computed its Longitude and Latitude. As for the Stars about the Poles he deduced them from the former by the the Praxis described in *Prop. 28*; tho' he verified the Place thus found by observing its distance from a third Fix'd Star first determined. And universally, he never rested satisfied with the Place of a Fix'd Star determined only by one observation; but each of them by the methods described before, by his Equatorial Armilla, and by the Praxis laid down in *Prop. 23*, making use of the best Clocks for measuring the time, Quicksilver and Lead being Chymically reduced into powder, and shewing the time by its running thro' a little hole; all which you may see accurately describ'd in the *Progymnasmata*.

Tycho's Catalogue of 777 Fix'd Stars made with such care and diligence as this for the Year 1600, and taken from the Heavens themselves, at last came forth, in the Year 1610, in *Astron. Instaur. Progymnasm.* And afterwards in the Year 1627, copied into the *Rudolphine Tables*, and increased by 223 Stars from some other observations of *Tycho*; so that this Catalogue in the *Rudolphine Tables* rises to the number of a thousand Stars. *Kepler* has added a second Class of Fixed Stars, that *Tycho* left out of the ancient Catalogue of *Hipparchus*, repeated and corrected by *Ptolemy*, and calls them *Semi-Tychonic*: He

subjoins a third containing 12 Celestial Images which are invisible in our Northern temperate Zone. These he has put into his Catalogue being named by the *Portuguese*, and corrected by *Peter Theodori*. 'Tis generally reported that these Southern Constellations were afterwards observed by one *Frederick Houtman*, a Dutchman and the *Celestial Globe of Bleau* corrected according to those Observations. But the Celebrated Mr. *Edmund Halley* was the first that took the pains about them as became an Astronomer while he was at *St. Helena*, and publish'd a Catalogue of 350 Southern Fix'd Stars to the Year 1677 compleat, which is a true Supplement to *Tycho's Catalogue*. He observ'd the Distances of the unknown Southern Stars from some of *Tycho's* and calculated their places from the assume places of some of the *Tychonic* ones (by *Prop. 28*, adding also the distances themselves, which he had observed.

The next after *Tycho*, that presented the World with an intire Catalogue of the Fix'd Stars of his own, was *R. P. Ricciolus*, in *Astron. Reform. Lib. 4.* to the Year of Christ 1700 compleat. But this Catalogue containing as many Stars as *Tycho's*, is taken from *Tycho's*, excepting in 101 Stars, which are all that he had observed in the Heavens. In the others, he recedes a little from *Tycho*, and has copied *Tycho's* manifest errors into his own, retaining at the same time some Stars as were really observed by *Tycho*, but were not visible in the time of *Ricciolus*, as *Hevelius* has observed in *Prodr. Astr. p. 133.* and the following.

Last of all came out a Catalogue of the Fix'd Stars, by the Illustrious *Jo. Hevelius* of *Dantzick* containing 1888 in all; namely 950 known by the Ancients, 603 which he calls his own because

because they were determined by none before himself as they should be by proper Instruments, and the 335 of Mr. *Halley*, invisible in the Horizon of *Dantzick*. In the making of this he solemnly professes that he had no regard to the observations of any Author, but entered upon the work, as if there were no Catalogue at all in being. Nay, he intirely laid aside all other foundations, and composed a-new the Theory of the Sun and Solar Tables, founded intirely upon his own observations. And lastly, he sincerely declares, that he has borrow'd nothing of any body, or put into his Catalogue the Place of the smallest Star, that he himself had not obtained by his different Distances and Meridian Altitudes.

Hewelius drew up two such Catalogues, the first or larger, containing as well the Longitudes and Latitudes, as the Right Ascensions and Declinations, to the Year of Christ 1660 compleat; to which are annexed the Places as to Longitude and Latitude, reduced to the same Year, of the Fix'd Stars observ'd formerly by *Tycho Brahe*, the Prince of *Hesse*, *Ricciolus*, *Ulugh Beigh* and *Ptolemy*; the second or less, exhibiting the Places of the same Fix'd Stars as to Longitude and Latitude, to the Year of Christ, 1700 compleat; to which he has annexed Mr. *Halley's* Catalogue of the Southern Stars, partly corrected as to their Longitude, from observations at *Dantzick*, and reduced to the same time: So that the larger Catalogue presents us at the same time with the Catalogues of all his Predecessors; by which means it is evident at sight, how far the observations of them all, agree or disagree. To these he subjoins a Table of the Right Ascension and Declination of several Fix'd Stars

to the Years of Christ 1660 and 1760, with their differences for that hundred Years.

By the help of such a Catalogue of the Fix'd Stars as this, a Celestial Globe may be easily made. For the Ecliptic and its Poles being marked out upon the surface of the Globe, let the places of the Fix'd Stars taken out of the Catalogue be in like manner marked out according to their Longitude and Latitude in regard of that Ecliptic, draw also the tract of the Milky Way, thro' the places of the Fix'd Stars, thro' which it passes; and add the Equinoctial Circle, cutting the Ecliptic in the point assumed for the beginning of the reckoning, and the opposite point, and duly inclined to it; as also let the Polar Circles and Tropics, and the other necessities be added; of which we have spoken before in *Prop. 13*. And to this very purpose did *Hevelius* design his lesser Catalogue, and from hence did he deduce his *Uranography* or Tables of all the Stars drawn upon a Plane, which was publish'd in the Year 1690, under the Title of the *Firmament of Sobieski*. For they differ no more from a Celestial Globe than Maps do from the Terrestrial. These *Uranographic* Schemes of the Constellations are delineated in *Plano*, just as if the convex part of the Heavens were seen by us, as it is in the Artificial Globe, as was usual among such as went before him, and made Catalogues of the Fix'd Stars. He says that *Bayer*, who in his *Uranometry* attempted the contrary, *thoughtlessly inverted all the Stars*; and that, *by his Figures he did but disorder the whole business of the Heavens*; and that, *all the Stars which are placed on the left hand in the Globes, appear on the right hand*.

The making of a Celestial Globe by *Hevelius's Firmament* is much easier, because the Figures of

of the Constellations are very aptly delineated in it, together with the Milky Way, and its different Streams as it were. But we are to expect the most ample Catalogue of the Fix'd Stars from Mr. *Flamsteed's* Observations, which have been continued for several Years with this view, and made with large Instruments, and at the same time very fit for that purpose.

PROPOSITION XXX.

TO give an account of some of the more remarkable Changes as have happen'd among the Fix'd Stars.

So long ago as the Age of *Hipparchus*, a new Star appeared among the Fix'd ones. And this, as *Pliny* informs us, was the reason why *Hipparchus* set upon numbering the Fix'd Stars for the use of Posterity, and reducing them to some Rule, having invented some Instruments for observing the Places and Magnitudes of each; so that it might easily be known by this, not only whether they perished or had a beginning, but whether any at all passed away or had any Motion; as also whether they increased or decreased, the Heavens being left in common to them all.

And tho' *Ricciolus* reckons no less than six new Stars between this and that famous one, which appeared in *Cassiopeia* in the Year 1572; yet since they were either scarce observed at all, or by Persons not over-skill'd, there hardly remains any thing to be said of them.

This new Star in *Cassiopeia*, a rival to the biggest in the Heavens, first appeared about the beginning of November, in the Year 1572, and continued until March in the Year 1574. This, just as that of *Hipparchus*, was the occasion of *Tycho's* observing the Fix'd Stars again, with new and large Instruments; as he informs us himself in the *Progymnasm.* where he treats expressly of this Fix'd Star.

In August 1596, *David Fabricius* discover'd a New one of the 3^d magnitude in Cetus, which disappeared about two months after.

In the Year 1600 a New Star appeared in the breast of Cygnus, first seen by others, but afterwards by *Kepler* himself, who has written an Astronomical account about it. This continu'd of the same magnitude viz. the 3^d until the Year 1659, as *Hevelius* assures us; (who had observed it from the Year 1638, with large Instruments,) but from the Year 1659 it sensibly decreased; and about the latter end of the Year 1661, it intirely disappeared. Yet five Years after, namely in September 1666, *Hevelius* observ'd it again with his naked Eyes, of the 7th or 6th magnitude, retaining the same place in the Heavens precisely.

In the Year 1604, about the beginning of October, a New Star was seen in the right Foot of Serpentarius, which appeared bigger than Jupiter and almost equal to Venus: It continu'd a whole Year; because it was not seen after October in 1605. Concerning this *Kepler* has written a Book full of Astrological and Physical disputes.

It was the Milky Way that presented us with these three New Stars, namely in Cassiopeia, Cygnus, and Serpentarius; it has therefore on that account been called by some, the Fund of new Stars.

There have been some other New Stars observed, namely in the Year 1612. that in the belt of Andromeda, by *Simon Marius*, who is the Author of the *World of Jupiter*, and another in Antinous by *Justus Byrgius* and others.

In the Year 1638, *John Phocyllides Holmwarda* of Franequer observed a New Star in the Whale's neck of the 3^d magnitude, or bigger. How it disap-

disappeared every Year, and then appear'd, but not always exactly at the same time, (tho' *Bulialdus* thinks otherwise,) and afterwards was invisible for four Years, is fully evident, from the History of it written by *Hevelius* and annexed to his *Mercury*, and from his *Climacteric Year*. Mr. *Cassini* has found by Observation, that the same Phases of this Star return at the end of 330 days; but so as that this return is sometimes retarded or anticipated by 15 days. This star seems to be the same with that, which *Fabricius* observed in the Year 1612; for their places are the same.

In the Year 1670 in *July*, *Hevelius* discovered New Star of the 3^d magnitude under the wan's head, and in his Catalogue reduces it to *Vulpecula*; but it was lessen'd so much in a short space of time, that about the end of *August* 1671 it was intirely extinguished: But next *March* it appeared again, at first like a very small Star, but by degrees it increased to one of the third magnitude, and afterwards grew as again sensibly as it did before; and in *September* 1672, totally disappeared, and was seen no more.

Not only New Fix'd Stars now-a-days make their first appearance, but others of the sixth, fifth and fourth magnitude, known by the Ancients and observed by *Tycho*, have intirely vanished, instances of which may be frequently met with in the *Catalogue of Hevelius*. He himself mentions four in *Prod. Astr.* namely a Star in the left thigh of *Aquarius*, the contiguous one preceding in the Tail of *Capricorn*, the second of the Belly of the Whale, and the first of the informed ones after the Scales of *Libra*. There are also other Observations which have been made about them by the *French Astronomers*.

mers. And even some considerable Fix'd Stars of the first, second and third magnitude are found to alter their brightness and magnitude sensibly, as is evident from the different sentiments of Authors about Stars of the first and second, or of the second and third magnitude, That Posterity may be more capable of judging about the change of the more considerable Fix'd Stars with greater ease and less mistake; *Hevelius* in his *Prodromus* has drawn up several things tending to that purpose observed by himself.

I pass over other changes in the Fix'd Stars observed by the help of a Telescope, such as are those said to be discovered lately by Mr *Cassini*; namely that some, as for instance, the first of Aries and the preceding head of Gemini, sometimes appear divided each into two equal to one another, and distant from one another the length of the diameter of either of them; others, as some of the Pleiades and the middle in Orion's Sword, appear triple, nay quadruple sometimes. For these Phenomena, besides that they are not to be seen by the naked Eyes (of which we only treat at present,) seem to have another original, and consequently are to be considered in another place.

PROPOSITION XXXI.

TO define the true precession of the Equinoctial Points, or the motion of the Fix'd Stars in consequence; and to determine from a given Catalogue of the Fix'd Stars, the place of any Fix'd Star at a given time.

Hipparchus of old, as *Ptolemy* informs us, Ch. 1. Book 7. of the *Great Construction*, from a comparison which he made of his own Observations with those of *Aristyllus* and *Tymocharis*, suspected that the Stars of the Zodiac had a motion

tion in consequentia : But *Ptolemy* from a comparison of the Observations of *Hipparchus*, and of others, with his own, has expressly asserted it of them all, and that it is made in Circles parallel to the Ecliptic ; which he proves in several of the more considerable Fix'd Stars, Chap. 2, and 3, comparing their places in regard of the Equator observed by such as lived before him and by himself. And he finds that this progressive motion is one degree in an hundred Years. *Albategnius* from the places of *Regulus* observed by *Menelaus* and himself, at the distance of 785 Years, gathers in Chap. 52 of his *Book of the Knowledge of the Stars*, that the motion of the Fix'd Stars is one degree in 66 Years. *Ulugh Beigh*, comparing his Observations with such as were made a great number of Years before, determines in the *Preface to his Tables*, that the Fix'd Stars move one degree in every seventy Solar Years. Later Astronomers comparing the Observations of the Ancients and Moderns, and the intermediate ones of *Albategnius* about this matter, find that this motion is equable, quicker than *Ptolemy* believed, but slower than *Albategnius*, and nearly the same with what *Ulugh Beigh* assigns. For *Tycho* in his *Progymn.* Book 1. gathers, that it is one degree and 25' in a hundred Years, or 51" in one Year. *Copernicus*, tho' he looked upon this motion to be unequal, made he mean to be, from the Observations of the Ancients and his own compar'd, to be about $1^{\circ}. 23'. 40''. 12'''$ in 100 Years. *Ricciolus* makes it something less, namely $1^{\circ}. 23'. 20''$. in that time, which Mr. *Flamsteed* retains. *Street* in his *Astronomia Carolina* makes it still less, as not above $1^{\circ}. 20'$. *Bullialdus* in his *Astronomia Philolaica* makes it $1^{\circ}. 24'. 54''$ in 100 Years ; whilst *Hevelius* lately in his *Solar. Tab. in Prodrum.* will have it to be only $1^{\circ}. 24'. 46''. 50'''$. For

For every Year therefore, forward from the the Year to which the Catalogue is fitted, add 50'' (the middle round number) to the Longitude of the Fix'd Star found by the Tables, and you will have the Longitude sought: The Latitude in the mean while continues invariable, since both the Fix'd Stars and the Ecliptic in the Heavens are immovable, and only the Equator (originally a Terrestrial Circle) being loosen'd as it were leaves its ancient seat. But for every single Year, backwards in respect of the given Catalogue, subtract 50'' from the Longitude in the Tables, if you would find the Longitude for such a given time. But if the order of the given Catalogue be such, as that the Longitudes of each of the Fix'd Stars are reckon'd from the first Star of Aries, (as it is in *Copernicus's* Catalogue of the Fix'd Stars, as also *Clavius's*, and of some annexed to the *Caroline Tables*,) to the number of the Longitude in the Tables, (because continuing always the same) add the Longitude of the first Star of Aries (which at the beginning of the Year 1701, according to *Hewelius* is $29^{\circ}. 00'. 58''$, and it is to be increased by 50'' every Year afterwards, but lessened by as many for every Year before, as was said before) for the time given, and you will have the Longitude sought from the Vernal Equinox, casting away 360 degr. if the sum exceed that. But if the Right Ascension and Declination at a time given be required, what is proposed may be done by the converse of *Prop. 27*, from the Longitude and Latitude found for the given time, as before.

SECTION VI.

Concerning the resolution of the more considerable Problems of the First Motion by Calculation.

PROPOSITION XXXII.

TO determine by observation the difference of the Meridians or Longitudes of two Places given upon the Surface of the Earth, and to make a Catalogue of the most noted Places of the Earth, with their Latitudes and the Differences of their Meridians, and to make a Terrestrial Globe.

Let any instantaneous Phænomenon (as the beginning or end of a Lunar Eclipse, or of an Eclipse of any of Jupiter's Satellites,) be observed by Observers in the given Places, noting the hour in both Places: The difference of the hours converted into an Arc of the Equator gives the Difference of the Longitudes or Meridians sought. And that of the Places proposed is more toward the East, that reckons more hours after Noon; But if they reckon the same, they are under the same Meridian. For since the same moment of time has differing names as to the hour in these two Places, the Noons of these Places or any other hours of the same name, will be as distant from one another, as these different hours in the same Place; and consequently the Arc of the Equator intercepted between the Meridians of the Places is rightly found by allowing 15° for every hour. This interval of Time is called the *Difference of the Meridians*, appositely enough, because it is so of the Noons.

In

In like manner, if a Clock that shews exactly the hours at one of the Places be carried to the other, to be compared with another shewing the hours there; the Difference of the hours reckoned by the Clocks, converted into an Arc of the Equator, gives the Difference of the Longitudes sought: For thus, as in the former case, you know the interval between the hours reckoned at the same moment in the two Places. For the Clock, that is transported, is supposed to shew the hours after the same manner, as it would have done, had it continued in its place.

To each of the more noted places that are put into a Catalogue, add the Latitude proper to it, expressed by the same number of degrees (by *Prop. 7.*) as is the Altitude of the Pole observed there by *Prop. 17.* Add also in another column the Difference of the Meridians expressed by hours and their parts (found by what has been said before) of the Place proposed and of the Place assumed, together with the mark of Addition or Subtraction, according as the Meridian of the one lies to the East or to the West of the Meridian of the other.

The Latitudes of the Places and the Differences of the Meridians being given, 'tis easy to make by them a Terrestrial Globe: For the Poles and a great Circle upon them being drawn upon a Sphere made for that purpose, which will represent the Poles of the Earth and the Equator, thro' the Poles draw a Circle representing the Meridian of the Place (to which the Meridians of the other Places in the Catalogue are referred,) and in it mark out a point distant from this feigned Equator, as many degrees as there are in the Latitude of the Place set down in the Table; and that point will there

herefore represent the said Place. Any other Place is to be put in a Circle passing thro' the Poles, between which and the Meridians already settled, there are as many degrees of the feigned Equator (and that according as the second Place is, towards the East or West,) as there are in the tabular difference of Longitudes found above; and as many degrees from the feigned Equator towards either Pole, as there are in the tabular Latitude of that Place. In like manner, the other more noted Places are to be marked upon the surface of the Globe. The intermediate or less noted Places are supplied by the common Chorography, and Itineraries, making use of the Latitude of some for correction; for this is more easily observed than the Difference of Longitude. The Globe being thus finished, either the Meridian of our own place, or any other passing thro' the Azores or any of the Fortunate Islands, may be reckoned for the first; that is, the degrees of the Equator are to be reckoned from the intersection of it with the said Meridian (or beginning) towards the East. Then the other Circles and necessary appurtenances described in *Prop. 14.* are to be added according to art.

PROPOSITION XXXIII.

THE *Elevation of the Pole, the Right Ascension and Declination of a Star, or any other point in the Heavens being given, to find its Ascensional Difference, and by that means its Oblique Ascension, Continuance above the Horizon, and Eastern Amplitude.*

Having laid down in what goes before, the method of determining the situation of the Circles of the Sphere for any habitation or place, and of finding by observation the places of the
Stars

Stars in regard of the said Circles in the Celestial Sphere, and the situations of Places on the Earth in respect of the same, considered on the surface of the Earth; we ought now, by the rules of Method, to shew how to resolve by calculation some of the chief remaining Problems, about what has been determined and found out already, and according to their Example may any others relating to the First Motion, be easily resolved after the same manner.

In the present case, let $E Q$ (Fig. 12.) represent the Equator, whose Pole is P ; HOR the Horizon, whose common intersection's, O and the opposite point, are the points of true East and West, namely the Poles of the Meridian PEQ . Let S be a Star rising or setting, and thro' it imagine a Circle of Declination PSA drawn. In the Triangle OAS there are given, besides the right at A , the Angle SOA , whose measure is HE , the Altitude of the Equator; the side AS , the Declination of the Star given; consequently OA , the Ascensional Difference sought, may be found: For O is the point of the Equator that arrives at the Horizon together with the Star; A is a point of the same that arrives with it at the Meridian. This Ascensional difference therefore subtracted from the Right Ascension of the Star declining towards the elevated Pole, but added to the Right Ascension of the Star declining towards the depressed Pole, gives its Oblique Ascension; and so on the contrary in the Oblique Descension. But this same Ascensional Difference added to the Quadrant, or subtracted from the same, gives the semidiurnal Arc of the Star, according as it declines towards the elevated or depressed Pole: And that turned into Time, gives half the continuance of the Star above the Horizon. In the Triangle OAS the side OS is also found, the Eastern or Western

By

By resolving the same Triangle OAS after the same manner may the converse of this Problem be solved; that is, the Right Ascension, Declination and Continuance above the Horizon of any Star or Point being given, to find the Elevation of the Pole.

PROPOSITION XXXIV.

THE Elevation of the Pole being given, if the Altitude of a Star be known by observation, whose Right Ascension and Declination are known, to find the moment of Time, and the Azimuth of the Star.

The same Circles of the Sphere being expressed by the same letters, let the Vertical ZSA (fig. 9.) be supposed to be drawn thorough the Star S . In the Triangle PZS , all the sides are given; namely PZ , the complement of the Elevation of the Pole; ZS , the complement of the Altitude of the Star above the Horizon; and PS , the distance of the same Star from the Pole, or the complement of the Declination: The Angle PZS therefore is found, which is measured by OA the Azimuth sought; as also the Angle ZPS , whose measure is the Arc of the Equator ER . But the Sun's place being given, its Right Ascension is also given: Consequently the difference between this and the Right Ascension of the Star, known also by supposition, is also given: Therefore the sum or difference of these Arcs will be given; namely the Arc of the Equator intercepted between the Meridian PEH and the Circle of Declination drawn thro' the Sun, which converted into time, gives the time before or after Noon, according as the Sun is to East or West of it.

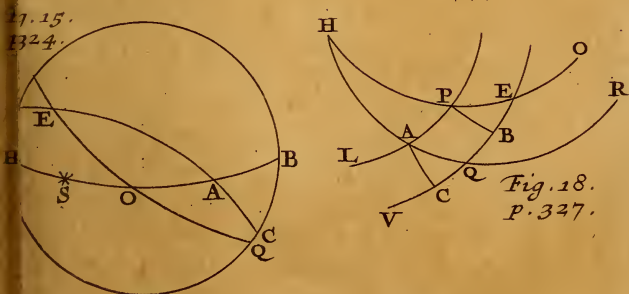
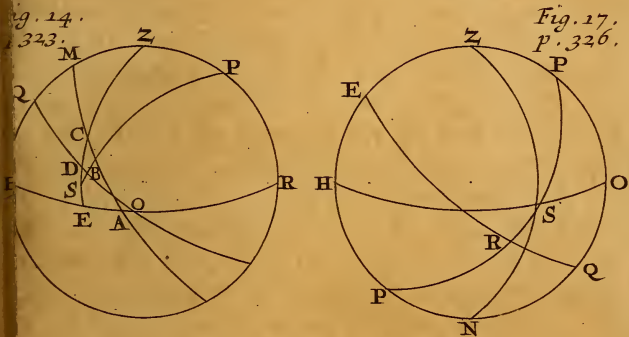
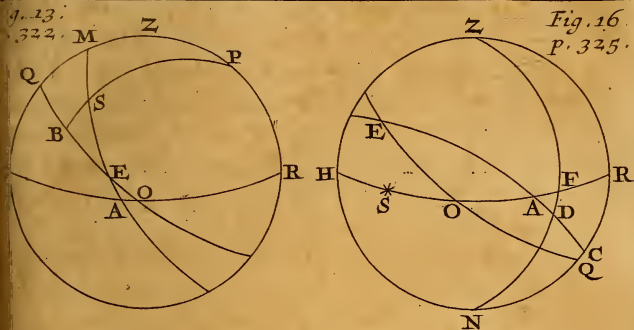
You will have the same, if instead of the Altitude of a Star its Azimuth be known by observation.

PROPOSITION XXXV.

THE Longitude of the Place, the Hour of the day, and the Sun's Place being given, to determine the situation of the Ecliptic in respect of the Horizon; that is, to find the Angle of the Ecliptic and Horizon, (or the Altitude of the Nonagesim degree of the Ecliptic from the rising point above the Horizon,) the rising point of the Ecliptic (and from thence the Nonagesim degree) and the point of the Horizon, in which the Ecliptic intersects it.

Things remaining as before, let S be the Sun in the Ecliptic $MSEA$ [Fig. 13.] Let the right Ascension for the given hour be found (by Prop. 20.) or (making PSB the Circle of Declination,) the point B in the Equator, or consequently its distance from E the next Equinoctial point, namely EB . And since the Hour is given, the Arc of the Equator BQ intercepted between B and the Meridian, and consequently BO its complement to the Quadrant of the Equator QO : But BE is given, from whence the Arc EO is known. Therefore in the Triangle AEO comprehended between the Ecliptic, Equator and Horizon, there are given the Angle AEO , the obliquity of the Ecliptic, and AOE the inclination of the Equator to the Horizon, and the side EO just found: From whence the rest will be known; of which EAO or its complement to two right MAH is the Angle that the Ecliptic makes with the Horizon, measured by the Altitude of the Nonagesim degree; EA the distance of the rising point of the Ecliptic from the Equinoctial point E , and from thence the highest point, or the Nonagesim degree from the rising point A becomes known; and OA the distance of the rising point A from the true East O , computed in the Horizon. $Q. E. F.$

Plate 3. Book 2.





PROPOSITION XXXVI.

THE same things being given, to find the situation of a Star, whose Longitude and Latitude are given, in regard of the Horizon, or the Altitude and Azimuth; as also to determine the point and Angle, in which the Vertical thorough the Star intersects the Ecliptic.

Let the place of the Star S given in regard of Longitude and Latitude (by the converse of *Prop. 27*) be reduced to the Equator; is, find its Right Ascension and Declination. And since the Sun's Right Ascension is given, the difference of these Ascensions will also be given. But from the Hour of the day given, the Arc of the Equator intercepted between the Meridian and Circle of declination drawn thro' the Sun is known: The Arc therefore (*Fig. 14.*) $\angle B$ of the Equator, between the Meridian and PS the Circle of declination passing thro' S , is also known; and consequently the Angle $\angle P B$, which it measures. In the Triangle ZPS , there are given the sides PZ , PS , (the one the complement of the Pole's Elevation, the other the distance of the given Star from the Pole, known by its Declination,) together with the Angle ZPS just now found; by which means there will be known the side ZS , and the Angle PZS : ZS is the complement of the Altitude ES , and RE the Arc of the Horizon, which is called the Azimuth, measures the Angle PZS ; and since RO is a Quadrant, the Arc OE the distance between O the point of the true East and the Vertical ZE drawn thro' S , is also found. Besides, in the Triangle ECA , comprehended between the Vertical ZSE , Horizon HOR , and Ecliptic ACM , the Angles at E and A are given, (the one being found above,

and the other a right angle,) together with the intermediate side EA , the sum or difference of the known Arcs OE , OA : Therefore the other sides are also given, namely AC , the distance of the point C from the rising point A , which is known, the side EC , and the Angle ECA , which the Vertical ZSE drawn thro' the Star S , contains with the Ecliptic. *Q. E. F.*

Tho' this Problem be no more than the converse of *Prop. 26*, yet because of its frequent use, I thought fit to insert it.

COROLLARY.

The same things being given, by the same labour may the Place on the Earth, to which the Star S is Vertical, be determined. For in the Triangle ZPS , in which all the Sides and Angles are known, the side PS denotes the distance of the Place on the Earth from the Pole P , from whence its Latitude is known; and the Angle ZPS is the distance of the Meridians of the Place given and sought. But if S be in the Ecliptic, the Problem becomes more simple; and is still the more so, if it be the Sun, or the Point opposite to it.

PROPOSITION XXXVII.

THE Elevation of the Pole and the Oblique Ascension of the Star being given, to find its Cosmical and Achronical Rising.

The Circles of the Sphere [*Fig. 15.*] being marked by letters, as oftentimes before, in the Triangle EOA , there is given the Angle at E , namely the Obliquity of the Ecliptic; the side EO the Oblique Ascension of the Star S (at least the distance of the point of the Equator that rises together with it from the next Equinox, from the Oblique Ascension given) and the Angle EOA the inclination of the Equator to the Horizon, or its complement to two right:

right: Therefore the side EA will also be given; and consequently that point in the Ecliptic that rises with the Sun. Therefore when the Sun is in this point of the Ecliptic, the Star S rises Cosmically; and the Sun being in the opposite point, the same Star rises Achronically. And after the like manner may the Cosmical and Achronical Rising of a Star be found from the Oblique Descension of the Star (found by Prop. 33.)

But if the Cosmical Rising and Setting signify the same thing with an Ascent above Horizon or Descent below it; the time of the Cosmical Rising and Setting is determined by the preceding: For the Altitude of the Star (being nothing in its Rising or Setting) is given, and therefore the moment of time will be known. And then the Achronical Rising will signify what it commonly does; namely the Evening one. And the Achronical setting all one with the Cosmical setting, as it is commonly called, or Morning one.

PROPOSITION XXXVIII.

TO determine the Heliacal Rising and Setting of a given Star.

Things remaining as they were, let D [Fig. 16.] be the place the Sun is in when the Star S rises Heliacally. Therefore in the Triangle AFD there is given, beside the right angle at F , the Side FD , viz. the Arc of the Sun's depression requisite to permit the Star of a given magnitude to appear; also the Angle DAF , which the Ecliptic comprehends with the Horizon, when the Star S rises, found by Prop. 35; therefore the Side AD will also be found. But this Arc added to the Arc EA (found by the preceding) gives ED the distance

the Sun from the Equinoctial point *E*, when *S* rises Heliacally. Therefore the time of the Heliacal Rising of a given Star *S* will be known; namely the same as that wherein the Sun possesses *D* the point of the Ecliptic found. And after the like manner the Heliacal setting of it may be found.

PROPOSITION XXXIX.

THE Latitude of the Place and the Sun's Place being given, to find the beginning of the Morning and end of the Evening Twilight.

Let *ZEPN* [Fig. 17.] express the Meridian; *E* $\mathcal{Q}$ the Equator; *HO* the Horizon, whose Poles are *Z* the Zenith and *N* the Nadir; *S* the Sun depressed below the Horizon: Thro' which let the Circle of Declination *PSR* and Vertical *ZSN* be supposed to be drawn. In the Triangle *ZPS* all the Sides are given; namely *PZ* the complement of the Height of the Pole; *PS* the distance of the Sun from the Pole, known by its Place; and the side *ZS*, made up of the Quadrant and depression of the Sun, necessary for the beginning or ending of the Twilight: Therefore the Angle *ZPS* will be known, whose measure is the Arc *RE*, which converted into Time, gives the Hours sought, to be reckoned after Noon to the end of the Evening Twilight; and their complement to 12 Hours, or the Arc *R* $\mathcal{Q}$, the Hours to be reckoned from Midnight to the beginning of the Morning Twilight.

PROPOSITION XL. LEMMA.

IF two great Circles of the Sphere cut another great Circle at equal Angles, the Arcs of all Circles parallel to this intercepted by the two cutting the third, are similar to one another; and the Arcs of the interceptants contain'd between two Parallels are equal.

Let two great Circles HEO , HQR [Fig. 18.] cut the great Circle EQV in E and Q , so as that the Angles HEQ , HQV may be equal. I say, the Arc AP of any Circle PAL parallel to EQV , intercepted between HEO and HQR is similar to EQ ; and the Arcs PE , AQ of HEO , HQR intercepted between the two Parallels, are equal. Thro' the Poles of EQV , draw the great Circles PB , AC . And the Sine of the arc AC , is to the sine of the arc AQ , so is (the Sine AQC to Radius, so is Sine PEB to Radius so is) Sine PEB to Sine PE . But the arcs PB , AC , are equal (by Prop. 10. B. 2. Spher. Theodos.) and therefore their Sines; consequently their Arcs AQ , PE are equal. Besides, in the Triangles PEB , AQC , in which the angles B and C (because right) and E and Q (by supposition) are equal, and the sides PE , AQ and PB , AC are already shewn to be equal, the sides EB , QC , will also be equal, and consequently the Arcs EQ , BC are equal. But AP , BC (by Prop. 10. B. 2. of the Sphaeric, of Theod.) are similar Arcs; therefore QE and AP are also similar. And because the Circle PAL is taken at pleasure, the Arcs of all the Circles parallel to EQV , comprehended by the Circles HEO , HQR , are similar to EQ , and consequently to one another. Q. E. D.

PROPOSITION XLI.

TO find the Parallel the Sun is in, at the time of the Least Twilight.

It has been long ago observed, that the proportion of the increase and decrease of the Twilight is very much different from the increase and decrease of the Artificial Days. For the Days are upon the increase all the while the Sun returns from the Winter Tropic to the Summer one; but the Twilights do not. For these are lessen'd after the Winter Solstice to a certain limit before the Vernal Equinox, where they are smallest; and are afterwards increasing while the Sun passes the Equator and moves towards the Summer Solstice: And from thence they are upon the decrease to a certain limit after the Autumnal Equinox; and then again they increase to the middle of Winter. Let that Parallel then be proposed to be sought, that is described by the Sun in its Diurnal Motion, when the Twilight is smallest at a given Place.

Let $ZSNM$ [Fig. 19.] be the Meridian, SEM the Horizon, to which let a lesser Circle CAR called that of the *Twilights*, be parallel, namely that wherein the Sun is at the beginning of the Morning and end of the Evening Twilight. Let AP be the Parallel sought. Thro' the point A , where it intersects CR , suppose the great Circle AQH to be drawn meeting the Horizon HO in O and H , which cuts the Equator EQ in Q , so as that the angle AQE is equal to the angle PED or TEK . And therefore (by the preced. Lemma) the arc EQ is similar to the arc PA ; and the arc ap of any other Circle parallel to the Equator, intercepted between OES and OAL , will also be similar to the same AP or QE ; and each of them

Fig. 19.
328.

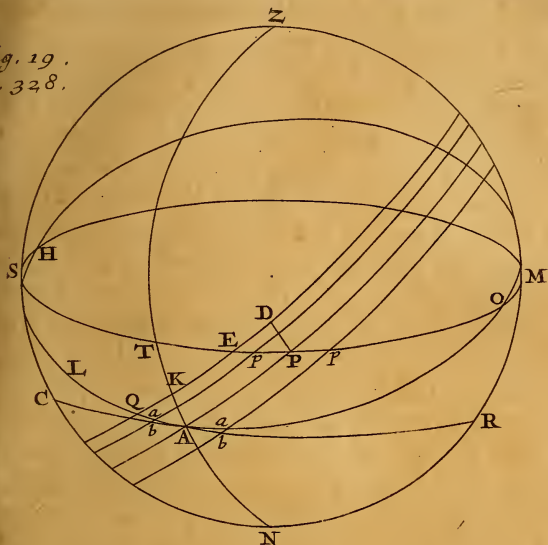


Fig. 20.
331.

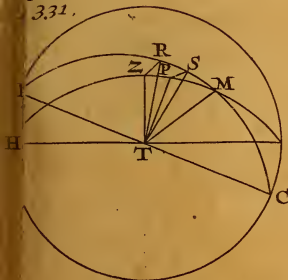
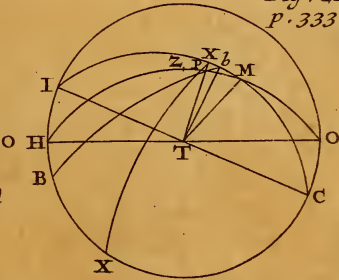


Fig. 21.
P. 333.





them will be the measure of the duration of the
 Twilight, while the Sun describes the Parallel
 PA . But because (by hypothesis) it is the
 least Twilight, none of these arcs, besides PA ,
 will reach to CR . (For while the Sun describes
 in its Diurnal motion any other Parallel, the
 measure of the duration of the Twilight is its
 arc pb , intercepted between the Horizon SEM
 and the Circle of Twilights CAR ; which since
 it is (by supposition) greater than the measure
 PA of the Least Twilight, 'tis evident that pa
 does not reach to CR , because pb which is
 greater, only reaches so far.) And therefore
 the Circle of Twilights CAR touches the
 great Circle $OA H$: and because part of it un-
 der the Horizon, (namely $OA H$) is a semi-
 circle, and CAR is parallel to the Horizon, 'tis
 evident that HLA or OA , is a quarter of a
 Circle. And therefore the Circle $NATZ$, de-
 scribed upon the Poles O and H thro' A , inter-
 secting the Equator at K , is a great Circle, and
 passes thro' the Zenith and Nadir, and meets
 the Horizon at right angles in T . Besides, in
 the Spherical Triangles ETK , $\triangle AK$, the an-
 gles ETK , $\triangle AK$, (by Prop. 15. B. 1. Theod.)
 are right and consequently equal, as also the
 angles TEK , $A\triangle K$, (by construction) are
 equal, and the rest TKE , $AK\triangle$ are likewise
 equal, (because vertical:) From whence (by
 Spherical Trigonomet.) these Triangles are mu-
 tually equilateral; that is, the arc TK is equal
 to KA , and TE to $\triangle A$: But (by the preced.
 Lemma) $\triangle H$ is equal to EP , therefore TE is
 equal to EP . Besides, in the Spherical Trian-
 gle ETK right angled at T , (by a known
 Prop. in Spheric. Trigonomet.) the Radius is to
 the Co-Tangent of the angle TEK of inclina-
 tion of the Equator to the Horizon, as the Tan-
 gent

gent of the arc TK to the Sine of the arc ET : And by permut. the Radius is to the Tangent of half the Arc of the depression of the Circle of Twilights below the Horizon, as the Tangent of the Pole's Elevation to the Sine of the arc ET or EP . Again, in the Triangle EDP , (making DP the Circle of Declination) right angled at D , the Sine of the arc EP is to the Sine of the Arc DP , as the radius to the Sine of the Angle DEP ; that is, to the Co-sine of the Elevation of the Pole: But in all arcs, the Tangent is to the Sine as the Radius to the Co-sine; wherefore the Sine of the Arc EP is to the Sine of the Arc DP as the Tangent of the Height of the Pole to the Sine of it. And by inverting and permutation, the Tangent of the Height of the Pole is to the Sine of the arc EP , as the Sine of the said Height is to the Sine of the arc DP . But (by what has been shewn before) in the case of the Least Twilight, as the Radius to the Tangent of half the arc of the depression of the Circle of Twilights, so is the Tangent of the Height of the Pole to the Sine of the arc EP ; and therefore the Radius is to the Tangent of half the arc of the depression of the Circle of Twilights, as the Sine of the Height of the Pole to the Sine of the arc PD ; namely of the declination of the Parallel that the Sun describes when the Twilight is Least.

Q. E. I.

SCHOLIUM.

If the depression of the Circle of Twilights below the Horizon be 18^d (as it is commonly determined,) the Radius will be to the Tangent of 9^d as the Sine of the Latitude to the Sine of Declination of the Sun towards the depressed Pole at the time of the Least Twilight. But the Decli-

Declination of the Parallel and Obliquity of the Ecliptic being given, the two points of the Ecliptic will be found (by *Prop. 20.*) and (by the Theory of the Sun) the times of the Year, when the Sun is in the given Places; that is, when the Twilight is Least. Tho' at *London* the Least Twilight is when the Sun declines from the Equator towards the South $6^d\ 7'$, and in Δ or $\Re$ $17^d\ 30'$, which happens in the present Age, about 22^d of *Febr.* and 27^{th} of *September* of the *Julian* Year. This was noted by *Blancanus* in his *Sphere*, who says that the shortest Twilights happen about the middle of *Libra* and the middle of *Pisces*. *Peter Nonius* has proposed this Problem and solved it (yet by two Proportions) in a *Book* concerning the *Twilights*, published in the Year 1541; who is followed by *Ambrosius Rhodius* and *Clavius*.

PROPOSITION XLII.

IN a Plane given by position, on which a Sun-Dial is to be drawn, to determine the Meridian Line, Angle of the Style, and the Substyle line.

Every Plane upon which a Sun-Dial is drawn, is supposed to pass thro' the Center of the Sphere, (because in respect of the distance of the Sun, its distance from the Center is insensible,) and consequently its intersection with the surface of the Sphere is a great Circle.

Let *IRSMCT* [Fig. 20.] be such a Plane passing thro' *T*, the Center of the World which produced to the surface, forms the Circle *IRSMC*. Let *HIO* be the Horizon of the Place, *HZO* the Meridian (in which is the Pole *P*, and the Zenith *Z*) intersecting the former Circle in *M*. The position of the Plane *IRSC* being given, its Declination is given, measured by the Angle *ITH*, or Arc *IH*, namely that whereby
the

the Horizontal Line IC of the Plane $IRSC$ declines from HO the Meridian line drawn on the Horizontal Plane, by *Prop.* 16. ZR is also given, namely the arc of the Vertical Circle perpendicular to $IRSC$, intercepted between the Zenith and the said Circle, the measure of the *Inclination* or *Reclination* of the said Plane. In the Spherical Triangle MZR rightangled at R , there is given ZR , and the the angle RZM , equal to the complement of the angle ITH to a right one: (For the Vertical Circle, ZR perpendicular to the Circle $IRSC$, declines from the prime Vertical (drawn thro' the Equinoctial East and West) as much as IT does from HT ; and consequently the complement of the one, namely RZM , is equal to the complement of the other, namely ITH .) Therefore the side RM will be found, or the Angle (which it measures) in the plane of the Dial RTM . But TR is given by position; (for it is succedaneous to a perpendicular on the same Plane, or a right line perpendicular to the Horizontal line ITC ;) wherefore the Meridian line TM sought in the Plane of the Dial, or the common intersection of the plane of the Meridian with the plane of the Dial, will be given. In the same Triangle MZR , are also found the side ZM and Angle RMZ , which will be of use by and by. Again, thro' the Pole of the World P , draw an arc of a great Circle PS , perpendicular to $IRSC$: And in the Triangle PSM rightangled at S , there is given the Angle PMS found just now, and the side PM , the sum or difference of the Arc IM found before, and of the distance of the Pole from the Zenith, namely ZP : Consequently there will be known both the arc PS , or the Angle, which it measures, PTS , the inclination of the Axis

PT to the Plane $IRSC$, that is, the Angle of the Style sought; and the side MS , or the rectilineal Angle STM , which it measures in the plane of the Dial, contained between TM the Meridian determined before, and the Substyle TS , namely that upon which the plane of the Gnomon PTS stands, whose other edge is the Axis of the World PT casting the shadow, and is perpendicular to the plane of the Dial. *Q. E. F.*

PROPOSITION XLIII.

TO find the Angle, that the line of a given Hour makes with the Meridian in the plane of the Dial; and thence to describe a Sun-Dial upon a given plane.

The same things remaining as before, let BPb [Fig. 21.] be the Hour Circle, the intersection Tb of whose Plane with the Plane IMC given by position is sought. The Hour of this Circle being given, the Angle HPB or MPb is given, contained by it and the Meridian. Therefore in the Spherical Triangle PMb , there are given (by preced. Prop.) the side PM , and the angle PMb , as also the angle MPb : Therefore the arc Mb is found; and the rectilineal angle, which it measures, MTb , which the Hour line Tb makes with the Meridian TM . Therefore the right line TM being given (by Prop. preced.) the Hour line sought Tb , in the plane of the Dial given, will be determined. *Q. E. F.*

And after this manner a Sun-Dial may be made, by drawing a Meridian and Substyle from a point assumed at pleasure for a Center, (by preced. Prop.) and erecting upon the Substyle a Style in an Angle found also by the preced. and drawing the other Hour lines in Angles

Angles with the Meridian TM , determined by this Proposition, and marking them with their proper numbers. Thus Tb is the eleven a clock Hour line after Midnight, if PB be a Circle to the East of the Meridian PZH , making with it an Angle of 15^d ; that is, if Tb be to the West of the Meridian line TM , and the Angle MPb in the surface of the Sphere of 15^d . And after the like manner TX , situated on the same side, will be the ten a clock Hour line, if MPX be 30^d , and so on.

PROPOSITION XLIV.

TO draw a given Hour line upon a Sun-Dial, whose plane passes thro' the Poles; and so to describe a Polar Dial.

Because all the Hour lines intersect one another in the Axis of the World, and this Axis is always in a Polar plane; 'tis evident that all the Hour lines, in such a plane, are to be found in that Right line: But that they may be distinguished, and not confounded with the Substyle, the Dial is not to be conceived as described in the plane IMC itself, [Fig. 22.] passing thro' the Center (as it is in all other cases,) but in another plane $im\gamma$, parallel and near to the former; so that the opaque right line that casts the shadow and coincides with the Axis of the World, is distant from the plane $im\gamma$, by the right line PS : And since the distance of the Sun is look'd upon as Infinite, any given Right line as PS , vanishes in respect of it, and the parallel Planes passing thro' IMC , $im\gamma$, are esteem'd as very near one another; and the portions Pm , Pb , &c. of the Hour circles HPm , BPb , &c. crossing one another in the Pole P , intercepted between the Pole and the Circle $im\gamma$, are to be looked upon as Right lines; and
the

Plate 5. Book 2.

Fig. 22.
P. 334.

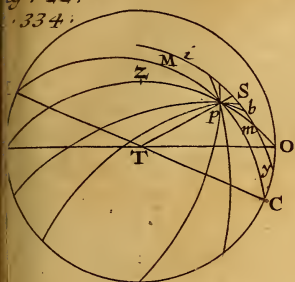


Fig. 25.
P. 339.

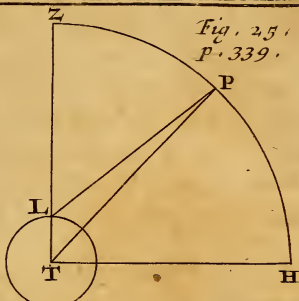


Fig. 23.
P. 337.

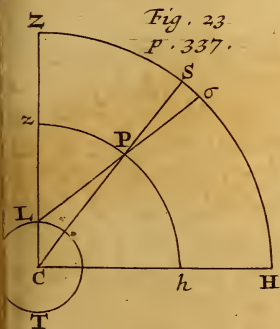


Fig. 26.
P. 340.

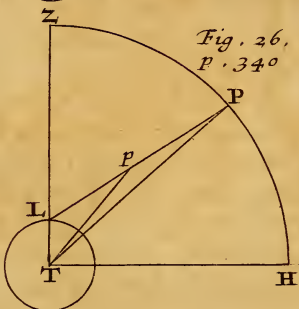


Fig. 27.
P. 341.

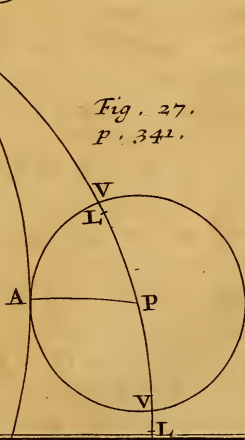
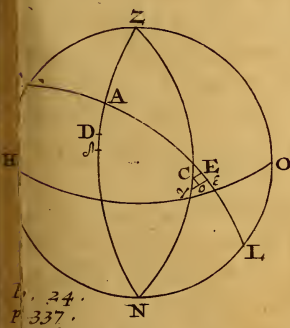
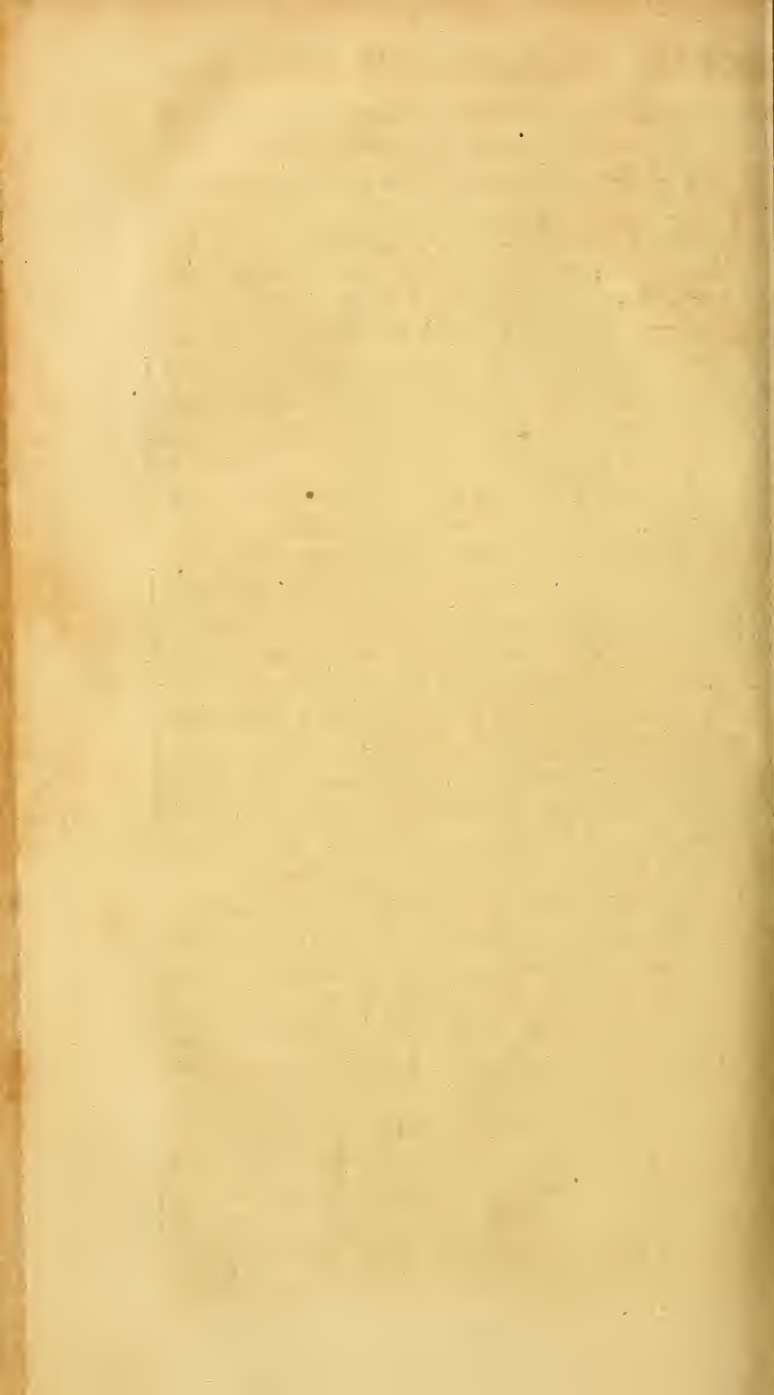


Fig. 24.
P. 337.





the part of the Spherical surface which these portions take up (being infinitely small in comparison of the whole) is to be look'd upon as a Plane.

In the Polar plane then, having found (by *Prop. 42.*) the Substyle, erect the Style of any breadth taken at pleasure, as PS , whose other edge plac'd in the Axis of the World is parallel to the Substyle; for in this case there is no Angle of the Style. To find the Meridian line, observe that in the rectilinear Triangle PSm , besides the right angle at S , there is given the side PS , namely the height of the Style; and the angle SmP equal to the angle MPZ , found at *Prop. 42*; hence you may find Sm the distance of the Meridian line from the Substyle in the right line Sm , which is perpendicular to the Substyle, because MP (parallel to iz ,) is perpendicular to PT ; and you may also find the side Pm : A right line therefore drawn thro' the point m , parallel to the Substyle (because, as was said before, the Hour lines and Substyle, which also is one of them, on this sort of Planes are parallel) will be the Meridian line in the Sun-Dial proposed.

If any other Hour line given is to be drawn, whose Circle for instance is BP ; in the Triangle Pbm there are given the angles Pmb , and mPb , the one found before, and the other equal to the vertical angle ZPB , given because the Hour of the circle PB is given;) as also the side Pm determined before: Hence therefore may mb be found, whose extremity m is given, consequently b also will be given. A right line therefore drawn thro' it parallel to the Substyle, will be the Hour line of the circle PB . And after the like manner may all the other
Hour

Hour lines be drawn, and a Sun-Dial made upon a Polar plane. *Q. E. F.*

There are other easy methods of making Sun-Dials; but we have taken the most obvious, and one that is the fittest for the illustration of the doctrine of the First Motion: And after this manner 'tis easy to describe upon a Sun-Dial any other such furniture as may be shewn by the shadow of a Gnomon.

SECTION VII.

Concerning the Parallax of the Stars.

PROPOSITION XLV.

TO describe the nature of the Parallax, and to enumerate the various sorts of it.

Thro' all this Book we have supposed the Phænomena, whose places we have been treating of, to be situated among the Fixed Stars; or (if they are placed between the center of the Earth and surface of Fix'd Stars, that they are referr'd to the Fix'd Stars by the help of right lines drawn from the center thro' the Phænomena; that is, that they are seen from the Center. And tho' the Earth in respect of the Sphere of the Fix'd Stars, is taken for a center, and puts on the nature of one, yet in respect of the distances of the other Celestial Bodies it does not happen so; but an intermediate Phænomenon, seen from the surface of the Earth, will have a place among the Fix'd Stars (or in an infinite distant surface of a concave Sphere concentric with the Earth,) different from that, which it seems to have when seen from the center; and this diversity of appearance

pearance is very fitly call'd the *Parallax*: Thus for instance, if LT [Fig. 23.] represent the Earth, whose Center is C ; L a Place on the surface, whose Zenith is Z ; ZSH the Sphere of the Fix'd Stars; the Phænomenon P seen from L will be placed among the Fix'd Stars at σ , but from the Center C , at S its true place. The difference between its true place S and the apparent one σ , or the Arc $S\sigma$, is the Parallax of the Phænomenon P : And it may be expressed by the Angle $SP\sigma$, or LPC ; for since LT in respect of the circumference ZSH vanishes, LP also (to which LT has a sensible proportion) is as a Center in respect of ZSH ; that is, the Arc $S\sigma$ may be considered as described upon the Center P . From whence it is evident that by reason of the Parallax, any Phænomenon appears nearer to the Horizon than it is, in a Vertical Circle, and that so much the more is (other circumstances being alike) it is less elevated; and that a Phænomenon seen from the surface in L , in the Vertex, as in z , is referred to the same place among the Fix'd Stars, as if it had been seen from the Center C , or that there is no Parallax in the Zenith.

By reason of the Parallax before described (which is call'd either *the Parallax of Altitude*, or also simply the Parallax) the Place of a Phænomenon is alter'd as to Longitude and Latitude: For let the true place of a Phænomenon be C , [Fig. 24.] the apparent γ , so that the Parallax is $C\gamma$; if you imagine the Circles of Latitude CE , $\gamma\epsilon$ drawn thro' C and γ , meeting the Ecliptic EL in E and ϵ , the apparent Place on the Ecliptic will be ϵ the true E ; and therefore the change of Longitude made by reason of the Parallax, is $E\epsilon$, which is also called the *Parallax of Longitude*. And the true Latitude CE

Z

is

is changed into the apparent one $\gamma \epsilon$, the difference of which $\gamma \circ$ (supposing $C \circ$ parallel to the Ecliptic) is the *Parallax of Latitude*. But if it happen that the Vertical, as ZDN , passing thro' the Phenomenon D , is also perpendicular to the Ecliptic (which comes to pass when it passes thro' the highest point of the Ecliptic A , or Nonagesim degree from the East) then the Parallax affects only the Latitude. For by the Parallax the Place is only changed in the Vertical, and because (in the present case) this coincides with the Circle of Latitude, the Place is only changed in this, that point of the Ecliptic continuing unchanged, to which it is referred by the said circle of Latitude: and in like manner only the Longitude is sometimes affected the Latitude continuing unchang'd; namely when the Phenomenon is in the Ecliptic passing thro' the Zenith. From whence it is evident, that the Longitude of a Phenomenon situated to the East of the Nonagesim degree of the Ecliptic, is increased by the Parallax, because it is removed more to the East upon this account; but that the Longitude of one situated to the West from thence is lessen'd: Wherefore besides all other motion which the Phenomenon may have, it will seem to be moved by reason of the Parallax, because change the Parallax and its apparent place will be changed. And this motion, because not real but apparent, is called by some *the Parallax of Motion*. Now what is to be understood of a Phenomenon in respect of the Ecliptic and its Secondaries, is to be understood after the like manner concerning the same in regard of the Equator.

PROPOSITION XLVI.

THE distance of a Phænomenon from the Earth is to the Semidiameter of the Earth, as the Sine of the apparent distance from the Vertex is to the Sine of the Parallax.

Let T be the center of the Earth; [Fig. 25.] the Place on the Surface L , its Zenith Z , Horizon TH ; and let the Phænomenon be in P . Now (from the known property of a rectilineal Triangle) TP is to TL as the Sine of the angle TLP (or of its complement to two right ZLP) to the Sine of the angle TPL ; that is, the distance of the Phænomenon from the center of the Earth is to the semidiameter of the Earth, as the Sine of the apparent distance from the Vertex to the Sine of the Parallax. *Q. E. D.*

Consequently the Parallax agreeing to a given Altitude being given, the distance of that Phænomenon from the Earth will also be given; and so *vice versa*.

PROPOSITION XLVII.

THE Sines of the apparent distances of a Phænomenon from the Vertex are as the Sines of the parallaxes.

For (by preced. Prop.) the Sine of the apparent distance from the Vertex is to the Sine of the Parallax, as the distance of the Phænomenon from the center of the Earth is to the semidiameter of the Earth; that is (in the same Phænomenon) in a constant Ratio. Wherefore, as the Sine of the apparent distance from the Vertex in the first Observation is to the Sine of the apparent distance from the Vertex in any other Observation, so is the Sine of the Parallax in the first observation to the Sine of the Parallax in the second Observation. *Q. E. D.*

PROPOSITION XLVIII.

THE Parallaxes of two Phænomena at unequal distances from the Center of the Earth, but whose apparent distance from the Vertex is equal, are reciprocally as the distances from the Center of the Earth.

Things being expressed as before, let the two Phænomena be P and p [Fig. 26.] in an equal apparent distance from the Vertex Z , measured by the Angle ZLP . 'Tis evident that the Sine of the Angle LPT , is to the Sine of the Angle PpT , or LpT , that is, that the Sine of the Parallax of the Phænomenon P , is to the Sine of the Parallax of the Phænomenon p as Tp to TP , that is, reciprocally as the distances of the Phænomena from the Center T .

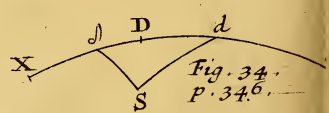
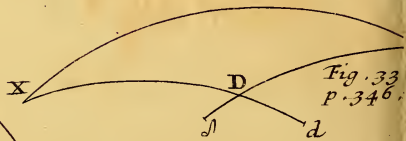
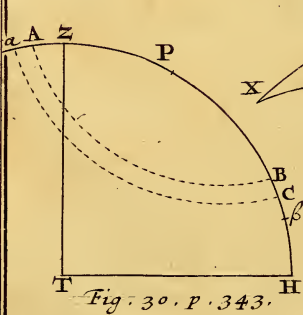
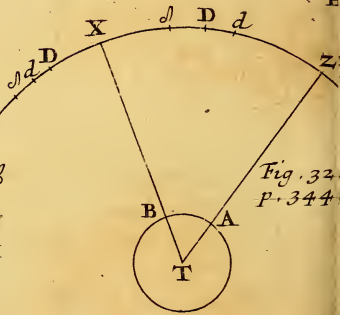
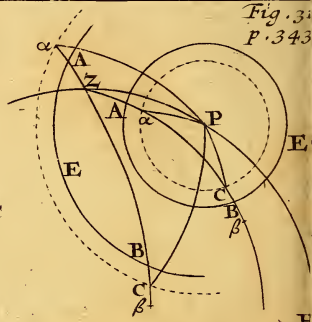
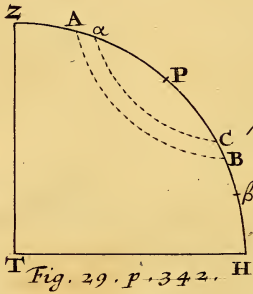
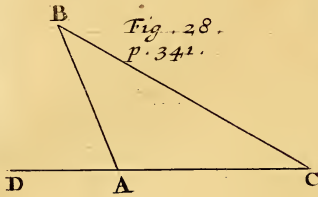
PROPOSITION XLIX.

THE ratio of the Sine of the Parallax of one Phænomenon to the Sine of the Parallax of another Phænomenon is compounded of the inverse ratio of the distances from the Center of the Earth, and of the direct ratio of the Sines of the apparent distances from the Vertex.

For when the distance from the Center of the Earth is given, the Sine of the Parallax is as the Sine of the apparent distance from the Vertex, (by Prop. 47;) and the apparant distance from the Vertex being given, the Sine of the Parallax is reciprocally as the distance from the Center (by preced.) And therefore when neither is given, the Sine of the Parallax is as the apparent distance from the Vertex directly, and the distance from the Center of the Earth reciprocally, conjunctly. Q. E. D.



Plate 6, Book 2.



PROPOSITION L.

TO find the Parallax of a Phenomenon that does not change its Declination, passing between the Pole and Vertex.

Let ZP [Fig. 27.] be the Meridian, and in it Z the Zenith, P the Pole; the true Place V , and L the apparent Place of the Phenomenon, whose Parallel described by the first Motion is VA ; the Vertical farthest off from the Meridian ZA , touching the Parallel VA , PA an Arc let fall perpendicularly upon it, from the Pole P . In the Spherical Triangle ZAP , right angled at A , by the Side ZP , the Complement of the Height of the Pole, and the Angle PZA , or Azimuth known by Observation being given, you will find the Side PA , or PV equal to it, the true distance of the Parallel from the Pole P ; the sum or difference of this and ZP is ZV the true distance (or that seen from the Center of the Earth) from the Vertex Z : And the difference between this and the apparent one ZL , namely VL , is the Parallax of the Phenomenon agreeing to the Altitude in V . Q. E. I.

PROPOSITION LI. LEMMA.

THE sum or difference of any two Arcs, together with the Ratio of their Sines, to find the Arcs themselves.

First, if the sum of the Arcs be given, the Problem is all one with that common Trigonometrical one, where two Sides AB , AC , [Fig. 28.] of a rectilineal Triangle being given, together with the Angle included BAC , the other two Angles B and C , are required: For when the Angle BAC is given, the Angle BAD is the sum of the Angles B and C themselves; and the ratio of the Sides AC , AB , is the same

with that of the Sines of the Angles B and C . This Problem therefore will be solv'd after the same manner as that; for 'tis known that the sum of the Sides AB , AC is to their difference, that is, the sum of the terms of the given ratio is to their difference, as the Tangent of half the sum of the Angles B and C , or of the Arcs that measure them, to the Tangent of half the difference of them: But the sum and difference of the terms of the given ratio are given, and the sum of the Arcs being given, the Tangent of half the sum is given; therefore the Tangent of half the difference will be known, and consequently half the difference it self; from hence, and half the sum, will the Arcs themselves be known. But if the difference of the Arcs be given, half the one and Tangent of the other will be given, and consequently (from the former Proportion) the Tangent of half the Sum of them, and the half Sum it self will be found; from whence the Arcs themselves will become known, after the like manner as before. *Q. E. F.*

PROPOSITION LII.

BOTH the Meridian Altitudes of a Phænomenon being given, which does neither change its declination nor set, to find its Parallax.

Let ZH be a quarter of the Meridian [*Fig. 29.*] reaching from the Zenith to the Horizon, in which the Pole is P ; the real place of the Phænomenon above the Pole A , the apparent α ; its true place below the Pole B , its apparent β . If the Phænomenon pass between the Pole and the Vertex, the difference between the observed distances from the Pole, namely $P\beta - P\alpha$, is the sum of the Parallaxes in α and β . For a Circle described upon the Pole P thro' A , passes also thro' B , because the true declination of the Phæ-

Phænomenon is always the same. If farther thro' α you describe upon the same Pole the Circle αC , then will $PC = Pa$; and therefore $P\beta - Pa = (P\beta - PC = C\beta =)$ to the sum of the Parallaxes in α and β . But the sum of the Parallaxes in α and β being given, and the ratio of their Sines, (being the same, by *Prop.* 47. with the ratio of the Sines of the apparent distances from the Vertex,) the Parallaxes themselves sought, will also be found by the preced. Lemma.

If the Phænomenon pass beyond the Vertex in respect of the Pole, then the difference between $P\beta$, Pa [*Fig.* 30.] the distances observed from the Pole is equal to the difference of the Parallaxes. For since $B\beta$ is the Parallax in the apparent place β , and $A\alpha$ or (the same preparation being made use of as before) BC in the place α , $C\beta$ (that is, $P\beta - Pa$) is the difference of the Parallaxes in the apparent Places β and α . But the difference of the two Arcs being given, together with the ratio of the Sines (namely the same with the ratio of the Sines of the apparent distances $Z\alpha$, $Z\beta$ from the Vertex,) the arcs themselves or the Parallaxes sought will be found.

But if the Phænomenon pass thro' the Vertex it self; that is, if the points Z , A and α coincide, also B and C , then $B\beta$ or $C\beta$ is the Parallax it self of the apparent Place β .

PROPOSITION LIII.

BOTH the Altitudes of a Phænomenon, that does not change its declination, being given, and observed in the same Vertical Circle, to find its Parallax.

The rest being expressed as before, Let ZAB [*Fig.* 31.] be the Vertical Circle, in which the Phænomenon has been twice observed,

namely in α and β , but whose true Places are A and B , and AEB the Parallel of the Phenomenon described by the first Motion. Upon the same Pole P , thro' α , describe another Circle αC , and the portions $A\alpha$, BC of the Verticals intercepted between them (by *Prop. 13. B. 2. Sphær. of Theodosius*) are equal; and therefore $C\beta$ is either the sum or difference of the Parallaxes $A\alpha$, $B\beta$. And to find it let the great Circles $P\alpha$, PC , be imagined to be drawn. In the Triangle $Z\alpha P$, ZP the complement of the Height of the Pole, $Z\alpha$ the observed distance of the Phenomenon in α from the Vertex Z , and the Angle contained αZP , from the Azimuth, known by observation, being given, you will find αP , to which the arc CP is equal. Again in the Triangle PZC , two Sides PZ , PC being given, together with the Angle PZC , ZC will also be known, whose defect or what it wants of the observed Arc $Z\beta$, is $C\beta$ the sum of the Parallaxes in the places α and β , when PZ exceeds $P\alpha$; or their difference, when PZ is less than $P\alpha$. But in both cases, when over and above, the ratio of the Sines of the said Parallaxes is given, (namely the same, by *Prop. 47.* with the ratio of the Sines of the apparent distances from the Vertex,) both the Parallaxes agreeing to the apparent places α and β will be found.

PROPOSITION LIV.

TWO Altitudes of the same Phenomenon observed at the same moment of time in the common Azimuth of two given Places upon the Earth, to find the Parallax of the Phenomenon.

Let the two Places on the Earth be A and B , [*Fig. 32.*] whose Vertices are Z and X ; the common Azimuth ZDX ; and let the true Place of the

the Phænomenon be D , α the place observed at A , d its place seen at B : And first let these points be between the Vertices Z and X . The sum of the apparent distances from the Vertex, (namely of the arcs $Z\alpha$ and Xd ,) known by Observation, exceeds ZX the distance of the Vertices (found by *Prop. 24*) by an excess equal to $d\alpha$, the sum of the Parallaxes. But if the points D , d , α , be without the arc ZX , the difference of the apparent distances from the Vertex (namely $Z\alpha$ lessen'd by Xd ,) exceeds the distance of the Vertices XZ by excess equal to the difference αd of the Parallaxes sought.

Besides the sum or difference of the Parallaxes found after the manner described before, there is given the ratio of the Sines of the said Parallaxes; namely the same (by *Prop. 47*.) with the ratio of the Sines of the arcs $Z\alpha$, Xd of the apparent distances from the Vertex. From whence (by *Prop. 51*.) in both cases the Parallaxes themselves will be found. Q. E. F.

PROPOSITION LV.

GIVEN two altitudes of a Phænomenon observed at given Places on the Earth, the same moment of time, even without the common Azimuth, to find the Parallax of the Phænomenon.

The Longitudes of the Places being given, the same moment of time necessary for the making the Observation in the two Places, is expressible by the number of Hours reckon'd from Noon or Midnight respectively, and their difference is equal to the difference of Longitudes turn'd into time, the more Eastern Place reckoning more hours; which is determined during the Observation in the preced. when the Phænomenon is found in the common Azimuth, unless both Places be in the Equator. Let XZ

[Fig.

[Fig. 33.] be the common Azimuth of the two Places upon the Earth, whose Vertices are X and Z ; from which at the same moment of time (tho' expressed differently,) let the Phænomenon be observed, whose true place let be D ; and its place seen from that Place whose Vertex is X be d , from the Place whose Vertex is Z be δ . In the Triangle XDZ , the angles X and Z being given (contained between the common Azimuth of the Places and the observed Azimuths,) and the Side XZ the distance of the Vertices (found by *Prop.* 24.) the Arcs XD , ZD may be found, and they subtracted from Xd , $Z\delta$ observed, leave Dd , $D\delta$ the Parallaxes sought. *Q. E. F.*

PROPOSITION LVI.

TO find very exactly the Parallax of a Phænomenon, by a method not much different from the two foregoing, by the help of some neighbouring Fix'd Star.

Since the Phænomenon whose Parallax is sought, is in the common Azimuth XDZ [Fig. 34.] of the two Places on the Earth pitched upon for this purpose, whose Vertices are X and Z ; let it be observed with some neighbouring Fix'd Star; and let S be the Place of the Fix'd Star, d the Place of the Phænomenon observed at the place below X ; consequently in this Observation there are given the arc Xd the apparent distance of the Phænomenon from the Vertex, the arc Sd the distance between the Phænomenon and the Fix'd Star, and the angle XdS contained between it and the common Azimuth. At the same moment of time from the place below Z , where the apparent place of the Phænomenon is δ , let $Z\delta$, $S\delta$ and the angle $S\delta Z$ be observed: And tho' the arcs Xd , $Z\delta$ can't

can't be observed exactly, (because there is need of a Quadrant to measure them;) yet they may do for our purpose: But if they could with perfect accurateness, the Parallax of the Phænomenon would (by *Prop. 54.*) immediately appear. Notwithstanding the arcs dS , δS and the angles at d and δ , may be observed very accurately, by projecting the image of the Phænomenon and neighbouring Fix'd Star S , by the help of Speculums or Lens upon a Plane, upon which a right line is drawn representing the common Azimuth XZ ; and being in the place of its image, (if it should project any;) as is evident to any one acquainted with Optics and Astronomical Observations. Therefore in the Triangle $dS\delta$, the sides $S\delta$, Sd with the angles subtended by them d and δ being given, the Base $d\delta$ will be found, which is the sum of the Parallaxes; from whence and the ratio of the Sines of the Segments δD , dD , (D being supposed to be the true Place of the Phænomenon,) namely the same with the ratio of the Sines $Z\delta$, Xd of the apparent distance from the Vertex (by *Prop. 47.*) the Segments themselves Dd , $D\delta$ the Parallaxes sought, will (by *Prop. 51.*) become known. Much after the same manner the differences of the Parallaxes may be found, and from thence the Parallaxes themselves (by the said Lemma,) if D , d , δ , be without XZ , but yet in the continuation of it. If the true Place D [Fig. 35.] of the Phænomenon be without the common Azimuth XZ of the Places, the same things being expressed the same way as before, thro' the Vertices X and Z and the respective apparent Places, let the Vertical Circles Xd , $Z\delta$ be drawn, whose intersection is the true Place D of the Phænomenon. Draw also thro' S , the place of the Fix'd Star, the

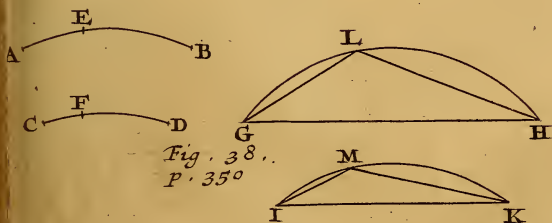
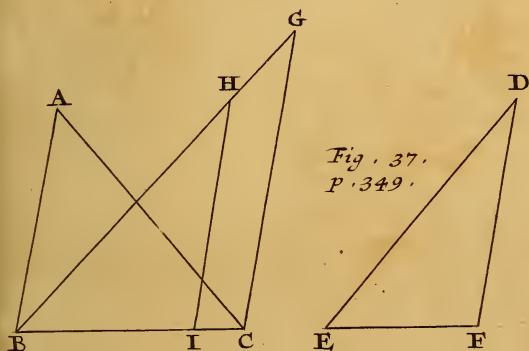
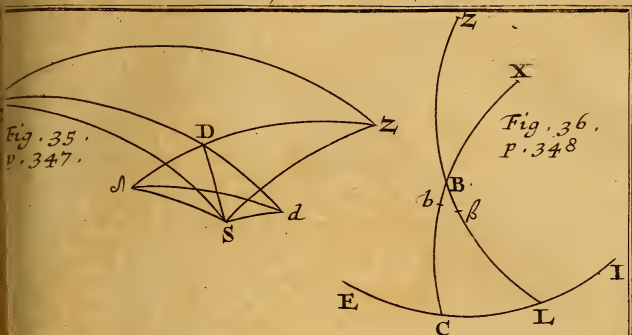
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Verticals XS , ZS , and join the points S , δ , d by the great Circles Sd , $S\delta$, $d\delta$. Let the Observer below X , observe accurately (besides the arc Xd) the angles XdS , XSd and the Side SD , either by the method insinuated above, or any other more exact; in like manner let the Observer below Z (beside $Z\delta$) do to the Angles $Z\delta S$, $ZS\delta$ and the Arc $S\delta$. Besides, the habitations being given (by *Prop.* 24.) ZX is given, and from the situation of this, by the Observation of the Fix'd Star S , the angles ZXS , XZS , are had; from whence the Angle XSZ will be known, which taken from the sum of XSd , $ZS\delta$, will leave $dS\delta$, from which and the sides Sd , $S\delta$, being known, the angles $Sd\delta$, $S\delta d$ and the side $d\delta$ are found. But SdX , $S\delta Z$, were known by Observation; consequently the others $Dd\delta$, $D\delta d$ are known, by which and the side $d\delta$ known in the Triangle $Dd\delta$, the sides Dd , $D\delta$, the Parallaxes sought, are found.

PROPOSITION LVII.

TWO Observations of a fix'd Phænomenon, that is, one that has only a diurnal motion, being given, to find its Parallax.

Let *ECLI* [Fig. 36.] represent the Ecliptic, *B* the true place of the Phænomenon, intirely fix'd in regard of the Ecliptic. And in the one Observation let ZBL be a Vertical Circle, in which Z is the Zenith, and β the apparent place of the Phænomenon; in the other, let XBL be a Vertical, X the Zenith, (for the Vertical point may be considered as moved in regard of the Ecliptic unmoved, as well as the Ecliptic in respect of the Vertex unmoved, and indeed more properly;) and b the apparent place of the Phænomenon. From the first Observation (by the Problems of the First motion con-



constructed before,) you may determine the point L of the Ecliptic, where the Vertical passing thro' B meets the Ecliptic; and the arc ZL , namely a portion of a vertical Circle, intercepted between the Zenith and the Ecliptic; and the angle ZLE contained between the said Vertical and the Ecliptic. And in like manner in the second Observation of the Phænomenon in β , the point of the Ecliptic C , the arc XC , and the angle $XC L$ will be known. Therefore in the Triangle $BC L$, the angles $BC L$, BLC being given, and the side CL that lies between them (namely the distance of the known points in the Ecliptic,) the sides BL , BC will be known, which taken from the arcs found before ZL , XC , will leave ZB , XB . And these again taken from $Z\beta$, Xb , the known apparent distances from the Vertex, leave $B\beta$, Bb the Parallaxes of the Phænomenon agreeing to the moments of the observations.

PROPOSITION LVIII. LEMMA.

LET there be two rectilineal Triangles, ABC , DEF [Fig. 37.] having the Angles ABC , DEF given; and let the ratios of AB to DE , AC to DF , and BC to EF , be given: To find both Triangles from the side AB given.

Make the angle GBC equal to DEF , and take BG such, as that the ratio of BA to BG may be equal to the ratio compounded of the given ratios of BA to ED , and EF to BC , which is done by taking BG equal to a fourth proportional to EF , BC and ED . Then in BC given by position, chuse such a point C , as that the ratio of the lines connected AC , GC is equal to that compounded of the given ratios of AC to DF , and EF to BC , which may be done as before. Then let BG be to BH in the

the ratio of BC to EF , and let HI be parallel to GC : I say the Triangles ABC , BHI , are those sought.

'Tis manifest from the Construction, that the ratio of BC to BI , is equal to the ratio of BC to EF ; and that the Angles ABC , HBI , are equal to the given ones. Besides, the ratio of AB to BG is compounded (by construction) of the ratios of AB to ED and EF to BC : But the ratio of AB to BG , is compounded of the ratio of AB to BH , and of the ratio of BH to BG , or EF to BC ; and therefore AB has the same ratio to BH as it has to ED . Again, the ratio of AC to CG (by construction) is compounded of the ratio of AC to DF , and of the ratio of EF to BC , or HI to GC : But the ratio of AC to CG is compounded also of the ratio of AC to HI and HI to GC ; and therefore AC has the same ratio to HI as it has to DF . Therefore the Triangles ABC , BHI , have the conditions required.

PROPOSITION LIX. LEMMA.

THE ratio of the Sines of the Arcs AB , CD , [Fig. 38.] belonging to a given Circle, and the ratio of the Sines of the Arcs taken away AE , CF , and the arcs remaining EB , FD being given; to find the whole arcs and those taken away, AB , CD , AE , CF .

In Circles equal to the given ones, let there be supposed to be taken the arcs GL , LH , IM , MK respectively the doubles of AE , EB , CF , FD : Wherefore GLH , IMK will be respectively the doubles of AB , CD . Compleat the rectilineal Triangles LGH , MIK , in which are given the ratio of GL to IM , the same with the given ratio of the Sine of the arc AE ,

to the Sine of the arc CF ; the ratio of LH to MK , the same with the ratio of the Sines of the arcs given EB , FB ; the ratio of GH to IK , which is the same with the given ratio of the Sines of the arcs AB , CD . There are also given the angles LGH , MIK insisting upon the known arcs, and therefore (by the prec. *Prop.*) the angles LHG , MKI will be known, and from thence the arcs GL , IM , and their halves AE , CF : But the arcs themselves EB , FD are given; therefore the wholes AB , CD are also known.

And after the like manner, the ratio of the Sine of the whole arc to the Sine of the arc taken away, and the remaining arc itself being given, the whole arc and the arc taken away may be found. For suppose GLH the double of the whole arc, and GL of the arc taken away; from whence you have LH double the remaining arc. Connect GH , GL , LH , and in the Triangle LGH , there is given the ratio of the sides GL , GH ; the same with the given ratio of the Sines of the whole arc and that taken away: The angle LGH is also given, insisting upon the given arc LH . And therefore (by 41. of the *Data of Euclid*,) the angles GLH , GHL , are given; and therefore the arcs on which they stand, *viz.* the complement of the arc GLH to the whole circumference, and the arc GL ; and consequently GLH and GL themselves, and their halves, namely the arcs sought.

PROPOSITION LX.

TWO Altitudes of a Phænomenon not altering its Declination, and the corresponding Azimuths being given, to find its Parallax.

In two observations of a Fix'd Star five things may be given; namely the two Altitudes, two Azimuths, and the Time between the observations; from any four of which the Parallax may be found, from whence arise these three following Problems.

For the solution of the first, let P [Fig. 39.] represent the Pole; L the true place of the Phænomenon; and in one of the observations let LM be the Vertical Circle, and M the place of the Phænomenon observed in it, and therefore LM the Parallax; in the other let LN be the Vertical Circle passing thro' L , and LN the Parallax of this observation. Let the great Circles PM , PN , NM be suppos'd to be drawn. From the Altitudes and Azimuths observed, and the Height of the Pole, the Arcs PM , PN the apparent distances from the P , and the Angles PML , PNL contained between the Circles of Declination and the Verticals are known: There is also given the ratio of the Sine of the Parallax LM to the Sine of the Parallax LN ; namely, the same (by *Prop. 47.*) with the ratio of the Sines of the distances observed from the Vertex. There are therefore given the ratio of the Sines of the whole Angles PMN , PNM , namely the same with the ratio of the Sines of the known Arcs PN , PM ; also the ratio of the Sines of the Angles LMN , $LN M$ taken away, the same with the ratio found before of the arcs LN , LM ; as also the remaining Angles PML , PNL found before by Calculation. Therefore the Angles themselves PMN , PNM , LMN , $LN M$.

Plate 8, Book 2.

Fig. 39.
p. 352.

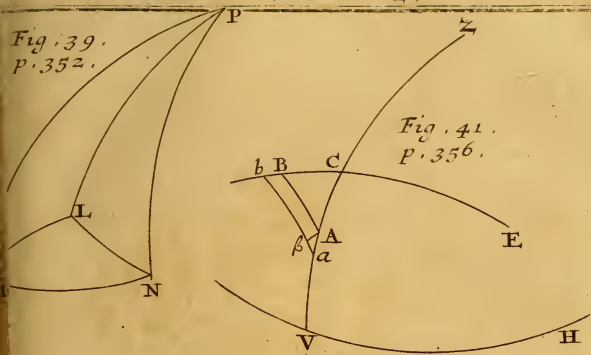


Fig. 40.
p. 354.

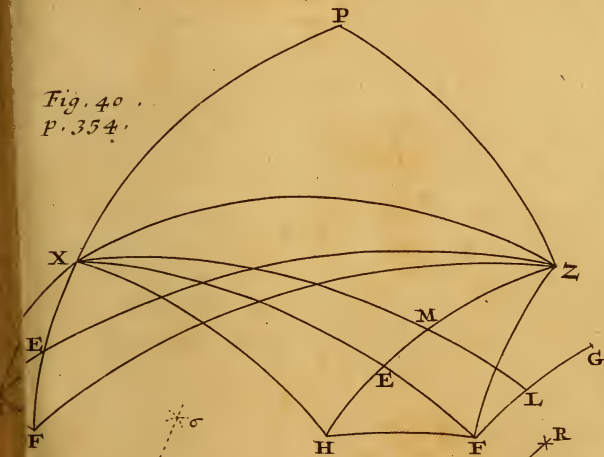
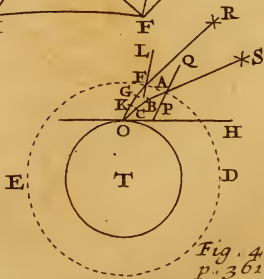
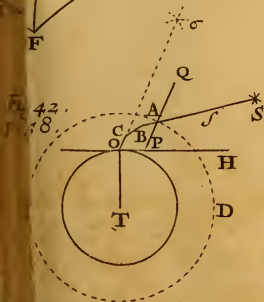
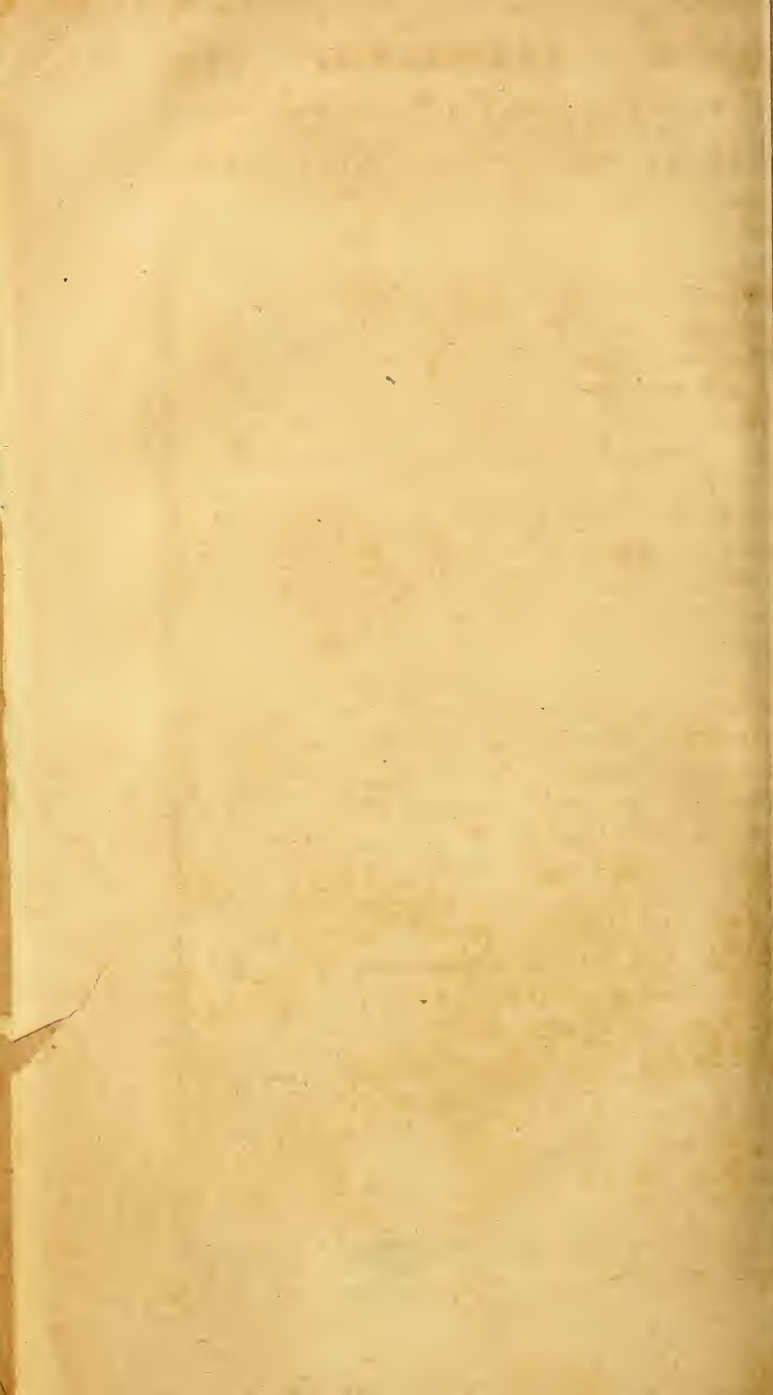
[illegible]

Fig. 43.
p. 361.



LNM will be known, (by the preced. Lemma.) Again, in the Triangle MPN , the Sides PM , PN being given, with the Angles subtended by them, the Base MN is found. And lastly, in the Triangle LMN , the Base MN , and the Angles adjacent LMN , LNM being given, the Sides LM , LN the Parallaxes sought will be found.

If one of the Observations was made when the Phænomenon was in the Meridian, then one of the Triangles PML , or PNL is changed into an Arc, and the Calculation becomes much simpler.

But if the Phænomenon L change its Declination, and that equably, (which will be done in one revolution of the Earth, or thereabouts,) we may still find its Parallax, by correcting the places of the Phænomenon, by a proportional part of the Declination thus: When the Phænomenon first, next after the first Observation, arrives at the Azimuth LM , observe its apparent Altitude, which will be different from that observed the Day before, otherwise it would retain the same declination. Let the difference be A . Call the distance of time between the two first Observations made in M and N ; B ; and the time between the two Observations of the Phænomenon in the Azimuth LM , C . And of the difference of the aforesaid Altitudes A , be an excess, to the apparent Altitude in M add $\frac{BA}{C}$; but if this difference be a defect, subtract it from the same. This sum or difference will be the corrected apparent Altitude, which put in the preceding Calculation (instead of the former Altitude in LM) gives the Parallaxes sought of a Phænomenon that does not change its declination.

PROPOSITION LXI.

Given two Altitudes of a Phenomenon, with the Azimuth of one of them, and the Time between the two observations, to find the Parallax of the Phenomenon.

Let P be the Pole, (Fig. 40.) E the true place of the Phenomenon, F the apparent place when X is the Zenith, and H when Z is the Zenith. There are given the two apparent distances from the Vertex, namely XF , ZH and the Angle PXF , since one Azimuth is given, and the Azimuth XPZ is known by the time. In the Isosceles Triangle PXZ , the equal Sides PX , PZ , and the Angle XPZ being given, PXZ , XZ are found: And since PXF is given, ZXF is also found. Therefore in the Triangle XZF , XZ , XF , ZXF are given; and therefore ZF , ZFX become known. Therefore the ratio of the Sine of the angle ZFH , to the Sine of the angle ZHF is given namely the same with that of the Sine of the arc ZH , to the Sine of the arc ZF ; and the ratio of the Sine of the angle ZHF to the Sine of the angle EFH , or the Sine of the arc EH , to the Sine of the arc HE , or the Sine of the arc XF to the Sine of the arc ZH . Therefore the ratio of the Sine of the angle ZFH to the Sine of the Angle EFH is given; and the difference of those Angles ZFE is also given and therefore (by Prop. 59.) the angles themselves ZFH , EFH become known; and consequently EHF also. And lastly in the Triangle EFH , HF , EHF , EFH being given, the Parallax sought EF , EH become known. Q. E. F.

PROPOSITION LXII.

Given two Azimuths of a Phænomenon, together with the Altitude of one of them, and the Time between the two observations of the Azimuth, to find the Parallax of the Phænomenon.

The same things being supposed in the same Diagram, after the same manner may PXZ equal to PZX , and XZ be found. The two Azimuths being given, the Angles PXE , PZE are given, and therefore the angles XZE , ZXE become known; by which, together with XZ , may XE be found. But because the Altitude is given, its complement XF is also given, and therefore EF the Parallax sought will become known. Q. E. F.

If, in the case of either of these Problems, the Phænomenon has any motion after an entire revolution of the Earth, when the Zenith X returns to the same point of the Heavens, the Phænomenon will be observed, not in F (as before) but in some other place, for instance, G ; and FG , XFG become known by observation. Make therefore, as a whole Circle is to FG , so is the Angle XPZ to FL , and draw a great Circle XL cutting ZH in M . In the Triangle ZFL , XF , FL , XFL being given, XL , FXL are also given; and therefore ZXL is also given. Lastly, taking the points L , M instead of F and E , HM the Parallax of the Phænomenon may be found as before. Q. E. F.

There are other sorts of Observations, whereby the Parallax of a Phænomenon may be Geometrically determined; as its ingress into the shadow of a Planet; and a Conjunction of Bodies seen between the Phænomena. But since these things do not belong to the Parallax considered in general, they more properly belong to another place in the following Books.

PROPOSITION LXIII.

THE place of a Phænomenon in respect of the Ecliptic, and the Parallax of a given Altitude, as also the place of the Sun, and the hour of the Day being given, to find the Parallax of Longitude and Latitude of the Phænomenon.

Let VH be the Horizon, (Fig. 41.) ECB the Ecliptic, and ZCV a Vertical Circle passing thro' the Phænomenon. Let A be the true place of the Phænomenon in it, a the apparent; and therefore Aa the Parallax of Altitude. Thro' A and a draw the Circles of Latitude $ABab$, meeting the Ecliptic in B and b . Now (by Prop. 36.) there are given the Angle VCB , and arcs VA , VC , and therefore their difference AC . But because the Parallax of a certain given Altitude is given, the Parallax of the given Altitude VA will also be given (by Prop. 47.) namely the arc Aa . Consequently the sum or difference Ca of these arcs CA and Aa are given. Therefore in the Triangle aCb , besides the right Angle at b , there are given the Side aC , and Angle aCb ; from whence the Sides Cb , ab will become known. But because the place of the Phænomenon A in respect of the Ecliptic is known and the point of the Ecliptic (by Prop. 36.) also known, the arcs AB and CB are known. Therefore the difference between ab and AB namely the Parallax of Latitude become known; and the difference between Cb and CB namely the Parallax of Longitude also. Q. E. F.

The converse of this Problem is solved after the same manner; for if the Parallax of Longitude Bb , or the Parallax of Latitude, the difference between ba and BA , be given, Cb or ab , (from the Place of the Phænomenon in respect of the Ecliptic already given, and from thence

thence the arcs CB and BA ,) will be given. And since besides the Angle at C in the right angled Triangle Cba being given, Ca will be given, and consequently Va , since VC (by Prop. 36.) is already given: But VA is known; therefore the Parallax of Altitude Aa will not be unknown. If the Phænomenon, as the Sun, be in the Ecliptic, the case is much more simple.

But because the Parallax of the Celestial Bodies, even the nearest, is a pretty small Arc, therefore some construct this Problem of finding the Parallax of Longitude or Latitude from the given Parallax of Altitude, more expeditiously thus: Thro' A imagine the line $A\beta$ to be drawn parallel to Bb . Therefore in the small Triangle $A\beta a$, which is looked upon as rectilinear, there being given, besides the right angle at β , the angle βAa equal to ACB , and one of the Sides Aa the Parallax of Altitude, the other Sides $A\beta$ the Parallax of Longitude and $a\beta$ the Parallax of Latitude are found, by the bare resolution of a plane Triangle.

But if not the absolute Parallax of the Phænomenon be given, but the relative in respect of another more remote Phænomenon, that is, the excess of the Parallax of the nearer Phænomenon above the Parallax of the more remote; such an excess in a vertical Circle being given, the excess in regard of any other Circle will also be given.

For the absolute Parallax of a Phænomenon, is the excess of the Parallax of the Phænomenon itself above the Parallax of the Circle of a Sphere, which is nothing, because it is supposed infinitely distant.

SECTION VIII.

Concerning the Refraction of the Stars.

PROPOSITION LXIV.

ON the account of an Atmosphere, which is denser than the *Æther*, circumfused about the Earth, each Star appears more elevated above the Horizon in the same Vertical Circle, than it would appear were there no Atmosphere at all.

Let *T* represent the Earth (in Fig. 42,) round which the Atmosphere *AED* is diffused; *S* any Star; *O* the Spectator, placed on the Surface of the Earth. If there were no Atmosphere at all, or if it were equal in density to the circumfused *Æther*, the Rays of Light would reach directly from *S* to *O*, nor would they ever be bent into a Curve line; because in an homogeneous Medium. But if the Rays, after their passage thro' the *Æther* *SQ*, enter into a denser Atmosphere in *A*, they will be refracted towards the Right line *QAP*, perpendicular to the Surface of the Atmosphere at *A*; as is evident from *Dioptrics*. But it is not probable that an Atmosphere very considerably denser than the *Æther* is immediately under it, and then continued all along to the Earth of the same density; or that all the Refraction of the Rays, that proceed from any Star, is perform'd in that single Surface which separates the *Æther* from the Atmosphere, (as it is commonly believed,) the portions of the Ray from *S* to the outmost Surface of the Atmosphere, and from thence to the Earth, continuing all the while Right lines; as with us a Ray passing out of Air into Water is only

only bent at the Surface of the latter, being straight both before and afterwards. The Atmosphere rather seems to extend itself to a very great height, and to be rarer in its higher and more remote parts, and denser in the lower; on which account a Ray proceeding from a Star S , suffers some Refraction towards the perpendicular, in a distance from the Earth much greater than is commonly believed; and in its progress (as at B , C &c.) it is refracted the same way, because the medium of the Atmosphere, that it enters into, is more and more dense, the nearer it comes to the Earth: And therefore a Ray after its entrance into the Atmosphere (by reason of the perpetual refraction at the Spherical superficies concentric to the Earth of a continually denser medium) is curved in all its parts, and concave towards T . But an Eye plac'd at O sees a Star in the last part of the Ray $ABCO$ from O produc'd, namely in σ ; for this part only affects the Sight. But the part OC of the Ray produced, or the Right line touching the Curve $ABCO$ in O , namely, $O\sigma$, is less inclin'd to the Horizon of the Spectator HO , than the Right line connecting S and O ; because the line $ABCO$ is concave towards T : And therefore, to an Eye at O a Star appears in σ , higher above the Horizon, by reason of the circumfused Atmosphere, than if there were no Atmosphere at all.

Besides, the Curve that the Ray propagated oblique thro' the Atmosphere, puts on, is in the same Plane produced thro' the Star and the Center of the Earth. For in *Dioptrics* 'tis demonstrated that the former part AB of the Curve (consisting, as it were, of Right lines) is in the Plane that the Right lines AS , AQ , (because in the Plane of Inflection;) that is, in the Plane

produced and passing thro' the Star S and Center of the Earth T ; for (by *Prop. 5. B. 1. Spher. Theodos.*) the Right line $\mathcal{Q}AP$ produced, passes thro' the Center T ; and its second part BC is in like manner in the Plane passing thro' BA , and the perpendicular thro' B ; that is, in the same Plane with the former: and so on in the other parts of the Ray $ABCO$. Therefore, that intire Curve is in the same Plane produced, passing thro' the Star and the Center of the Earth; and therefore the Right line $O\sigma$ touching the Curve, (because a small particle of it produced in a Right line) is in the same Plane: But this Plane is vertical to the Observer O , (for it passes thro' the point O , and the Center of the Earth) therefore the Star, by reason of the Refraction made in the Atmosphere, appears in the same Vertical, but higher above the Horizon, $\mathcal{Q}. E. D.$

SCHOLIUM.

It appears, from the demonstration of the preced. *Prop.* that the Refraction is the same, whether the Star be nearer the Earth, or farther off (for instance, in f or in S ;) provided their apparent Altitude above the Horizon be the same: For in this case the Right line σO revolving above the Axe TO , reaches to the apparent place of f . Besides, 'tis probable that the Atmosphere spread about the Earth does not extend it self to the Moon, much less to the more distant Stars; and therefore the strait part fA of the line $SfABCO$, (before it begins to be bent into a Curve by the Atmosphere) will graze upon the next Star f . And consequently the Ray proceeding from it (namely fA) is bent in the Atmosphere into $ABCO$, after the like, nay, after the very same manner, as SA propagated from the more distant Star; since it is

all

all one (as to the Refraction made in the Atmosphere) whether the Ray begin from S or s . Therefore 'tis contrary to the nature of Light, that the Rays of a nearer Star should be refracted otherwise than (all other circumstances remaining the same) that of a more distant one. 'Tis true that the Angle σOS is greater than the Angle σOs , but the difference of them $\angle OS$ is owing to the excess of the Parallax of the nearer Star above the Parallax of the more distant Star, and not to the Refraction.

PROPOSITION LXV.

THE lower Stars are more elevated by reason of the Refraction at the Atmosphere, than the higher, other circumstances remaining the same.

In Dioptrics 'tis demonstrated, that in the same Mediums, the more oblique the incidence of the Ray is, the greater the Angle between the refracted Ray and the incident Ray produced; that is, in the present case [Fig. 43.] that the part AB of the Ray $ABCO$, proceeding from the lower Star S , contains a greater Angle with SA produced, than the part FG of the Ray $FGKO$ proceeding from the higher R does with RF produced; because $\angle SAQ$ is a greater Angle than $\angle RFL$, supposing that LF produced falls upon T . After the like manner BC will contain a greater Angle with AB produced, than GK with FG produced, and so on; that is, the line $ABCO$ is more curved in all its parts than $FGKO$ in its corresponding parts, or such as are equally distant from the Earth. And therefore a Right line touching the Curve $ABCO$ in O contains a greater Angle with the Right line OS , than a Right line at the same place, touching $FGKO$ with the line OR . The
lower

lower Star therefore is more elevated by the Refraction than an higher. *Q. E. D.*

COROLLARY 1.

The Place of a Phenomenon when it is in the Zenith is not changed by the Refraction: For the Rays in this case falling perpendicularly upon all the Surfaces of the Mediums, of which the Atmosphere consists, are not refracted; but going directly to the Eye in *O*, make the Phenomenon appear in the same Right line as it would appear were the Atmosphere away. And for the same reason the Eye being placed in the Center *T*, the Refraction made at the Atmosphere would have no power to change the apparent place of the Star; for every point in the Heavens would be a Zenith to it.

COROLLARY 2.

An account may from hence be given, why the Sun or Moon appears of an Oval Figure near the Horizon. For since the upper Limb appear but a little more elevated than it should be, and the lower a great deal, the latter by the Refraction, will appear to approach the former. And therefore the erect or vertical Diameter of the Luminary seems contracted or shortened, while the transverse or horizontal one is not so, because its extremities are equally elevated by the Refraction. For the same reason the distance of two Fix'd Stars by observation is sensibly less, (if it be measured by an Instrument,) when, both being in the same Vertical, one of them is nearer the Horizon, than when both of them, by reason of their greater Altitude, are free from all Refraction.

COROLLARY 3.

On this account it is also, that the Sun and the other Stars rise sooner above the Horizon, and set later, than they ought, according to
their

their places determined by the aforesaid Methods. For the Ray flowing from such points of the Heaven as are but a little below the Horizon, are so incurvated in the Atmosphere, that the Right lines touching them at their extremities, where they enter the Eye, being produced, are above the Horizon. For which reason the Moon has been seen to be eclipsed, tho' the Sun was above the Horizon; whereas one of the Luminaries at that time being opposite (by *Prop. 18. B. 1.*) to the other, and above the Horizon, the other must necessarily be below it. 'Tis to the same Atmosphere diffused round the Earth, that its Inhabitants owe the Sun's shining to them longer than it would do were there no Atmosphere; and that when the Sun is kept from inlightening any given Inhabitant by reason of the Spherical Surface of the Earth, thick darkness does not immediately cover him, but he enjoys a Twilight for a considerable time, the Light being gradually extinguished.

PROPOSITION LXVI.

TO define how much a Star appearing in a given Altitude is elevated by the Refraction; and to make a Table of the Refraction of the Stars.

Chuse a Star that has no sensible Parallax; namely some one of the Fix'd Stars that is elevated much above the Horizon of the Observer. Let its Place be determined (by *Prop. 26.*) by observations made when it was elevated pretty much, and therefore when it was out of the Effects of a sensible Refraction. Let the time be marked of its coming afterwards to the given Altitude, either ascending or descending, and find then by Calculation, the Altitude, for the time marked, of the Star (whose place is known,

known,) and that will want of the observed Altitude, as much as the Star is elevated by the Refraction in that given apparent Altitude.

After this manner may a Table of the Refraction of all the Stars be best constructed, if the true Altitudes of the Fix'd Stars, agreeing to the times exactly reckoned by a Pendulum Clock, be compared with the Altitudes of the same Star, in the same apparent points of time: For the difference between the true and apparent Altitude is the Refraction agreeing to that Altitude. And since the Refraction of a Fix'd Star at the greatest distance, and of any other Star at a less distance, is the same, in the same apparent Altitude (by *Schol. Prop. 64.*) 'tis evident that a Table of the Refraction of all the Stars is made. *Q. E. F.*

SCHOLIUM.

If the *Dioptric* density of the Atmosphere were known in any given distance from the Earth, or the ratio of the Sine of the Angle of incidence to the Sine of the refracted Angle, in the passage of a Ray thro' its Surface concentric with the Earth, whose distance from the Center is known; then by Geometry you may find out the nature of the Line, into which the Ray is bent by reason of the continued Refraction of the Atmosphere. Consequently, the Angle being given by observation, by which a Star in a given Altitude is elevated more than it should be on the account of the Refraction, the Angle may be defined, by which it is elevated in any other Altitude proposed more than it should be, the Atmosphere continuing the same; that is, a Table of Refraction may be made for every degree of a Star's Elevation. But because our ignorance of the Law of Refraction observed in the Atmosphere hinders all
this

this, 'tis better to make the Table by Observation, as in the preceding. But besides, the diversity of the Atmosphere in different places, and the change of it in the same place, on the account of which it happens that the same Table is not fitted for all places, is an impediment to both methods of making a Table of Refraction. For in places near either Poles the Refraction is greater in the same distance from the Horizon, (as is well known from the observation of some *Dutch Men* who Wintered in *Nova Zembla*;) and in the same place at different times of the Year, 'tis certain, not only from the observation of the *Dutch*, but of other considerable Observers, who make the Winter Refraction greater than the *Æquinoctial*, and that again greater than the Summer one, and form intire Tables of Refraction built upon this change. But the hourly change of the Refraction is demonstrated by the Tops of Mountains being sometimes visible at a given place, and sometimes invisible, tho' there be no darkness or mist between: To this purpose also do the Experiments mention'd by *Hugens*, in his *Treatise of Light*, tend and serve; namely that the lower part of a Tower or any other thing that is immovable is seen in the Morning and Evening, but the upper at Noon thro' a Telescope continuing fix'd all the while; and that at the same place, sometimes the top of a more remote Tower has been visible, rising above the top of one nearer, and sometimes it is invisible. Now all these things, besides the hourly change of the Refraction of the Rays in the Atmosphere, demonstrate the Curvity of them, (from their nature, as was shewn above,) since they are propagated obliquely from an higher to a lower place, and *vice versa*.

It must be confessed that all this change of the Refraction is made within pretty narrow limits: For the before said observations of the change of the apparent Altitude of the Sun, Mountains and Towers are made only in a small Elevation above the Horizon, nor do they reach much beyond three degrees above it. The vicissitude and changeableness near the Earth, of Vapours arising from it, as also of Heat and Cold, by which they are rarified and condensed, is so great, that nothing certain can be laid down about the density of the Medium. Besides, the inclination of the Rays of Light is so very oblique at their incidence on the Atmosphere, when the Phenomenon is not elevated above three or four degrees, that its place is changed very much by the Refraction. Nay, so dense yet pellucid a Cloud as it were of Vapours lies sometimes upon the Surface of the Earth, that to an Eye placed above without it, the visible Objects below it, namely the very low ones, continue unseen, because the Rays do not penetrate the Surface of the upper and rarer Medium, but are reflected by it downwards; as it is evident from *Dioptrics*, that it ought to be: But such things as are immediately above and without it, namely such as are a little higher, besides that they are seen by the direct Ray, appear also inverted by a reflection made at the Surface of the above-mentioned Cloud, just as if it were an Horizontal Speculum. But these things neither belong to this place, nor directly to Astronomy. They show in the mean while, that no credit is to be given to any Table of Refraction of the Stars below the third degree of Altitude; nor consequently to Observations made in that

Alti-

Altitude for determining the place of the Phenomenon.

If a Person has a mind to make a Table of the Refraction of the Fix'd Stars, not from Observation, but upon the Principles of *Dioptrics*, he may proceed the following way very safely. Let the Atmosphere, which is extended 40 or 50 Miles high, be supposed to be divided by 8 or 10 parallel Superficies, into as many Mediums of different densities; so as that the Medium which lies between the two next Superficies, may be of the same density, that at the said Superficies may be changed instantly into a rarer above, and a denser below: Not that it is really thus in nature; for no doubt the density of the Atmosphere in its descent, is increased by the smallest degrees. Let these 8 or 10 Mediums be so tempered in regard of one another by trials, that in one, two or more Altitudes (for the more they are, the more accurate will the Table be) the total Refractions made at all these Mediums, may be the same with the Refractions that are found by the accuratest Observations, to agree to those respective Altitudes. In which case the Refraction made in any assumed Altitude at the said Mediums, and known by Calculation, according to the Laws of *Dioptrics*, is very nearly equal to the Refraction in the Heavens, agreeing to the same Altitude. After this manner also will the Line be determined very nearly, into which the Ray is bent in its passage thro' the Atmosphere. But in finding, by this method, the Refraction corresponding to an Elevation three or four degrees less, the lower, or two or three lower, ought to be supposed a little rarer; for they really are such, in the vicinity of the Earth, which is heated.

SECTION IX.

Of the Tables of the First Motion, and of the Fix'd Stars, depending upon the Second Book.

THO' all the Particulars that belong to the First Motion may be found, by the Methods demonstrated in this Book, as occasion requires ; yet because there are some things of more frequent use, *Tables* are made for them, that they might be more expeditiously taken out of them, which would otherwise require a new Calculation every time they are wanted. And besides *Tables* taken from other disciplines, as of Right lines inscribed and circumscribed in a Circle, of Logarithms, &c. Astronomical ones are also made ; and such of them as depend upon this Book are of two sorts ; namely such as are the same to the Inhabitants of the whole Earth ; which regard either the First Motion, or the Fix'd Stars ; or such as only hold in some certain place of the Earth.

PROPOSITION LXVII.

TO describe *Tables common to all the Inhabitants of the Earth, relating to the First Motion.*

The Equator and Ecliptic are the same to all the Inhabitants of the Earth, and they are of considerable and frequent use in Astronomy : The former, because it is the mean, and as it were the Standard of the First or apparent Diurnal Motion of the Heavens ; the latter, because it is the Way and Path of the Sun, (who is the governor of the Days and Seasons.) The
mutual

mutual habitude and respect of these Circles to one another in all their points is disposed into Tables: That is, to every degree of the Ecliptic, marked with its proper name, (that is, the number it bears in the Sign) are annexed to the corresponding Declination from the Equator, Right Ascension, and Angle that the Ecliptic comprehends with the Meridian; as is shewn in *Prop. 20.* And therefore these three Tables are put together in one very commodiously, under the universal Title of *The Declination, and Right Ascension of every degree of the Ecliptic, and of the Angles of the Ecliptic with the Meridian.* And because in such Tables, either the same number recur, or the same increased by a Quadrant, a Semicircle or three Quadrants of a Circle; in the making of them there will be no need of carrying the Calculation beyond one Quadrant, since the remaining part of the Table flows naturally from the former, either by the change of the first Column or by Addition only.

Besides, since the intire apparent revolution of the Celestial Equator makes a natural Day, and proportionally in the parts of this revolution, by reason of the equability of the diurnal Motion of the Earth; they make a Table, out of which the time corresponding to each part of the revolution of the Equator, and on the contrary, the part of the revolution corresponding to any time, may be taken. And because among politer Nations, where Astronomy is cultivated, the natural Day is divided into 24 Hours, and the Hour in 60 Minutes, every one of which again divided into 60 Seconds, and so on; the afore-said Tables are accomodated to this division of the Time.

PROPOSITION LXVIII.

TO describe the Tables which relate to the Fix'd Stars, that are common to all the Inhabitants of the Earth.

The Places of the Fix'd Stars, in regard of Longitude and Latitude, are disposed of by the methods explain'd in *Prop. 29*. Such a Table (or Catalogue) as this, is the foundation of the whole Science of the Stars: And because the places of the Planets are found, by knowing the places of the Fix'd Stars, Astronomers are very careful in noting the places of the Fix'd Stars, as are in the Zodiac; not only those as are only seen by the naked Eye, but such also as are seen by a small Telescope.

Because the Longitude of the Fix'd Stars continually increases, by reason of the regress of the Equinoctial Points, the *Table of this Regress* is annexed to the Catalogue of the Fix'd Stars; that is, 'tis shewn how much it is in a Year, any parts of a Year, or collection of Years, so that the Tabular Longitude of a Fix'd Star may be very easily corrected by this means. *Copernicus* and others following him, reckon the distance of the Fix'd Stars in the Ecliptic toward the East, from the preceding of the two in the Horn of Aries, as was said before: By adding to this Arc (in any constant Fix'd Star) the distance of the Vernal Equinoctial point, perpetually augmented from the said first Star of Aries, to be taken out of that other Table, you make the Longitude of the Fix'd Star from the Vernal Equinox.

Because the Right Ascension and Declination of some of the more considerable Fix'd Stars are of more frequent use; and to find them out by the Longitude and Latitude given (by *Convers. Prop. 27.*) as often as there is occasion of them, would

would be very tedious; such as have a mind to spare others so much trouble, form to a given time, a *Table of the Right Ascension and Declination of the more considerable Fix'd Stars*, adding the difference of each for a certain number of Years, in which they suppose their change, as to sense, to be equable.

To the abovementioned Tables of the Motion of the Fix'd Stars is also added a *Table of the Refraction of the Stars*, made for the different Altitude of them above the Horizon (by Prop. 66.) The Altitude of any Star observed is to be lessened by the corresponding Refraction, that it may become true; or the true Altitude (found from the true place, at a given time, by Calculation) is to be increased by the said Refraction; that it may become the apparent Altitude. This different Altitude of the Star from Refraction induces a diversity of its Place in regard of Longitude or Latitude, or in any other Circle, after the same manner with that shewn before in the Parallax. We have reckoned this Table of Refraction here among the Tables of the Fix'd Stars, tho' it might more properly be refer'd to the preceding, as what is common to every Celestial Phenomenon: Others in the mean while will think it ought to be reckoned among the Tables that are to be describ'd in the following Proposition, since different Tables of Refraction are requisite for the different density of the Atmosphere in different places.

PROPOSITION LXIX.

TO enumerate the more common Tables, wherein the situation of the more considerable Places upon the Earth, in regard of one another, and in respect of the Equator, is described, and those wherein the Time reckoned in any of them is reduced to the Time reckoned according to the custom of any other.

In the Catalogue of the most considerable Places, you have the Elevation of the Pole in that place set down by it, (determined by *Prop. 17*;) to which the Latitude of the Place is equal. There is also set down the difference (found by *Prop. 32.*) of the First Meridian (whether it be supposed to pass thro' the place where the Astronomer lives, or through the *Azores* or *Fortunate Islands*) and of the Meridian of that place, and it is expressed in parts of the Equator, or in Hours and parts of an Hour. Thus we have the situation of these places to one another, and to the Equator settled, and may make use of it as occasion requires in Geography or Hydrography. The Hour reckoned in one of the Places, with the difference of the Meridians, expressed in Hours and parts of an Hour, added to it, gives the Hour reckoned in the more Eastern place; but subtracted from it, shews the Hour reckoned in a more Western place, provided both places make use of the same sort of Hours; but if they use a different sort, they are to be reduced to the same by a proper reduction.

You have also Tables for the more expeditious reduction of any Time in one of the places expressed by any Calendar, and reckoned from any known Epoch, into Time of any given form in the other place. For since (by *Prop. 11.* and *12.*) both the Epochs may be compared together, and the various forms of the Years among different Nations, the grounds and method of constructing these Tables (which are as necessary for Men of Business and Historians, as Astronomers) is evident.

Besides, the Circles distinguishing the Climates and Parallels are reduced into Tables by some. For if you begin at the Equator, the quantity of the longest Day is had, at the end of each Climate

mate and Parallel, or the beginning of the next; that is, the continuance of the Solstitial Point towards the visible Pole above the Horizon. There are also given the Declination of the Solstitial Point (namely the Obliquity of the Ecliptic) and the Right Ascension: The Elevation of the Pole in that place therefore is also given; that is, the distance of the Circle bounding the Climate or Parallel proposed from the Equator.

PROPOSITION LXX.

TO describe the more common Tables, by which the Problems of the First Motion, in different places of the Earth, are more readily solved.

For the finding out the places and times of the Rising and Setting of the Fix'd Stars and other Celestial Bodies, whose Places are known, and solving other Problems about them, Calculation is commonly used, as occasion requires. But because the use of the Sun's Motion is more considerable and frequent, and the Sun itself in its Annual course, is in different degrees of the Ecliptic; Tables are made for all Latitudes, to shew the respect of these points to the Horizon: that is, for every degree of Latitude from the Equator, the Difference between the right and oblique Ascension of every degree of the Ecliptic (found by *Prop. 33.*) is added to that degree of the Ecliptic expressed by its name, out of all which a Table is made, or rather a heap of Tables; for indeed every degree of Latitude should have a Table, extending to the several degrees of the Ecliptic. These sort of Tables are called *Tables of the Ascensional Differences*.

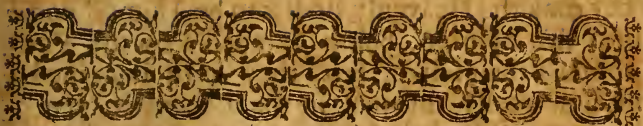
The Ascensional differences being thus reduced into Tables, the oblique Ascensions and diurnal Arcs of every one of the degrees of the Ecliptic are in like manner easily reduced into Tables by

bare Addition and Substraction; and from thence the Hours of the Rising and Setting of the Sun, when in those points. But these Operations are so easy, as that they are usually left to the pains of every one that makes the Calculation; excepting, that generally an Astronomer, that is pretty much taken up in supputations, has them already calculated for the place of his own Habitation. And sometimes also he has by him Tables (made by *Prop. 33.*) of the *Ortive Amplitudes* of the degrees of the Ecliptic, either for several Latitudes, or at least for his own.

There is another sort of *Table* (to be made by *Prop. 35.*) of the *Angle of the East*, namely that which the Ecliptic makes with the Horizon, for the different points of the Ecliptic rising in a given Latitude, or (which comes to the same) of the Altitude above the Horizon of the Nonagesim Degree from the Eastern point. And because in Calculating Eclipses of the Sun and other things, there is very great use of this Angle, Tables of it are made for every degree of Latitude from the Equator to the Pole.

And sometimes Tables of the Beginning and Ending of the Morning and Evening Twilight are made (by *Prop. 39.*) for the degree of the Ecliptic that the Sun is in, and for different Latitudes, or at least for the Latitude of their own Habitation.

In making all these sorts of Tables, there are several compendiums of the calculation and pains that occur, such as are shewn in *Prop. 67*; and a shorter and more elegant way of ordering these Tables may be had from thence.



THE
ELEMENTS
OF

Astronomy,
Physical and Geometrical.

The THIRD BOOK.

*Of the Theory of the Primary
Planets.*

IN the First Book, we have shewn what Laws a Planet must move by, to describe an Ellipse about the Sun placed in one of its Foci, which Figure only the Phenomena will admit of, as *Kepler* has shewn at large, in his *Commentaries of the Motions of Mars*, and all Astronomers now allow of. The ancient Astronomers themselves, together with *Ptolemy*, (who were for admitting nothing but perfect Circles in the Heavens, which seem'd the most proper for Bodies that were

not subject to generation or corruption, such as they took the Heavenly Bodies to be,) almost come into this opinion, being compell'd by the Phænomena themselves, since they have imagined a motion really unequal in the Excentric, but equal in the Equant Circle, whose Center was as far beyond the Center of the Excentric, as the Center of the true Motion was on this side of it; that is, since they bisected the Excentricity, and described the Orbit of the Planet, from the point of bisection as a Center. For such an Excentric circular Orbit of a Planet does not differ from an Elliptic Orbit, whose Foci are the said Centers, excepting that the former is something thicker about the middle, but the latter something smaller; which difference of the thickness of the Orbit is almost insensible in most of the Planets, by reason of the smallness of the Excentricity, and not distinguishable by the observations of the Ancients. Tho' *Kepler*, in *Chap. 44.* of the said *Commentaries*, has made it out very clearly and unquestionably, from the observations of *Tycho*. 'Tis true *Ptolemy* uses an Excentric in the simple Theory of the Sun, and does not think that the Excentricity of the Orbit is to be bisected; but in every one of the other Planets, besides the Epicycle (by which he explains the Phænomena really arising from the motion of the Earth, which he did not acknowledge) he uses (*Ch. 5. B. 9.*) an Excentric described upon the Center bisecting the Excentricity. Nay, *Copernicus* himself took the perfection of the motion and way of the Planets to be so firm, that upon this account he thinks (*Ch. 2. B. 5.*) the bisection of the Excentricity ought to be rejected, because it is absurd to grant that an equality of circular motion can be made upon a strange Center and not upon its own, (which that it is was once *Ptolemy's* opinion, is evident



Plate 1. Book 3.

Fig. 1.
p. 377.

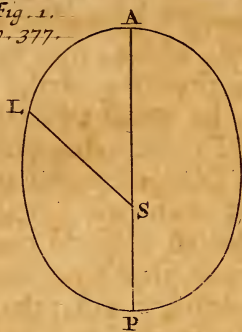


Fig. 2.
p. 378.

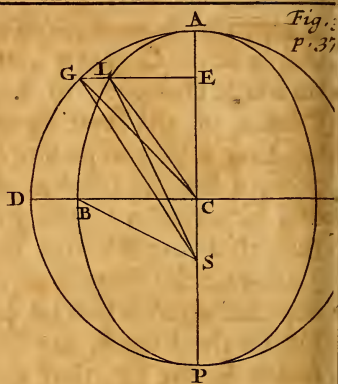


Fig. 3.
p. 378.

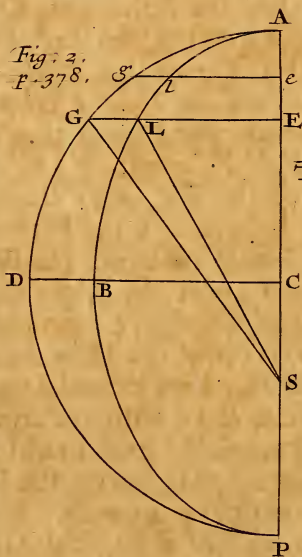


Fig. 4.
p. 383.

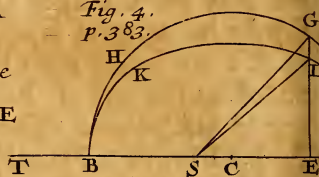
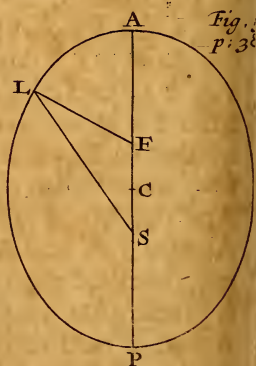


Fig. 5.
p. 383.



dent from Chap. 3. Book 3. where he says that all the motions of the Stars are equal and circular by nature; that is, (as he himself explains it) that all the lines, which are understood to carry about the Stars or their Circles in all simply equal Times, intercept equal Angles at the Centers of each Circulation, tho' he afterwards changed, in B. 9. already quoted,) and Chap. 4. of the same Book, he would rather have an Epicycle and a simple Excentric introduced, performing the same as an Excentric and Equant do; or even a Concentric with two Epicycles. But those exact and repeated observations of Tycho had not then been made, that Kepler wholly depends upon when he demonstrates that the Orbits of the Planets are Elliptic; tho' Ricciolus insinuates by the by, that Kepler began to suspect that the Way of a Planet thro' the Æther was Ovi-form; his natural sagacity taking the first hint of it from seeing the Oval Figure that Reinholdus added in the end of Purbachius's Theorics belonging to the Moon. Therefore in such a true Elliptic Orbit of a Planet ALP , where S [Fig. 1.] one Focus is the place of the Sun, ASP the line of the Apfides, A the Aphelion, P the Perihelion, L the Place of a Planet taken at pleasure, the Area ALS contained under the Right lines SA , SL , and the Elliptic Curve AL (computed from SA in consequentia) is called the Mean Anomaly, that is, Equable; for this Area increases equably, namely in the same ratio with the time: And the Angle ASL , contain'd by the line of the Apfides SA , and Radius SL that carries the Planet, is called the Coequate or True Anomaly. For it is proper that the Cultivators of a Physical Astronomy, should recede as little as may be from the ancient Terms of Art; being not only established by use, but really the fittest for that purpose.

SECTION I.

Some General Things belonging to the Theory of all the Planets.

PROPOSITION I. LEMMA.

IF upon the greater axe PA, of the Ellipse PBA [Fig. 2.] whose Center is C, as a Diameter, you describe the Circle PDA, and from a point G taken at pleasure in the Circumference of the Circle, a Perpendicular GE be let fall on PA, intersecting the Ellipse at L; and from some point in the Axe AP (for instance from one of the Foci as S) to G and L, the Right lines SG, SL be drawn; the trilinear Figure AGS contained under the Right lines SA, SG, and the Circular Curve AG, will be to the whole Circle, as the trilinear ALS contained under the Right lines SA, SL and the Elliptic Curve AL is to the whole Ellipse.

Draw eg parallel to EG , meeting the Diameter in e , the Ellipse in l , and the Circle in g . Then (by Prop. 21. Book I. of the Conics of Apollonius) el will be to CB in the subduplicate ratio of the rectangle under Pe , eA , to the Rectangle under PC , CA , or CD^2 : And eg to CD is also in the subduplicate ratio of $Pe \times eA$ to CD^2 ; therefore (by 11 Elem. 5.) el is to CB , as eg to CD ; and therefore $el : eg :: CB : CD$; that is, in a given ratio. And since this is always done, all the el insisting upon EA , will be to as many eg ; that is, the Elliptic Semi-segment ALE to the Circular Semi-segment AGE , in the same given ratio of CB to CD . Again, (by Prop. 1. Elem. 6.) the Triangle LSE , is to the Triangle GSE as LE to GE ; that is, (as has been already shewn) as CB to CD ; from whence (by Prop. 12. Elem. 5.) ALE and LSE together, i. e. ALS is to AGE and

GSE

GSE together, or AGS , in the ratio of CB to CD : And after the like manner, in that same ratio of CB to CD are all the el insisting upon the whole PA to as many eg ; that is, the Semi-ellipse PBA to the Semicircle PDA . And therefore ALS is to AGS , as the Semi-ellipse PBA to the Semicircle PDA , or as the Ellipse to the Circle. And on the contrary the trilinear ALS is to the whole Ellipse, as the trilinear AGS to the whole Circle. *Q. E. D.*

Consequently, if the Semi-ellipse PBA were proposed to be divided into two Segments having a given ratio, by a right line SL proceeding from a given point S in the Axe, it is sufficient to divide the Semicircle PDA (in some respect more known than an Ellipse) by a right line SG in the said ratio, and to let fall a perpendicular GE from the point G , on the Axis PA meeting the Ellipse in L : For the right line SL divides the Semi-ellipse PBA in the same ratio as the Semicircle PDA is divided by the right line SG : Or if ABP be an Elliptic Orbit of a Planet, in one of whose Focus S , the Sun is; as the Mean Anomaly of the Planet in L is represented by the trilinear figure ALS , so it may also be represented by the trilinear figure AGS .

PROPOSITION II.

THE Orbit of a Planet being given, to determine by calculation, as many Mean Anomalies and corresponding True ones, as you please.

Let the Elliptic Orbit given be $ALBP$, [Fig. 3.] in one of whose Foci S the Sun is placed. Let A be the Aphelion, P the Perihelion, C the Center, CA or CP the Mean distance of the Planet from the Sun, CBD perpendicular to AP . In the Circle ADP , described about the greater axe as diameter, take at pleasure from A , the arc AG , or angle ACG , expressed by

by the number of Degrees and Minutes (as is usual.) And because (by 23 *Elem.* 6.) the arc AG is to the whole circumference, as a sector AGC to the whole circle, if the Circle as well as the Circumference be supposed to be divided into 360 equal parts, or circular sectors corresponding to the degrees of the circumference; the same number that expresses the arc AG , expresses also the circular sector AGC ; the latter in small sectors, the former in small arcs, either of these *Kepler* calls the *Anomaly of the Excentric*. But from the dimensions of a Circle, (accurate enough for Calculation,) the ratio of the square of the Semi-diameter to the whole Circle is given; and from the species of the Orbit given, the ratio of the mean distance BS or CA , and also of the lesser Semi-axe BC to the Excentricity SC is given; and from thence the ratio of the Triangle BCS to the square of the Radius; and from the ratio of the arc assumed arc AG to the quarter of the Circumference AD , is given the ratio of the Sines of them, *viz.* GE to DC , and consequently of BC to GE , that is, (by 1. *El.* 6.) the ratio of the Triangle BCS to GCS . In the parts therefore, of which the Circle contains 360, there are given the sector AGC and the Triangle GCS , (whose analogous one LCS is call'd the *Physical part of the Equation*, as the angle SLC the *Optical part*;) and therefore the trilinear aggregate of them AGS , which in the preceding *Prop.* is shewn to be proportional to the trilinear ALS , and consequently may justly represent the Mean Anomaly. For if the whole area ABP of the Elliptic Orbit be imagined to be divided into 360 equal parts, the part of it ALS , namely the Mean Anomaly of the Planet in L , will contain as many parts of it, as AGS contains similar parts of the circular Area ADP ; and so will be expressed by the same number.

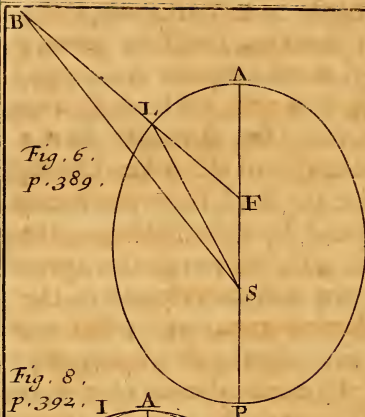


Fig. 6.
p. 389.

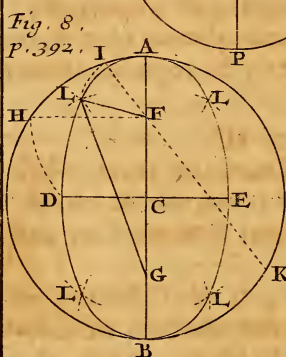


Fig. 8.
p. 392.

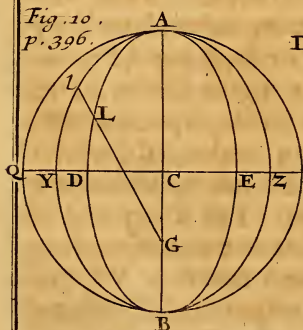


Fig. 10.
p. 396.

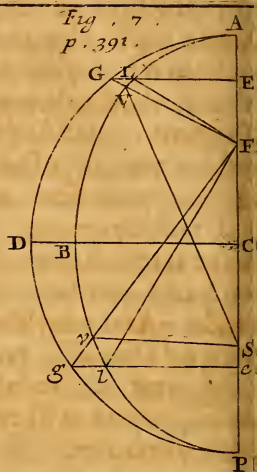


Fig. 7
p. 391.

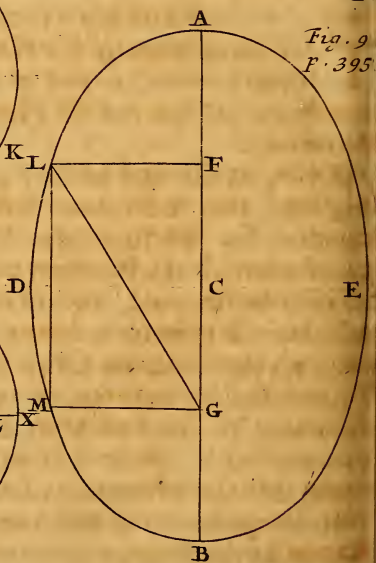


Fig. 9
F. 395

ber. Again, in the triangle GCS , the ratio of the Sides GC , CS being given, (the Mean distance of the Planet from the Sun to the Excentricity) and the contained Angle GCS , (the complement to two Right of the Angle ACG assumed,) the Angle GSC (by 41 of the *Data*) will be given; and therefore also ASL the Coequate Anomaly, namely the Angle whose Tangent EL is to EG the Tangent of the Angle ASG , found before, in the ratio of the lesser semi-axe CB of the given Orbit, to the greater semi-axe CD . And since the mean and true Anomaly, found by calculation, agree to the same assumed Arc AG , they will also agree with one another; or one will correspond to the other. And after the same manner any other arc may be assumed, and the agreeable Anomalies, (*viz.* mean and true) determined thence by calculation, which will consequently be correspondent to one another.

PROPOSITION III.

THE mean Anomaly of a Planet being given, whose Orbit is given, to find by calculation the true Anomaly and Distance of the Planet from the Sun, and the converse.

Kepler, at the end of Part 4 of his *Celestial Physics*, says, that there is no direct method of Calculation for solving this Problem. How the calculation of the Problem may be perform'd by an indirect method, and fit enough for Astronomical uses, is now to be shewn. In the Circle ADP , (*Fig. 3.*) touching the Orbit in the Aphelion and Perihelion, let the arcs AG or angles ACG of the Anomaly of the Excenter, be taken, exceeding one another by single degrees. For every one of them (by the preceding) find the corresponding Anomalies, *viz.* the mean and the true. When a Canon of the Mean and corresponding Coequate Anomaly, for a given Orbit of a Planet, is made

to each particular Arc of the Anomaly of the Excentric, look among them for the mean Anomaly given; the corresponding true Anomaly is that which is sought. But if the mean Anomaly proposed, is not found among those in the Canon, the true Anomaly sought will be found (after the manner commonly known) by a proportional part. If you are to work without Tables, *Kepler* advises the use of the Rule of Positions (call'd also the Rule of False;) by supposing the Anomaly of the Excentric AG an arc, or ACG an angle, or AGC a Sector, (for these three are expressed by the same number) to be of as many parts as you please, and by adding or subtracting from it, so assum'd as there is need, an analogous Triangle congruous to the Physical part of the Equation, namely GSC , to make the mean Anomaly AGS , for if it comes out as great as that proposed, the Anomaly of the Excenter was well put, and the true Anomaly that agrees to it is the Anomaly sought; because fitting the given mean Anomaly; but if it does not come out so great, the Position is to be corrected by that which does come out, and the operation is to be repeated. For finding the Distance of the Planet from the Sun, you have in the Triangle LCS , the Angle LSC given, namely the above found coequated Anomaly; also the Angle LCS , the Supplement of the Angle LCA , whose Tangent is to the Tangent of the Angle ACG of the Anomaly of the Excentric, as the lesser axe of the Orbit to the greater; the Excentricity SC is also given, from whence the distance SL of the Planet from the Sun becomes known.

And after the like manner, may the Mean Anomaly be determined from the given True one, either by Tables already made, or by the Rule of False Position.

PROPOSITION IV.

TO divide a Semicircle in a given ratio, by a right line drawn from a given point of the diameter.

Let AGB [Fig. 4.] be a Semicircle, C its Center, AB its Diameter, and S a point given in the Diameter; from whence a right line as SG is to be drawn, cutting the circumference in G , so as that the Area AGS may be to the Area $BHGS$ in a given ratio, namely as p to q . Suppose it done, and from G on AC let fall the Perpendicular GE . Let the Semidiameter AC or CB be called r ; AE , x ; let m be a fourth proportional to the Right lines SC , CA , and the Semicircumference AHB . Then Semicircumf. $\times$ Radius = twice

the Semicircle = $SC \times m$, or $m = \frac{2AHB}{SC}$. Again,

since by supposition, area AGS : area $BHGS$:: p : q , th. Semicircle AHB : area AGS :: $p + q$: p ;

th. $p \times AHB = p + q \times AGS$, and conseq. $2AGS$,

= $\frac{p \times 2AHB}{p + q}$. Put a for $\frac{pm}{p + q}$; then is $a = \frac{p \times 2AHB}{p + q \times SC}$

= $\frac{2AGS}{SC}$; Or, $a \times SC = 2AGS$. Towards B take

CT , so to CB , as CB is to CS ; and call AT , b ,

then is $CT = b - r$, and th. $SC = \frac{rr}{b - r}$; and $SC \times a$

= $2AGS = \frac{arr}{b - r}$. And since $CE = r - x$, th.

$SE = r - x + \frac{rr}{b - r}$; Also $2AGS = 2AGE$

+ $2GSE = 2AGE + GE \times ES$. But, by

the Doctrine of Series, in a Circle, whose radius is r , the Segment whose versed Sine is x , that is,

$$2AGE = \frac{4}{3}\sqrt{2} \times r^{\frac{1}{2}}x^{\frac{3}{2}} - \frac{2r^{-\frac{1}{2}}x^{\frac{5}{2}}}{5\sqrt{2}} - \frac{2r^{-\frac{3}{2}}x^{\frac{7}{2}}}{28\sqrt{2}} - \frac{2r^{-\frac{5}{2}}x^{\frac{9}{2}}}{72\sqrt{2}} \&c.$$

$$\text{And } GE = \sqrt{2xx - rx} = \sqrt{2} \times r^{\frac{1}{2}}x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}}x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}}x^{\frac{5}{2}}}{16\sqrt{2}} - \frac{r^{-\frac{5}{2}}x^{\frac{7}{2}}}{64\sqrt{2}} \&c.$$

Hence

Hence $2 \triangle SEG = (SE \times EG = r + x + \frac{rr}{b-r} \times EG)$

$$= r + x \times GE = \sqrt{2} \times r^{\frac{3}{2}} x^{\frac{1}{2}} - \frac{5r^{\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} + \frac{7r^{-\frac{1}{2}} x^{\frac{5}{2}}}{16\sqrt{2}} \&c.$$

$$+ \frac{rr}{b-r} \times GE = \frac{rr}{b-r} \times \sqrt{2} \times r^{\frac{1}{2}} x^{\frac{1}{2}} - \frac{r^{-\frac{1}{2}} x^{\frac{3}{2}}}{2\sqrt{2}} - \frac{r^{-\frac{3}{2}} x^{\frac{5}{2}}}{16\sqrt{2}} \&c.$$

But $\frac{rra}{b-r} = 2 \triangle AGS = 2 \triangle AGE + 2 \triangle SEG$, therefore, by reducing the Equation according to Art, you will find that

$$a = \sqrt{2} \times br^{-\frac{1}{2}} x^{\frac{1}{2}} - \frac{2r^{-\frac{1}{2}} x^{\frac{3}{2}}}{3\sqrt{2}} + \frac{\sqrt{2} \times br^{-\frac{3}{2}} x^{\frac{3}{2}}}{12} - \frac{\sqrt{2} \times r^{-\frac{3}{2}} x^{\frac{5}{2}}}{20} \&c.$$

$$\text{Th. } aa = \frac{2b^2}{r} x + \frac{b^2}{3r^2} - \frac{4b}{3r} x x^2 + \frac{2}{9r} + \frac{4b^2}{45r^3} - \frac{14b}{45} x x^3 \&c.$$

$$\text{th. } x = AE = \frac{ra^2}{2b^2} + \frac{r^2}{6b^5} - \frac{r}{24b^4} x a^4 + \frac{r}{720b^6} - \frac{13r^2}{360b^7} + \frac{7r^3}{72b^8} x a^6 \&c.$$

Taking therefore AE equal to so many terms of this Series as you please, and from E erecting EG perpendicular to AB , it will meet the Circle in the point G sought. Q. E. F.

And (by *Prop. 1.*) the same EG will meet any Ellipse AKB , describ'd upon the greater axe AB , in L , so as that SL being joyn'd will divide the Semi-Ellipse AKB into two parts ALS , $BKLS$, having a given ratio to one another.

The less the distance of the points C and S (that is, the less the excentricity of the Ellipse) and the ratio of p to q , the fewer terms of this Series will suffice: For the ratio of p to q ought to be small, that the Series, expressing the value of the Segment AGE , may soon converge; that is, that few of its terms may be very near equal to the Segment.

I formerly publish'd this Solution of the Problem before us, in a *Geometric Exercitation concerning the Dimensions of Figures*, Printed at Edinburgh in the

the Year 1684; but I thought it would not be amiss to insert it again here, as its proper place.

'Twas first thus done by Mr. James Gregory, and communicated to Mr. John Collins, in a Letter dated April the 9th 1672, from St. Andrews.

PROPOSITION V.

TO divide an Ellipse that is not very excentric, by a right line thro' one of the Foci, so as that the portion of the Elliptic area contained between the greater Axe and the Line to be drawn, may be very nearly to the whole Ellipse in a given ratio.

The method delivered in Prop. 3. of finding the true Anomaly by the mean, is reckoned tentative and indirect: But any Geometric one, however elegant, (as is that formerly given us by the protracted Cycloid by Dr. Wallis,) is not so fit for Astronomical Calculations; and that which does it by Series (as Prop. 4.) is too laborious; on which account Astronomers have recourse to Approximations, especially the following one.

Let the Ellipse proposed be ALP , [Fig. 5.] whose Center is C , one Focus S , the other F . From the Focus S a right line SL is to be drawn, so as that the Trilinear ALS , may have very nearly a given ratio to the whole Ellipse $ALPA$.

At one of the Foci F , make an angle AFL , having the same ratio to four right, as the trilinear Figure ALS ought to have to the whole Ellipse, whose Leg FL meets the Ellipse in L , and the Line SL will perform the thing proposed very nearly. For if the Foci S and F coincide with the Center C , it will be exactly, as is evident from Prop. 33. Elem. 6; the error therefore is as great as that which proceeds from the Excentricity: If that therefore be small, that is in an Ellipse not very excentric, the area ALS is to the whole Ellipse as the Angle AFL to four right angles; that is in a given ratio.

S C H O L I U M.

Kepler himself, who first shew'd that a Planet described an Ellipse, so as that in equal times equal areas were described, being cut off by right lines drawn to the Sun placed in the Focus, took notice also of this Approximation, and describes it in *Chap. 3. Part 2. B. 5. Epit.* thus, *as if L were the Planet, the Angle AFL might be instead of the mean Anomaly almost.* But he did wholly forbear using it, both because in the Planet Mars it plainly differs from the Heavens, and especially because his elegant demonstrations of the motions from their Physical causes by this means would lie wholly neglected and uncultivated, because intirely unknown; he was therefore willing to persist in the true Physical Theory of the Planets, tho' forced to use an indirect method, as in finding the coequated Anomaly from the true, rather than to follow one foreign from the natural one, tho' the thing could be done by it directly and very easily.

Afterwards the Celebrated Astronomer *Seth Ward* made use of this Theory of the Planets, as one that Nature it self took into her service; and assumes that the Orbit of any Planet is Elliptical, in one of whose Foci is the Sun, which he acknowledges is the true and Physical instrument of all the Planetary motions; in the mean while the motion of each Planet is so tempered about the other Focus as to form at it, Angles proportional to the times; and upon this Hypothesis he has built his *Geometrical Elliptical Astronomy* (a very excellent work, and what all sound Astronomy is very much indebted to.) For he has applied Geometry (with which he was very well acquainted) so happily to Astronomy, as not only to find directly and Geometrically the true Anomaly from the mean one given, but to determine the Orbits of all the Planets, as well in regard of

of their figure and situation, as in regard of their magnitude, (taking the Orbit of the Earth as a measure.) But still he *has not wiped off that* Ἀκατέργητος, *which still cleaves to Astronomy*; For he solves this Problem of Astronomy, not according to the true and genuine System, whereby *Kepler* had challenged Geometers to solve them; but by a fictitious one, that comes very near the true one. Notwithstanding, the method of his Astronomy is of very great use: Since the Orbits of the Planets are not very Excentric Ellipses, but such as come pretty near to Circles, nor does *Ward's* Hypothesis, wherein he makes the motion about the Focus, which the Sun is not in, equable, differ much from the accurate one; and will be of very great service in determining the Orbits of the Planets very nearly, which may (by repeating the Calculation, as shall be shewn below,) again be made more accurate at pleasure.

'Tis true that *Ismael Bullialdus*, in his *Philolaic Astronomy*, while he makes every Planet move in an Ellipse, and cuts this Ellipse out of a Cone, such as that the Axis of the Cone passes thro' one of the Foci of the Ellipse; and lastly, while (*Chap. 15. Book 1.*) he makes the motion of the Planet about the Cone and in Circles parallel to the Basis of the Cone, to be equal; he has supposed the equable increment of the Angles at the Focus in which the Sun is not; (namely that through which the Axis of the Cone passes,) that is, the Hypothesis of *Ward*, without knowing of it: Nor did he understand it, till *Ward* inquired into the foundations of his Astronomy, and demonstrates it in the first *Chap.* of his *Inquiry*. And *Bullialdus* himself, who had commiserated *Kepler* on the account of his mistakes and defects in Geometry, there meets with what he deserved at the hands of *Ward*, who shows *Bullialdus* (besides the several er-

rors, which he had committed and ingenuously confesses in his *Found. of Phil. Astr. asserted against S. Ward*;) that he did not thoroughly understand his own Hypothesis, while he supposes, without knowing it, that the motion about the Focus of the Ellipse, in which the Sun is not, is equable; which notwithstanding had he thoroughly known it, he would have finished the greatest part of his Astronomy much better and easier.

And Count Pagan has also seriously embraced this same Hypothesis, namely of an equable angular motion of a Planet about the other Focus of an Ellipse, in which the Sun is not placed, for true and genuine, and has published in *French* at *Paris* *An.* 1657, a *Theory of the Planets* built upon it; and at the end of his Theory, has boldly thrown upon the mistakes of the Observers, the disagreement of it with the Heavens, which is greatest in the Octants from the Apfides. And tho' this *Theory of Pagan* comes a full Year after *Ward's Geometrical Astronomy*, and three at least after the *Inquiry into the Foundations of Bullialdus's Astron. Philolaica*, (where he had laid down the foundations of all his Astronomy, and constructed most of the Problems;) yet *Pagan* goes no farther than the determining the place of a Planet in its Orbit being given at an assigned time, and its distance from the Sun; that is, the establishing the principles themselves of Astronomy; omitting all the comparative Astronomy, in the cultivating of which by Geometry, the Accurate *Ward* has well employed his pains.

PROPOSITION VI.

Supposing the approximation lately described to be accurate enough, (that is, in the Hypothesis of *Ward*) the Mean Anomaly being given, to find the coequated Anomaly, and the Planets Distance from the Sun in a given Orbit.

Let

Let ALP [Fig. 6.] be the Elliptic Orbit of the Planet about the Sun placed in S as a Focus; let its Aphelion be A , Perihelion P , greater Axe AP : From the Mean Anomaly the angle AFL given, the coequated Anomaly is to be found, namely the angle ASL , and the Distance of the Planet in L from the Sun, viz. SL ; that is, the ratio of this right line to FS or AP .

Produce FL to B , so that LB may be equal to LS ; and join BS . And because (by *Prop. 52. B. 3. Conic of Apollonius*) FL and LS taken together are equal to the axe AP , FB will be equal to the same AP . And therefore in the Triangle BFS , the Sides BF , FS , and the Angle intercepted BFS , the complement of the Mean Anomaly given AFL to two right or four right angles, (according as L is in the first or second Semicircle of Anomaly;) the Angles FSB and FBS or LSB will become known, whose difference ASL is the coequated Anomaly sought. Besides, in the Triangle LFS at first the angle LFS and side FS are given, and now the angle FSL is found, and the remaining one FLS ; therefore the Side LS will be found in the parts of the given distance of the Foci SF ; that is, the Distance of the Planet from the Sun.

Q. E. F.

In finding the angles FSB , FBS , in the triangle BFS , when BF , FS , and the angle BFS are given; (by the common Rule of Trigonometry,) BF and FS together are to BF less'n'd by FS , as the Tangent of half the Angles B and FSB , to the Tangent of half the difference of the same. But BF and FS taken together are equal to AB and FS taken together; and BF less FS is equal to AP less FS , or twice SP ; and the sum of the angles B and FSB is equal to the external AFL , namely to the Mean Anomaly; and their difference is equal to FSL the coequated Anomaly. And

therefore the sum of AP and FS is to $2PS$, or the half of the one SA is to the half of the other PS ; that is, the distance of the Planet when in its Aphelion is to its distance when in the Perihelion, as the Tangent of half the mean Anomaly to the Tangent of half the coequated Anomaly: In which proportion the two first terms are constant in the given Planet.

After the like manner the mean Anomaly may be found, the coequated being given. But if you don't want the coequated Anomaly ASL that answers the given mean one AFL , but the angle FLS (called *the Elliptic Equation*;) it will be the double of the angle FBS found before. And this angle subtracted from the mean Anomaly AFL , the Planet descending from the Aphelion to the Perihelion, or in the first six Signs of the Anomaly, leaves the coequated ASL : But if it be added to the mean Anomaly in the last six Signs of the same it makes the coequated Anomaly; from whence it receives the name *Prosthaphæresis*.

PROPOSITION VII.

TO explain the correction of the preceding approximation made by Bullialdus; and to shew how (that being supposed) the coequated Anomaly and the distance of the Planet from the Sun may be found.

Ismael Bullialdus, in the defence of his *Philolaic Astronomy* against Ward, in a Book intituled, *The Foundation of the Philolaic Astronomy explained*, Chap. I. and 2. shews from four Observations made by Tycho, that the Hypothesis of Ward lately described does not agree to Mars, that in the first and third Quadrant of the mean Anomaly is more forward than it ought to be by this Hypothesis, but in the second and last Quadrant of the mean Anomaly is less than this Hypothesis requires. But even now neither has he so much as looked

back

back upon the sagacious *Kepler*, according to whose Physical System, the matter ought to be thus in every Ellipse, and in Mars sensibly, (because its Orbit is sensibly excentric;) and as the very numbers themselves which *Kepler* brings do plainly shew. But being resolved upon correcting *Ward's* Hypothesis, in requital of the like-kindness from him, he brings the following correction of the mean Anomaly, found as above, in *Chap. 4.* of the said Book. Let *ALBP* [*Fig. 7.*] be the Elliptic Orbit of a Planet, whose Foci, Apfides, Center, &c. are expressed by the same letters as they have been frequently before: And let the angle *AFL*, the mean Anomaly in the Hypothesis of *Ward*, be to four right angles, as the time in which the Planet describes the Elliptic Arc *AL* to the intire periodic Time. And because the point *L* in the first and third Quadrant of the mean Anomaly (that is, while the Angle *AFL* is less than a right, or greater than two, or less than three) is not so forward as Observations require, but in the first and fourth Quadrant too forward; that is, because the angle *AFL* is in the first case too little, in the second too great; therefore *Bullialdus* corrects this angle thus. Thro' *L* draw *LE* perpendicular to the Axe *AP*; meeting it in *E*, and the Circle described upon the diameter *AP* in *G*, join *FG*; let the Angle *AFG* be taken for the mean Anomaly instead of *AFL*; the angle *AFG* *Bullialdus* takes the liberty of calling the *True-Mean Anomaly*, leaving the name of the *Mean* to the angle *AFL*, which is proportional to the Time: And therefore the Place of the Planet in its Orbit corrected will be the point *V*, where *FG* meets the Orbit. The angle of the *True-Mean Anomaly* *AFV*, corresponding to the angle of the mean Anomaly *AFL* is found very readily, by taking an angle whose Tangent is to the Tangent

of the mean Anomaly, as the greater axe of the Ellipse is to the less; and this will be that which is sought. For GE , the Tangent of the angle GFE , is to LE the Tangent of the Tangent of the Angle LFE , to the same radius FE , as DC or CA to BC . And in the Triangle VFS (just as in the preceding *Prop.* in the Triangle LFS) the Side FS , the external angle AFV , and sum of the Sides FV and VS being given, the coequated Anomaly FSV is found, and the Side VS the Distance of the Planet from the Sun corrected.
Q. E. F.

And this is the correction of *Ward's Hypothesis* which *Bullialdus* brings; and 'tis well enough, if it be taken only for a correction of the approximation to the true System, as it ought to be; because *that by which he had effected, that at length the coequated Anomaly might be found from the mean à priori, and at the same time the calculation satisfy observation; which no one had done before him* (according to *Mercator's* judgment) in the *Elliptic Hypothesis*. But when *Bullialdus* puts it upon us as the true and genuine System, and derives the Physical causes of it (*Chap. 3. and 4.*) after his manner from the Cone, he not only leaves the Foundations of his *Philolaic Astronomy* unexplain'd, but disputes as if there were no Physical foundations of Astronomy at all.

PROPOSITION VIII.

TO describe the Orbit, wherein that Celebrated Astronomer *Dominicus Cassini*, thinks a Planet is carried about the Sun, and to explain its agreement and disagreement with Natural causes and the Phænomena.

Let the Diameter AB of the Circle $AKBH$ [*Fig. 8.*] be divided into two equal parts in the point C , and thro' it draw an indefinite line DE perpendicular to AB . Divide AB again in G ,
 so

so that AG may be to GB as the distance of the Planet (whose Orbit is to be described) from the Sun when it is in its Aphelion, to the distance from the Sun when it is in its Perihelion; and take CF equal to CG . From F on AB let a Perpendicular FH be erected, meeting the Circle in H . On the Center F , at the distance FH , describe a Circle cutting the indefinite right line DE , in the points D and E , which are the extrems of the lesser Axe of the Orbit to be described. To find its other points, draw thro' F any how, the right line IFK meeting the Circle in I and K . On the Center F at the distance IF or KF , let a Circle be described; and on the Center G , at a distance equal to one of the Segments of the right line IFK , namely KF or IF , another Circle: the intersections of these Circles, L, L, L, L , will belong to the Orbit. And after this manner, because two Circles are drawn upon both Centers, four points of the Orbit will be determined at the same time. After the like manner its other points may be determined, by drawing other right lines passing thro' F , and bounded by the Circle on both sides. Because the rectangle under the segments of the right line drawn thro' the point F and bounded by the Circle is given, being equal to the rectangle under AF, FB (by *Prop. 35. Elem. 3*;) 'tis evident that the rectangle under the right lines FL, GL , drawn from the points F and G , to any point L of the Curve ALB is also given, and equal to the rectangle under AF, FB or under AG, GB ; and therefore that AG is to GL as FL to GB or FA . As in the common Ellipse the sum of the right lines from the Foci inclined to any point in the Circumference is given, so the rectangle under the right lines drawn from the points F and G to any point in the Curve $ADBE$ is given, on which account

Cassini gave the name of *Foci* to these points *F* and *G*.

He would have this Curve to be the Orbit of a Planet, which it describes about the Sun placed in the Focus *G*, so as that the angles at the other Focus *F* are proportional to the times; which he therefore calls the Focus or Center of the mean Motion, as he does the other *G*, of the true Motion, where he places the Center of the Sun and of the World. On which account any two distances of a Planet from the Focus of the true Motion are reciprocally as its distances from the Focus of mean Motion. In this Orbit, the true Anomaly, namely *AGL* is determined from the mean Anomaly or Angle *AFL*, if in the Triangle *LFG*, the Base *FG*, and one of the adjacent angles *LFG* (the complement of *AFL* the mean Anomaly,) and the rectangle under the two other sides *FL*, *FG* being given, the other things may be found.

This *Cassinian* Hypothesis, briefly delivered in a *Treatise of the Origen and Progress of Astronomy*, has this Physical disadvantage, that tho' a Planet, by the action of a centripetal Force of a certain kind toward the Sun, (whose Law may be known by *Cor. Prop. 37. B. 1.*) may describe the Orbit *ALDBE* about the same placed in *G*, yet this force will be intirely different from that used in most of the operations of Nature, and from that whereby the common Ellipse, having the same Foci and Vertices, is described about the Sun placed there; and this over and above, that a Planet describing the Curve *ALD*, as *Cassini* supposes, (namely so as that a line *FL* being drawn every where to the Planet, the angle *AFL* is made proportional to the time) does not all observe that universal Law demonstrated in *Prop. 11. B. 1.* so as to make the area *ALG*, which the radius *GL* proceeding from the

the

the Sun, describes, proportional also to the Time. On which account this Hypothesis, as well as the above describ'd Hypothesis of *Ward* and *Paganus*, is Physically impossible. 'Tis utterly impossible that a Planet, tho' it be acted upon by any centripetal Force tending toward the Sun, should describe the Perimeter of the common Ellipse, or of this *Cassinian* Ellipse, so as that the angle at the Focus where the Sun is not be also proportional to the Times. For this being supposed, the areas at the Sun would not be proportional to the Times, (as has been shewn above of the common Ellipse, and shall be shewn hereafter of both) which notwithstanding is done, as has been demonstrated in *Prop. 9. B. 1.*

But we will demonstrate that in the Figure above described ALDBE (or in any other that is concave every where toward the Center, whose four parts made by the two axes are equal and similar, and the two Foci F and G equi-distant from the Center,) every angle at the Focus F is not to four right angles as the area at the Focus G to the whole Figure ADBE.

Let us suppose the angle at the Focus *F* to be to four right as the area at the Focus *G*, to the intire Figure [Fig. 9.] Draw the right line *FL* perpendicular to *AB*; by which means the angle *AFL* will be a fourth part of four right, or half of two right; and therefore the area *ALG* will be by supposition a fourth part of the intire Figure *ADBE*, or a half of the half *ADB*, that is, equal to *LDBG* what it wants of half. From *G* draw *GM* parallel to *FL*, meeting the Orbit in *M*; and then *GM*, from the nature of the Orbit, will be equal to *FL*; and consequently by *Prop. 33. El. 1.* *ML* (which intirely falls within the Curve *ADBE*, since by supposition it is every where concave towards the Center *C*) is equal and parallel to *GF*; and (by *Prop. 34. El. 1.*) the Triangle

angle MGL is equal to the Triangle GFL . And since the Figure AFL is, from the nature of the Orbit, equal to the Figure BMG ; the Figure ALG is equal to the Figure $BMLG$, contained under the right lines BG , GL , LM , and the Curve BM . But it was just now shewn, that the Figure $BMDLG$, contained under the right lines BG , GL and the curve $BMDL$, is equal to the Figure ALG ; and therefore the Figure $BMLG$ is equal to the Figure $BMDLG$; that is, a part equal to the whole; which is absurd. Therefore the Figure ALG is not a quarter of the whole Figure $ADBE$, when the Angle AFL is a quarter of four right; or every angle at the Focus F is not to four right angles as the area at the Focus G to the whole Figure. Q. E. D.

Besides the aforesaid Hypothesis labours under this difficulty, that the true Anomaly, and the corresponding mean Anomaly, and the distance of the Planet from the Sun, do not in the least agree with the Phænomena in the Heavens. For since the common Ellipse does agree with these Phænomena, that other, the *Cassinian* one, which is of a different nature, cannot agree with them. For the place of a Planet, at a given time, in the common Ellipse $ATBZ$, [Fig. 10.] will differ from that in the *Cassinian* Orbit $ADBE$, described with the same Vertices A , B , and the same Foci F , G , as well in regard of the true Anomaly or Angle made with AB at G , as of the distance from the Sun, in respect of the distance of the Aphelion GA . And tho' in Orbits, where the Excentricity is very small, the difference is not very sensible; where the Excentricity is great, things are otherwise. In the former case, the Circle $AQBX$, described upon the diameter AB , almost satisfies the Phænomena. Yet all Astronomers now-a-days acknowledge that a circular Orbit is too broad

broad about the mean Longitudes, as $\mathcal{Q}$ and X . But this *Cassinian* Orbit $ADBE$, on the contrary is much too narrow in those places. Suppose for instance, $ARBZ$ to be the common Ellipse, whose greater axe is to the less as 5 to 4; let a *Cassinian* Orbit $ADBE$, be describ'd upon the same Vertices and Foci; its lesser axe DE will be less than the lesser axe RZ of the true Elliptic Orbit, above a third part, whereas in the mean while $\mathcal{Q}X$ the breadth of the Circular Orbit exceeds RZ by a lesser part; namely only by a fourth part. Therefore this Orbit, since Physical reasons don't suit with it, (because it is intirely impossible to be described after the manner that *Cassini* its Author requires, namely so as that the Angles at the other Focus, than where the Sun is, may be proportional to the Times; for thus the area described by the radius would not be proportional to the Time: Nay, to describe it at any rate, a Centripetal force towards the Sun is necessary that is altogether different from that which is used by nature, which, like every other natural Virtue, propagated from a Center or to a Center, in right lines thro' the circum-ambient spaces, is reciprocally as the Square of the distance,) and since the Orbit does not agree with the Phænomena and Celestial Observations, by reason of the shortness of the lesser axe, and the narrowness of the middle Orb arising from thence; it ought, I say, to be thrown out of Astronomy.

Since the celebrated Mr. *Cassini*, in his before-mention'd Treatise, propos'd to Astronomers this Curve, as the Orbit of a Planet, there have been various disputes about the nature thereof, and the Law of Gravity requisite to describe it. In my second thoughts upon it, its different Species, and some properties of them not sufficiently known, presented themselves to me.

'Tis sufficiently known that this is the nature of this Orbit. If from two given points *F* and *G* [Fig. 11.] to any point *H* of the Curve, the right lines *FH*, *GH* be drawn; the rectangle under *FH*, *GH* is equal to a given space. The right line *FG* produced both ways till it meets the Curve, shows the Vertices *A* and *B*; and *AB* is the principal Axe; and the point *C* which is the middle between the Vertices, is the Center of the Figure, and *DE* thro' *C* perpendicular to *AB*, the lesser axe; and the points *F* and *G* the Foci.

In this Figure if the lesser axe exceed the distance of the Foci, the Curve terminating the Figure is every where concave towards the Center, as it is commonly taken to be. If, the principal Axe remaining the same, the distance of the Foci is lessened, the lesser Axe will be encreased, but it continues less than the Axe of the Ellipse described upon the same principal axe and the same Foci; till at length the Foci meeting it becomes equal to the greater axe, and the Figure turns into a Circle. But if, on the contrary, the distance of the Foci be increased, the lesser axe will be lessened, and will become equal to the said distance, since the distance is to the principal axe as unity to a mean proportional between unity and three.

If the distance of the Foci be farther encreased, the lesser axe will still be lessened, and the Curve will be no longer concave towards the Center but convex, at the extremities of this axe, as in Fig 12; till the distance of the Foci being so far increased, that it is to the greater axe as the Side of a Square to the Diagonal of the same, the lesser axe will become nothing, and the Curve reach to the Center on each side.

If the distance of the Foci be greater than in the said ratio, the lesser axe is impossible, and the Figure turns into two conjugate ones, as in

Fig.

Fig. 13. which, upon the increase of the distance of the Foci, will be lessened, till at length the Figure run into two conjugate points.

The distance of the Foci increasing, the two conjugate Figures do again emerge, and they increase after the same manner as they decreased before, differing from the former in the order of the Foci and of the Vertices, and are increased till they become infinite. And afterwards this System will again approach to a Circle by the same degrees as it receded from it.

From what has been said, 'tis evident at first sight, that this Figure is by no means fit to be the Orbit of a Planet. Not to mention the cases wherein it passes into two conjugate Figures, and lays aside the nature of an Orbit, namely where-soever its Excentricity is so great, as Comets (if they revolve round the Sun like Planets, as is most probable) require to describe their course: to pass over these cases, I say, there are also in those cases, where it returns into it self and makes an Orbit, some with an Excentricity so large, as that the Curve near the points *D* and *E* [*Fig. 12.*] becomes convex towards the Sun; and therefore the Planet would need a Centrifugal force from the Sun to describe this part of its Orbit, whereas in the mean while, in the nearer and more remote places *B* and *A*, a Centripetal force towards the Sun is requisite. That is, that the circumsolar Bodies may be moved by that Law, it must be granted, that in equal distances from the Sun, here the force is centripetal, but there centrifugal; which, how foreign it is to all the Laws of Nature, any body may easily see. And tho' the Excentricity of no Planet be so great, yet 'tis well known to Geometers, that of a Figure, all whose Species beyond a certain limit are unfit for the discharging any office in nature, the remaining Species

Species on this side of that limit, can't be admitted as fit for that office. 'Tis necessary *that this Cassinian Curve be thrown out of Astronomy*, not only upon the account of the reasons mention'd before in this Proposition, namely, *because it does not agree with Celestial Observations, by reason of the shortness of the lesser axe, and no Physical reasons answer it, since to describe it there would be need of a centripetal force toward the Sun, intirely different from that made use of by Nature*; but because of its absolute impossibility. For it is impossible that any Species of this Figure can be describ'd by a Planet, so as that the angles at the other Focus, where the Sun is not, may be proportional to the times; for thus the area described by the radius that carries the Planet along would not be proportional to the Time. For 'tis not true that increasing the angle at one Focus by equal increments, the increments of the area made at the other at the same time will also be equal, as I thought not long ago.

In the *Figures 12 and 13*, the greatest breadth of the Figure is found, by describing a Circle upon the Center C thro' the Foci; for it will cut the Curve in the points sought L, L . And the greatest Ordinate KL is a third proportional to the right lines GF and FD , in the first of them, or a fourth proportional to the right lines GF, GA , and AF in both.

If DE remains in being, the Ordinate from the Focus FP is equal to the lesser Semi-axe CD , when the lesser axe is to the distance of the Foci, as the Side of a Square to its Diameter. If the distance of the Foci be greater than in that ratio, FP will exceed CD .

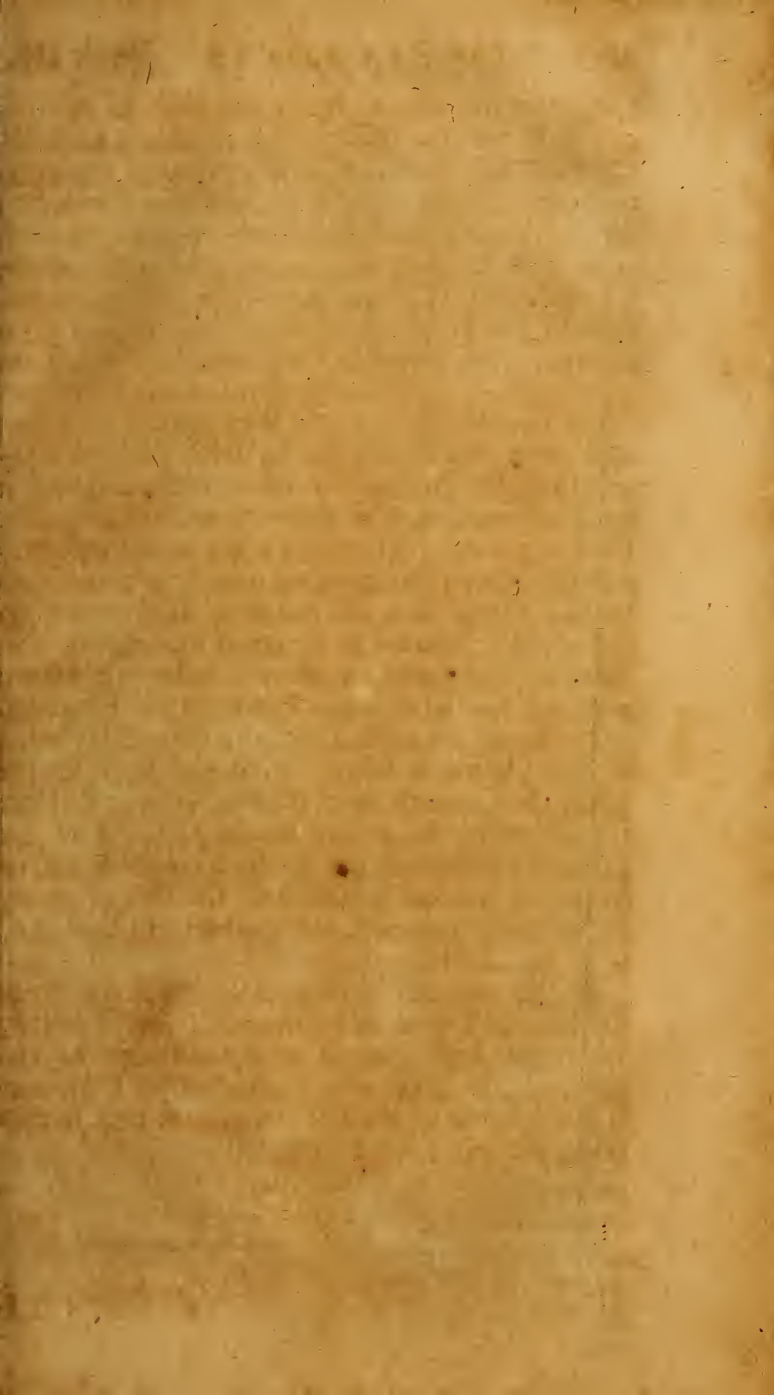


Fig. 14.
P. 401.

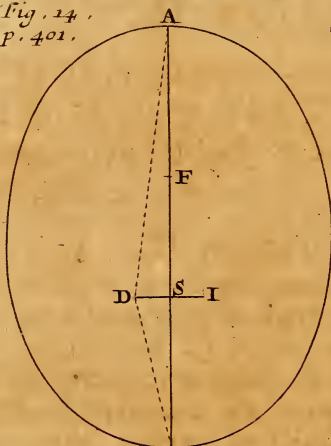


Fig. 15.
P. 404.

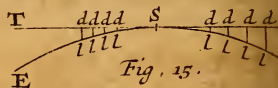


Fig. 16.
P. 405.

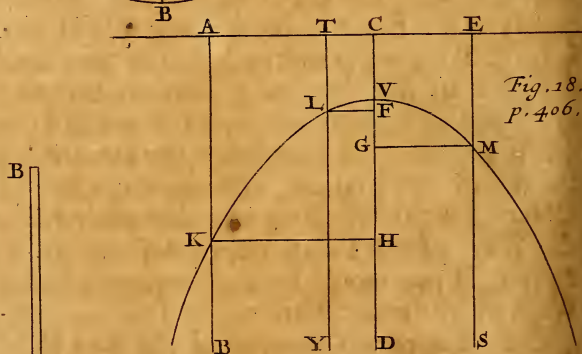
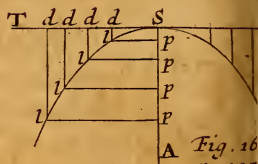


Fig. 18.
P. 406.

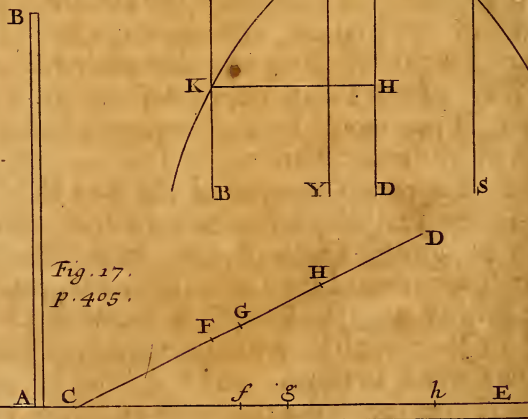


Fig. 17.
P. 405.

SECTION II.

Concerning the determining the Orbit of the Earth, and the Theory of the Earth seen from the Sun, or of the Sun seen from the Earth.

PROPOSITION IX.

TO define by observation, the Species of the Orbit of the Earth and the Position of its Axe; as also the Periodic Time of the Earth.

One design of this Book being to determine the Species of the Orbits of the Primary Planets about the Sun; the Orbit we ought to begin with is that of the Earth about the Sun, as well because it is the same with that which the Sun seems to describe in the space of a Year about the Earth, as especially because the Species and Position of this is necessary to determine the same things in the rest of the Planets. For since the Eye of the Observer is carried along together with the Earth, the Way of the Earth ought first of all to be known, if you would be well acquainted with the Ways of the other Planets.

Let the apparent motion of the Sun be observed in the Ecliptic, when he is swiftest; that is, when in a given space of time it describes the greatest arc toward the East; for in that case (by *Cor. 2. Prop. 41. B. I.*) the Earth is in the Perihelion *P* [Fig. 14.] but when its motion is found to be slowest the Earth is in the Aphelion *A*. But the place of the Earth seen from the Sun is exactly opposite to the place of the Sun observed from the Earth, and therefore the place in the Ecliptic of the Aphelion and Perihelion of the Earth seen from

the Sun is had ; that is, the Position of the line of the Apfides of the Earth. But in Book 2^d we shew'd how the places of the Fix'd Stars in respect of the Ecliptic, that is, the cardinal and other points of the Ecliptic might be determined in respect of the Fix'd Stars, from whence the Position of the line of the Apfides of the Earth among the Fix'd Stars will be found. The same may be done by bisecting the angle contained between two right lines drawn to two places, where the apparent motions of the Sun from the Earth performed in a given time are equal ; for such points are equidistant from the points of the Aphelion and Perihelion. To find the Species of the Orbit, you must observe (by *Cor. 3. Pr. 41. B. 1.*) that the apparent motion of the Earth in the Aphelion *A* seen from the Sun in the Focus *S*, is to the apparent motion of the same in the Perihelion *P*, as SP^2 , to SA^2 . And since the apparent motion of the Earth seen from the Sun, is the same with the motion of the Sun seen from the Earth ; from the observation of this, while the Earth is in the Aphelion and Perihelion, the ratio of SA^2 to SP^2 will become known, and consequently its subduplicate SA to SP ; by compounding and dividing them, you will have the ratio of $SA+SP$ to SP , and $SA-SP$ to SP ; or of AP to SP , and (supposing *F* to be the other Focus) of SF to SP , and consequently the ratio of AP to FS is also given,

The same things may be determined from the observation of the apparent Semidiameter of the Sun : For the Earth is then in the Aphelion, when the apparent Semidiameter of the Sun is the least ; and consequently the place of the Sun opposite to the place then observed, is the place of the Aphelion seen from the Sun : and after the same manner of the Perihelion. The middle
point

point also between the two points, where the Sun is of the same apparent diameter, shews the place of the Aphelion or Perihelion of the Earth; all which are too plain to insist upon.

To determine the Species of the Ellipse, let DI be the diameter of the Sun situated in the Focus S . And the angle DAI is to the angle DPI , as SP to SA ; but the two first terms of this Proportion are given; therefore the ratio of SP to SA is known: From whence, as before, the ratio sought of AP to FS , determining the Species of the Ellipse, may be drawn. Instead of the apparent diameters or semidiameters of the Sun, measured by the Angles or Arcs, the Tangents of the angles $DA S$, $DP S$, of the apparent semi-diameters may be taken, as being evidently more exact.

Besides, the space of time between the moments immediately following, wherein the Earth was at the same Apfis, to be determined from what has been said in this Proposition, is the periodic Time of the Earth. But this will be better determined, if such moments are used as are at the greatest distance from one another as may be, and the intermediate time be divided by the number of the revolutions the Earth has made in the mean while about the Sun; for then the Time of one revolution of the Earth will come out.

PROPOSITION X.

TO determine the Time of either Equinox by Observation.

Just about the Equinox observe (by *Prop. 18. B. 2.*) the Sun's declination, which if it be nothing at all, will then be the Time of the Equinox itself: But if any thing, then from thence (by *Prop. 20. B. 2.*) the place of the Sun in the Ecliptic will be found. Let the same be repeated again, or even a third

time, at some distance of time between. And by the help of the places thus found and of the intermediate time, the Time of the Equinox itself will be found.

In the performing this and such like Problems, the observations of the Sun are to be corrected; (as always) by the Parallax and the corresponding Refraction, and all other things are to be cautiously avoided, that may render the calculation uncertain; *v. g.* not two observations made on the same side of the Time which is to be determined; but rather two observations, whereof one precedes and the other follows the Equinox, are to be made choice of.

PROPOSITION XI.

TO determine the Time of either Solstice.

Because the Tropic touches the Ecliptic in the Solstitial point, the 15th Figure will represent the little portion of the Tropic and Ecliptic near the Solstice; where TR is the Tropic; EC the Ecliptic touching it in the Solstitial point S ; and the little right lines dl , dl perpendicular to TR , will represent the deviations of the points of the Ecliptic l , l from the Tropic, or the changes of their Declinations: In which case, the little right lines dl , dl are (by *Prop. 24. Book 1.*) as the squares of the Arcs Sl , Sl , or of the right lines Sd , Sd , being nearly equal to them respectively. On which account a small portion of the Ecliptic near the Solstice S , will not sensibly differ from the Parabola ESC ; that is a Figure, whose property it is to have dl every where as the square of the right line Sd respectively. Again, the Arcs Sl , Sl are as the times wherein they are described by the Sun: For, since the Apfides of the Elliptic Orbit, which
the

the Sun describes as to sense, continue at this time pretty near the Solstitial points, the apparent motion of the Sun will be equable; that is, the arcs Sl, Sl will be as the times wherein they are described. From whence it may be fairly inferred, that a portion of the Ecliptic near the Solstice (suppose that portion which the Sun describes in about ten days, five before and as many after the Solstice,) does not at all differ from the Parabola Sl , [Fig. 16.] in which the Abscissa's SP, SP are what the Declination of the Sun every moment wants of the greatest Declination; consequently their differences PP, PP are as the differences of their Declinations themselves; that is, as the intervals of the Altitudes of the Sun above the Horizon, (or of the distances of the same from the Vertex) in the Meridian or any other vertical: And the Ordinates Pl, Pl are respectively as the times wherein the Sun acquires the foresaid Declinations, and the intervals of the one are as the intervals of the other.

These things being demonstrated, let a Gnomon AB [Fig. 17.] be erected, whose shadow made by the Sun when 'tis near either of the Solstices and received upon the Plane CD (to which a right line BH , connecting the Sun and the Vertex of the Gnomon B , is almost perpendicular) marks out the point H : Some few days after (for instance, three or four) observe again the point where the shadow of the extremity B falls, when the Sun is upon the same Vertical; and let that be F : And let the like observation be repeated again, and the point observed be G : Now from these three points H, F, G being given, the time of the Solstice itself is to be found. 'Tis evident that the right lines HF, GF are the intervals of the distances of the Sun in the same Vertical from the Horizon, or from

the Vertex; (that is, by what has been shewn above, the changes of the Declination.) For FGH does not in the least differ from an Arc of a Circle described upon the Center B with the distance BG , because the Angles FBG , HBG , are very small. And if instead of the Plane CD , to which BG is perpendicular, the shadow should be taken upon any other Plane CE inclined to CD by the angle DCE , not a very great one; for instance, in the points b, f, g ; the right lines bf , gf will have the same ratio to one another, as HF , GF , because the right lines fF , gG , bH , are almost parallel; since they meet only in the point B , which is at a distance great enough in respect of the distances fF , gG , bH . The Problem therefore for finding the Solstice from the Points F, G, H given, together with the moments of Time, wherein the Sun, placed in the same Vertical, casts the shadow of the point B to them, is reduced to the following Problem, which is purely Geometrical.

The distances AT , TE , [Fig. 18.] of three parallel right lines AB , TY , ES , lying in the same Plane being given; to describe the Parabola KLM , whose Axis CVD is parallel to AB , TY , ES , that cuts them in the points K , L and M , so as that, drawing the Ordinates to the Axe, KH , LF , MG , the portions FG and FH of the axis intercepted by them, may be equal to given right lines.

For by this means, (if FG and FH are equal to FG , FH marked out by the observation described above, and the ratio of AT to TE the same with the ratio of the Time between the observation of the points H and F , to the Time between the observation of the points F and G) KH , LF , MG will represent the Times between the observations of the points H , F and G , and the moment itself of the Solstice; that is, as the points

A ,

A , T , and E representing the moments of the Time wherein the Shadow of the Gnomon was observed in the points H , F , and G , the point C will represent the moment of the Solstice itself. Call the Line AT , a ; TE , b ; FH , c ; FG , d ; TC , x . Therefore AC or $KH = a + x$, and EC or $GM = b - x$. Call the principal *Latus rectum* of the Parabola, r . Then from the nature of the Parabola $x^2 = VF \times r$, and therefore $VF = \frac{xx}{r}$. After the like manner $VH =$

$$\frac{a^2 + 2ax + x^2}{r}, \text{ and } VG = \frac{b^2 - 2bx + x^2}{r}.$$

Wherefore $c = (FH = VH - VF =) \frac{a^2 + 2ax}{r}$;

and $d = (FG = VG - VF =) \frac{b^2 - 2bx}{r}$. And

getting away r , by the help of these two Equations; you will find $\frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$. And

ordering the Equation as is usual, x will be $\frac{b^2c - a^2d}{2ad + 2bc}$; that is, the space of time between the known moment represented by T , (namely, when the shadow of the Gnomon was observed in F), and the moment of the Solstice sought; from whence the time of the Solstice itself will be given. Q. E. F.

But if the time between the observation of the Shadow in H , and the Solstice itself be sought, add AT to TC found, and the sought $HC = (a + x =) \frac{b^2c + 2abc + a^2d}{2ad + 2bc}$. After the like manner will EC

be $= (b - x =) \frac{b^2c + 2abd + a^2d}{2ad + 2bc}$. But if the or-

der of the observations be such, as that the observation of the Shadow of the Gnomon in F is ex-

actly in the middle between the observations of the same in H and G ; that is, if $AT = TE$, or $a = b$, the Equation will be more simple; for the the distance of the Times of the Solstice and of the Shadow observed in F will be equal to $\frac{ac - ad}{2d + 2c}$ the distance of the Solstice from the time of the observation of the Shadow in H equal to $\frac{3ac + ad}{2c + 2d}$ and from the moment of the observation of the Shadow in G equal to $\frac{ac + 3ad}{2c + 2d}$; that is, in the Figure, as $2FH + 2FG$ to GH or $4HF - 2GE$ to GH , or $2HF - GH$ to $\frac{1}{2}GH$, so is the half of the time between the first observation of the Shadow in H and the last observation of the same in G , to the time between the middle observation of the Shadow in F and the moment of the Solstice itself sought.

The sagacious Mr. *Edm. Halley* first found out and published this method of determining the times of the Solstices, in the *Philosop. Transact.* for the beginning of the Year 1695; and illustrated it by the example of two Solstices determined by this, from the Observations of *Walther* and *Gassendus*; where he takes notice of several things well worth observing in the Practice, concerning the situation and advantageous form of the Gnomon, and concerning the accurate observation of the Shadow, and other things.

PROPOSITION XII.

TO define the quantity of the Tropical Year.

Since the Tropical Year is that space of time wherein the same Seasons of the Year return again; that is, wherein the Sun departing from any point of the Ecliptic (for instance that where-
in

in it touches the Tropic) returns to it again; 'tis evident that its quantity will be determined, if the Sun be twice observed in the same point of the Ecliptic, (for instance, twice in the same Equinox, by *Prop. 10*, or in the same Solstice, by the *11th*), and the intermediate time be divided by the number of revolutions of the Earth about the Sun made in the mean time. If there be any error in determining the moment of the Equinox or Solstice, that it may not disorder the measure of the Tropical Year, 'tis proper to make use of such Equinoxes or Solstices as are at the greatest distances from one another, so that the error divided into so many parts (by the division made by a great number of intervening Revolutions) may become insensible.

But because the Times of the Solstices seem'd difficult to the Ancients to be discerned, as *Ptolemy* himself says, who therefore passed over the Solstices of *Meton* and *Euctemon* and *Aristarchus* himself; which also seem'd so true to the Moderns, that *Ricciolus* asserted that *Ptolemy* was too confident in hoping that neither himself nor *Archimedes* had erred so much as a quarter of a day in their observation or computation in the Solstitials; and *Hevelius*, *Chap. 4. Prod. Astron.* says, *That the Solstices, tho' they be observed by the best and greatest Instruments, and even the most experienced Observer, can never be determined in their smallest parts*: Astronomers have made use of the Observations of the Equinoxes, whose moments they thought more accurately determinable, on the account of the vast and sensible change of the Declination of the Sun about the times of the Solstices. But, by the help of the preceding Method, the times of the Solstices themselves may hereafter be determined, at least as accurately as those of the Equinoxes.

Whereas the Sun seems to move sometimes slower and sometimes swifter, (according to the different velocity of the Earth about the Sun,) if instead of that, it be imagined to move equably, so as to describe the Ecliptic in the time determined by this Proposition; such a motion is very fitly called its *Mean Motion*, (being really such an one between the swiftest and slowest,) and the place the Sun is thus in, is called its *Mean Place*.

PROPOSITION XIII.

TO determine the quantity of the *Sydereal Year*.

By comparing the place in the Ecliptic a Fix'd Star was formerly in, with its present place, find the Annual precession of the Equinox; that is, how much each point of the Ecliptic recedes from a given Fix'd Star towards the West in the space of one Year, which was done in *Book 2. Prop. 31*: And by the preceding Proposition determine the space of Time requisite for the Sun to describe an arc of the Ecliptic equal to the Precession thus found. This therefore added to the quantity of the Tropical Year, gives the quantity of the *Sydereal Year*, which consists of the Tropical Year, wherein the Sun departing from a point of the Ecliptic returns to the same, that has in the mean while moved a little way forwards to meet it, and of that besides wherein the Sun describes that arc thro' which the said point of the Ecliptic moved to meet the Sun; that is, wherein the Sun departing from an immovable point (for instance, a Fix'd Star) seems to return again to it.

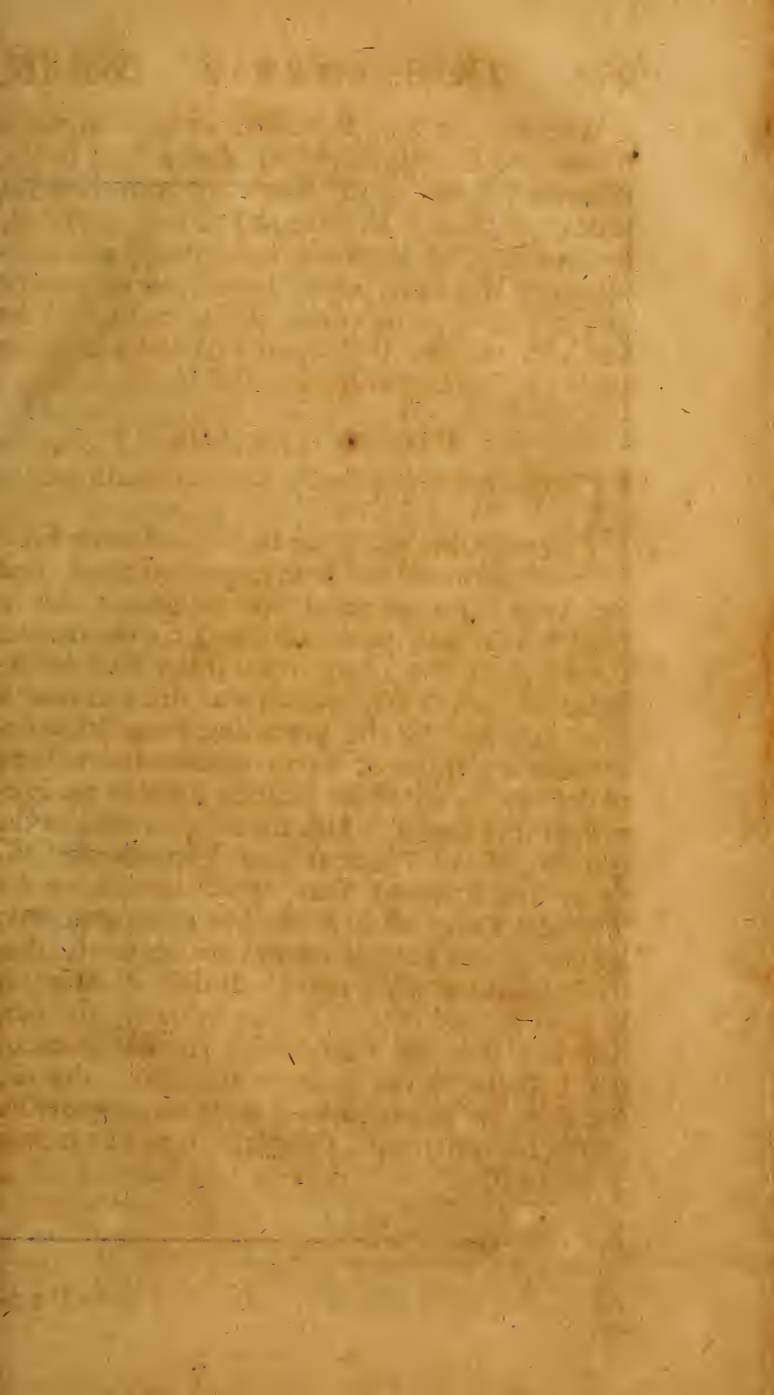


Fig. 19.
P. 411.

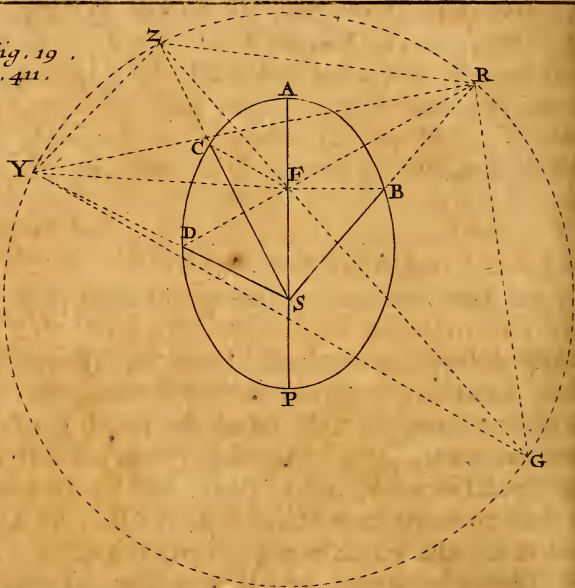


Fig. 11.
P. 398.

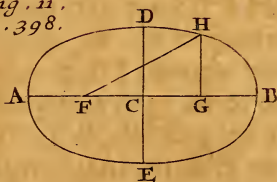


Fig. 12.
P. 398
P. 400

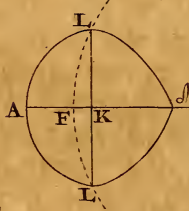
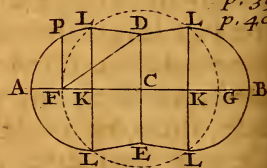
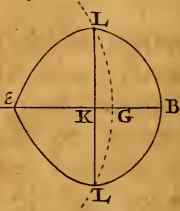


Fig. 13.
P. 399.
P. 400.



PROPOSITION XIV.

Three places of the Sun, determined by Prop. 20. B. 2. and the Periodic Time of the Earth, which is very nearly equal to the Syderial Year, determined by the preced. Prop. being given, to find the Species of the Orbit of the Earth, the Situation of the line of the Apsides, and the Time when the Earth is in the Aphe-
lion.

The methods laid down in Prop. 9. for finding the Species and Position of the Orbit of the Earth, require such accurate observations that they are hardly fit to be put in practice: We shall therefore add the following one better fitted for Astronomic uses. Let there be given three Places of the Sun by observation, namely when the Earth is in the three points B, C, D of its Orbit [Fig. 19.] Therefore the three lines SB, SC, SD are had by position, or the three angles BSC, CSD, BSD . And the Period of the Earth being given, together with the Times between the observations, the ratios between the whole area of the Ellipse $ACPB$ and its parts BCS, CDS and BDS are also given. The Problem therefore is reduced to this, to describe such an Ellipse upon a given Focus S , as that its whole area should be to its parts contained under the right lines given by position, SB, SC, SD , and the Curve of the Ellipse, in given ratios.

Let AP then be the Orbit of the Earth, S and F its Foci, A its Aphelion, P its Perihelion. On the Center S imagine a Circle described by a radius equal to the greater Axe, the Circumference whereof cutting the right lines SB, SC, SD produced in R, Z and γ . Join the right lines $FB, FC, FD, FR, FZ, F\gamma$; and produce one of them ZF , untill it meet the Circumference again in G , and join $RG, G\gamma, \gamma Z, ZR$.
If

If $ACDPB$ be the true Orbit of the Earth, then (by *Prop. 5.*) the Angles BFC , CFD , BFD , will be respectively to four right angles nearly as the Times between the observations to the Periodic Time: And on the contrary, if the said ratio hold in these angles, $ACDPB$ is the Orbit of the Earth sought. What remains therefore to be done is, to find the Position of the right line SF or AP , and the Species of the Ellipse, by having given, besides the angles BSC , CSD and BSD , the ratios also of the angles BFC , CFD and BFD to four right angles.

Since there are given the angles RSZ , ZSY , RST , namely the observed differences of the Longitudes, and the angles BFC , CFD , BFD , namely the Mean motions of the Earth almost for the spaces of the times between the Observations; therefore there are also given RFZ , ZFY , RFY Arithmetical Proportionals between BFC and RSZ , CFD and ZSY , BFD and RST respectively. For since SR is equal to AP , by construction, and the sum of the right lines SB and BF equal to the same, (by *Prop. 52. B. 3. of the Conics of Apollonius*) BR and BF will be equal; and after the like manner may CZ , CF , as also the right lines DY , DF , be shewn to be equal. Consequently BRF and BFR , CZF and CFZ , DYF and DFY will be equal. But the angle $BFC = RFZ + BFR + CFZ$; and $RFZ = RSZ + BRF + CZF$. Since therefore BFC exceeds RFZ as much as that does the angle RSZ ; 'tis plain that these are Arithmetically proportional. After the same manner are CFD , ZFY , ZSY ; also the angles BFD , RFY , RST are Arithmetically proportional. Besides, in the Triangle YFG all the Angles are given; namely YFG equal to the complement of the angle YFZ , to two right angles a mean Proportional between CFD and CSD , which

which are known, and $\angle RGF$ half the observed Angle $\angle ZSR$: And therefore supposing RF to be of any number of parts whatever, FG will be given in those parts. In the Triangle RFG all the angles being given, (namely $\angle RFG$ the complement of $\angle ZFR$ to two right, and $\angle FGR$ half the angle $\angle ZSR$) and the side FG , FR will be given. And in the Triangle FR there being given FR and FR , and the angle $\angle FR$, (because a mean Arithmetic proportional between BFD and BSD ,) TR will be given and the angles $\angle FRY$, $\angle FRR$. Then, in the Isosceles Triangle YSR all the angles are given (because $\angle YSR$ at the vertex of the Triangle is given) and TR , therefore YS also will be given. Lastly, in the Triangle FYS the angle $\angle FYS$ is given (the difference between the known ones $\angle RYS$, $\angle YRF$) and the sides FY , YS ; therefore the angle $\angle YSF$ and side SF will be found. But the Position of the right line YS is given, namely the Place of the Sun in the third Observation, when the Earth was in D ; therefore the Position of the line of the Apfides AP will be given, because inclined in a known angle $\angle YSF$ to YS given by position. But in the same parts, in which YS is expressed, the right line SF is found; therefore the ratio between YS or what is equal to it AP the greater Axe of the Orbit, and SF the distance of the Foci is given; that is, the Species of the Orbit. Nor can any thing else be determined by the Observations: For every Elliptic Orbit similar to $ACDP$, described upon the Focus S , whose greater Axe is the same in position with the right line SF , will satisfy the Observations proposed.

In the Triangle DFS , the side SF , the angle $\angle DSF$, and the angle $\angle SDF$, the double of $\angle SYF$ lately found, being given; DFS is found, and consequently DFA contiguous to it. But the
Time

Time which is to the whole Periodic Time as the angle $DF A$ to four right, is that which happens between the moment of the observation in D , and the moment that the Earth is in its Aphelion: this therefore becomes known.

The method of determining the Species of the Orbit, and the Position of the Line of the Apfides here delivered, agrees as well to any other Planet revolving about the Sun as to the Earth, provided there are but three Observations, as much to the purpose in this other Planet, as those three used about the Earth.

PROPOSITION XV.

TO correct at pleasure the Position of the Line of the Apfides, and the Species of the Orbit determined above.

Let the Position of the line of the Apfides and the Species of the Orbit found by the foregoing, be taken for true and accurate, (for they do not differ much from thence by *Prop. 5.*) from them and the Periodic time and the times between the observations being given, find the angles corrected at F , agreeable to the times of the observations made in the points B , C and D ; and if you will use only the first correction (namely *Bullialdus's*;) this may be done by *Prop. 7*; and you may go to what exactness you please by *Prop. 4*. By the help of these angles thus corrected, instead of the former, that are simply proportional to the times; that is, in the language of Astronomers, by the help of the angles of the mean Anomaly thus equated, instead of the angles of the mean simple Anomaly made use of before, let the calculation of the former Proposition be repeated, and let the Position of the Line of the Apfides and the Species of the Orbit be found, and they will be more accurate.

COROL

COROLLARY.

From whence the Motion of the Apfides of the Orbit of the Earth may be determined: Namely by comparing of the Place of the Aphelion at present with its Place some Years ago, (both which are found by *Prop.* 14, and corrected by *Prop.* 15,) and by dividing the arc made in the mean while by the said Aphelion by number of the intervening Years, the Annual Motion of the Aphelion will come forth, which also will be extended to parts of a Year.

PROPOSITION XVI.

TO find the Place of the Earth seen from the Sun, and its Distance from the Sun at any given Time.

The species of the Orbit of the Earth, and the place of the Aphelion for any known time being defined by the two last Propositions, find the place of the Aphelion for the time given: And if it be since the time to which the place of the Aphelion was determined, add the motion of the Apfides made in the mean while, and defined by the Corollary of the preced. *Prop.* to the place of the Aphelion determined before, or let it be subtracted, if the time given be before; and you will have the Place of the Aphelion for the given time. Then let the time when the Earth once was in the Aphelion be determined (by *Prop.* 14,) and since by the 13th the Periodic time of the Earth may be known; from thence also will be known the time when the Earth was in its Aphelion next before the Time proposed, and consequently the ratio between the Periodic time of the Earth and the time elapsed since the last Aphelion is known; that is, in the Figure *ALP* (*Fig.* 1.) representing the Orbit of the Earth, the ratio of the intire Ellipse to the area *ALS*: And consequently (by *Prop.* 3, 4, 6 and 7.) the angle *ASL* will become known.

And

And since the position of the right line AS is known, the position of the right line SL will also be known; that is, the Place of the Earth seen from the Sun. Besides, by the Propositions above cited, the ratio of the right line LS to the right line AP becomes known; that is, the Distance of the Earth from the Sun in the parts of the greater Axé.

PROPOSITION XVII.

TO *explain the inequality of the natural Days, and to shew the Equation of time.*

Since it is the business of an Astronomer to shew the motion of the Stars, and this is performed in Time, and can't be conceived without it; and since the most sensible part of Time is a natural Day, by the repetition of which a Year, and by the division of which an Hour and the parts of an Hour are made; and since in computing their motions, all days ought to be supposed equal; as far as this supposition is from truth, so far even the Celestial motions taken out of the Tables will disagree with the Phænomena, if no account of this inequality be taken. To understand this, let it be observed, that a Natural day is that space of time, that intervenes between the Sun's leaving a given Meridian in the Heavens and returning to it again; that is, the space wherein a revolution of the whole Celestial Equator is performed, and over and above, of that part of it, that answers to such a portion of the Ecliptic, as the Sun describes in the mean while by its Annual motion towards the East. But because this portion of the Equator, added to the whole Equator, is not every where equal, as well on the account of the Obliquity of the Ecliptic, as that the apparent Annual motion of the Sun about the Earth is not equable; neither will these Natural days be equal: And would therefore then be equal, if the

the Sun seen from the Earth in its Annual motion moved both equably and in the Equator; and 'tis evident in this case, after that the whole Equator has passed the Meridian, there remains that portion still to pass, which the Sun in the mean while would describe by its Annual motion towards the East, which portion would always be equal to itself, since we suppose this motion to be equable; and therefore even the Time measuring the passage by the Meridian of the whole Equator and this additional part, would be always equal to itself. Let us therefore suppose another Sun, as was said before, moving in the Equator, and consequently measuring the equal or mean Time: 'Tis evident that the difference of the Time, between the appulse of the true and imaginary Sun to the Meridian, or any other Hour Circle, is that which makes the mean Noon of the apparent Noon, and the apparent of the mean; that is, the *Equation of Time* itself. And because the true Sun and that point of the Equator, where its Right ascension ends, arrive at the Meridian together; and the Equation of Time is that space of Time, that flows while the Arc of the Equator contained between the extreme point of the Right ascension of the true Sun and the place of the imaginary Sun, passes the Meridian; if this Arc be converted into time, you will have the Equation of time sought. Besides, the place of the imaginary Sun is as far distant in the Equator from the point of the Vernal Equinox, as the mean place of the Sun in the Ecliptic is from the same: And therefore the foresaid arc of the Equator, the measure of the Equation of the time, is equal to the difference between the Right ascension of the true Sun and the distance of the mean place of the Sun from the Vernal Equinox. But it is commonly known, that the difference of any

two Quantities A and B is equal to the difference between A and any assumed quantity X , and to the difference between this X and B together. Assuming therefore a third arc, namely the distance of the true place of the Sun from the Vernal Equinox, the Arc measuring the Equation of Time will be the sum of the two arcs, one of which is the difference between the Right ascension of the Sun and its true place in the Ecliptic, the other the difference between the true and mean motion of the Sun. But sometimes it happens, that the difference of the two Quantities is equal to the sum of them taken simply; namely when the one is negative and the other positive: Therefore an Arc to be converted into Time, to produce the Equation of Time, is always the sum or difference of two arcs, one of which is the difference of the true Motion of the Sun and its Right ascension, and the other is the difference of the true and mean Motion of the Sun.

But because this Arc consists of two parts, we will consider them separately. And first let us abstract from the unequal motion of the Sun under the Ecliptic; that is, let us consider it a little while as equal; and therefore the motion of the imaginary Sun in the Equator will be equal to the motion of the true Sun in the Ecliptic, or the true and imaginary Sun will be equally distant from both Equinoxes. And because in the first Quadrant of the Ecliptic, or from the beginning of γ to the beginning of $\oplus$, the distance of the Sun from the beginning of γ is greater than its Right Ascension; the imaginary Sun will be farther distant from the beginning of γ toward the East, than is the Right Ascension of the true Sun, and therefore the true Sun arrives at the Meridian sooner than the imaginary, because more to the West; that is, the apparent time precedes the

the mean : hereupon when the Sun is in the first Quadrant of the Ecliptic, that part of the Equation of Time, which is the difference of the true Motion of the Sun and its Right Ascension, turn'd into Time is to be subtracted from the apparent, to make the mean or equable Time. But when the Sun is in the second Quadrant of the Ecliptic, its Right Ascension is greater than the distance of the Sun from the beginning of γ , (because the complement of the Right Ascension to a Semicircle is less than the distance of the Sun from the beginning of γ ;) and therefore the imaginary Sun (which, by supposition, is as far distant in the Equator from the beginning of γ , as the true Sun in the Ecliptic is distant from the same) is less distant from the beginning of γ than the true Sun; and consequently arrives at the Meridian sooner, and therefore the mean Noon precedes the apparent; that is, the mean Time reckons 12 Hours, when the apparent does not yet reckon so many, tho' it be the same moment, but differently called. And therefore in this case, when the Sun is in the second Quadrant of the Ecliptic, this part of the Equation of Time (which arises from the obliquity of the Way of the Sun to the Equator, and is the difference between the Sun's motion and its Right Ascension turned into Time) is to be added to the apparent, to make the mean time, or to be subtracted from the mean to make the apparent. But if the Sun be in the third Quadrant of the Ecliptic, the same is to be done as if it were in the first; for here the distance of the Sun from the beginning of γ is greater than its Right Ascension, as in the first: And the Sun being in the fourth Quarter, the same holds as to this part of the Equation of Time as if it were in the second, and for the same reasons.

Let us next consider the Inequality of natural Days arising from the unequal motion of the Sun in the Ecliptic, or the Equation of time arising from thence, laying aside a little while the Obliquity of the Ecliptic; that is, considering the Sun as moved after the like manner, and with the same celerity in the Equator as it is found really to move in the Ecliptic, and that these two Suns the true and imaginary, left the Apogæum together. And since the imaginary one moves with the mean Motion, it must go before the true one, while it goes forward from the Apogæum to the Perigæum; that is, the imaginary one is more to the East than the true one: The true one therefore arrives at the Meridian sooner; therefore the apparent Noon precedes the mean. And the difference between the places of the true and imaginary Sun, or between the true and mean Motion of the Sun, converted into Time, is the Equation of the time arising from this inequality of the Sun's motion: This Equation therefore, while the Sun goes from the Apogæum towards the Perigæum, or (in the language of an Astronomer,) in the six first Signs of the Anomaly, is to be subtracted from the apparent Time to produce the mean, or added to the mean to produce the apparent out of it. In the Perigæum the true Sun overtakes the imaginary, and from thence to the Apogæum is more to the East: The imaginary one therefore in all this time, namely in the six last Signs of the Anomaly, arrives at the Meridian sooner; that is, the mean Noon precedes the apparent Noon, by as much as the arc between the imaginary and true Sun spends in passing by the Meridian; that is, as much as that arc, namely the difference between the mean and true motion of the Sun converted into time makes. This part of the Equation of time therefore

fore in the present case is to be added to the apparent time to make the mean, or to be subtracted from the mean to make the apparent.

And when both these causes of the Equation of Time hold, (as they really do) both these things are to be done; that is, to equate the Time you must consider the Sun two ways, *viz.* both in what Quadrant of the Ecliptic it is in, that one part of the Equation may be added to or subtracted from the apparent, as was said before; and in what Semicircle of Anomaly, that the part depending upon that, may be added or subtracted according as the nature of it requires. And on this account it is, that the Equation of natural Days (which, from what has been said above, absolutely speaking, is the Difference of the mean place and the right Ascension of the Sun's true place turned into time) is made up of two parts, And because the vernal Equinox and Apogæum of the Sun, the heads or beginnings of these parts, don't always keep the same position to one another, but the former being carried towards the West leaves the latter, (as was shewn in B. I.) comes to pass, that the (absolute and only) Equation of Time made up of these is not perpetual, but useless in a few Years, the Sun not possessing the same degree of the Anomaly, or the same situation in respect of the Apfides, when it returns to the same point of the Ecliptic.

Ptolemy expressly demonstrates the above mention'd Equation of Time, Chap. 10. B. 3. of *Great Construct.* notwithstanding *Vendeline* and others have said that it is all useless, and use the apparent and mean Time promiscuously. *Kepler* on the contrary suspected a third cause of equating the time, arising from the diurnal Motion of the Earth's not being intirely equable, but sometimes quicker sometimes slower, according as it is more or less distant

from the Sun, the original of its motion ; and calls the part of the Equation depending upon it, the *Physical Part*, leaving the name of the *Demonstrative* to the other described above, which is made up of two parts. *Tycho* also had a suspicion of the acceleration or retardation of the First Motion, and was for ballancing the other part of the Equation depending upon the Sun's Anomaly, by means of it ; for he only made use of the other arising from the Obliquity of the Ecliptic, which he calls the *Empyric*, because it would suffice for the finding by Calculation the Eclipses he had observed. But *Street* thought this Inequality of the First Motion, to be sufficient for the making use of the part neglected by *Tycho*, under a contrary Title, as he has put it in his *Astronomia Carolina*.

PROPOSITION XVIII.

TO correct the Place of the Sun, seen from the Earth, found by Prop. 16.

If the given Time is the mean, the Place found needs no correction ; for it is hitherto the true. But if otherwise, let the apparent given Time be converted into the mean (by the preced. Prop.) by taking the Place of the Sun found above, Prop. 16. as accurate : (For that Place of the Sun is really accurate enough for the determining the Equation of Time, since it continues very nearly the same, whether the Place of the Sun be that which is found above, or the true and most correct :) And to this mean, again (by the said 16th Prop.) find the Sun's place, and the place thus found will be the correct and accurate one. For the mean Time only is fit for Astronomic uses, on which account it is call'd the *Astronomic Times* ; this therefore should be made use of : But to determine the mean when the apparent is given, the Sun's place is necessary ; that therefore was first to be found, by

by using the apparent Time, as if it were the mean, to find the Equation of the Time. But this once found, and the apparent Time reduced to the Astronomic, seek for that Time (which is the same with the moment proposed, but expressed by a number fitted for an entrance into the Tables) the Sun's place (by the same Prop. 16.) and the Distance of the Earth from the Sun, which is evidently the true and correct one.

The converse of this Problem may be constructed much after the same manner, and the Time may be found, when the Sun seen from the Earth will be in a given Place.

SECTION III.

Concerning the determining the Orbits of the rest of the Primary Planets, and composing the Theory of them seen both from the Sun and the Earth.

PROPOSITION XIX.

TO determine by observation the Position of the Line of the Nodes of a Planetary Orbit, and the Distance of the Planet itself, when in the Node, from the Sun. [Fig. 20.]

In the two Figures fitted to the two cases of a superior and an inferior Planet, let S represent the Sun; Tt the Orbit of the Earth, whose Focus is S ; Nn the Line of the Nodes of the Planet proposed. And first let the Earth be in T , and from thence let the Planet be observed when in the Ecliptic, and therefore in its Node, suppose P : For the Planet will never be seen in the Ecliptic from the Earth, being in the Plane of the Ecliptic, unless when it is also in the Plane of

the Ecliptic. After an intire revolution, let the Planet be observ'd in the same Node, when the Earth arrives at t . Let the right lines ST , Pt , be imagined to be drawn, as also St , PT intersecting one another in the point X . In the Triangle STX there are given the angles TSt by the Theory of the Earth and Time between the observations, and STP the observed Elongation of the Planet from the Sun, and the Side ST the Distance of the Earth from the Sun in the former observation. Therefore SX is found; and so Xt since St the Distance of the Earth from the Sun in the second observation is also given: Again, in the Triangle tXP the angle tXP equal to the angle TXS in the former is given, and the angle StP the elongation of the Planet from the Sun in the second observation, and the side tX lately found; and consequently the side Pt will be found. Lastly, in the Triangle StP , the sides tS , tP being given, together with the angle contained PtS , SP the Distance of a Planet in a Node from the Sun will be found, and the angle tSP that the Centric Place (or that seen from the Sun) contains with St given by position, namely, the Place of the Earth seen from the Sun at the time of the second observation; and the opposition to this is the Centric Place of the other Node; that is, the Position of the Line of the Nodes Nn .

By this means the Motion of the Nodes of the Orbs of the Primary Planets will be determined, by comparing the Place of the Node of each Planet found out by the observations of the Ancients, with the Place of the same determined by the observations of the Moderns.

After the like manner, if a Planet be observed twice in any other point of its Orbit from the Earth, the Place of the Planet seen from the Sun and its Distance from the Sun are determined.



Plate 5, Book 3.

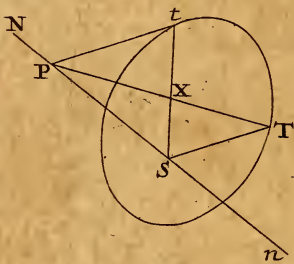


Fig. 20.
P. 423.

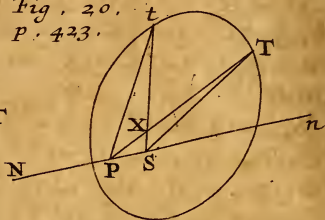


Fig. 21.
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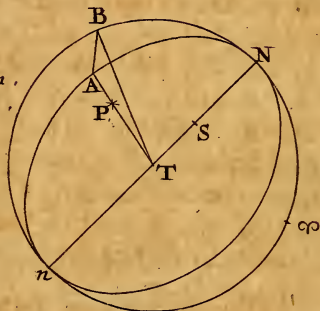
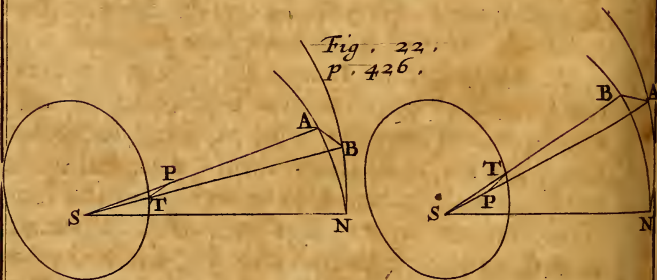


Fig. 22.
P. 426.



PROPOSITION XX.

TO define by observation the Inclination of any Planetary Orbit to the Plane of the Ecliptic.

Let S represent the Sun; the Circle NBn [Fig. 21.] the Ecliptic among the Fix'd Stars; and $NA n$ the Path of the proposed Planet, seen from the Sun among them: And the right line NSn will be the Line of the Nodes known by position, (by the preced. Prop.) Whenever the Earth arrives at the said Line of the Nodes, which it does twice a Year, the Time of which (by Prop. 18.) is known; let the Geocentric Place A of the Planet P be observed (by Prop. 27. B. 2,) whose Latitude is AB an Arc perpendicular to the Ecliptic, and Longitude VB . Since the Longitude of the Sun VN is known, the difference also of these Longitudes NB will be known. In the Spherical Triangle ANB , right angled at B , the Sides AB , NB are given; and therefore the angle ANB , the measure of the Inclination of the Plane $NA n$ of the Orbit of the Planet to the Plane of the Ecliptic NBn , will be known.

PROPOSITION XXI.

From the observation of a Planet in Opposition or Conjunction with the Sun, to get away its Second Inequality and find its Distance from the Sun.

Besides the true Inequality in the motions of the Planets visible to an Eye in the Sun, placed in the common Focus of all the Orbits, there is another Optical one, arising from the annual motion of the Earth, relating to us living on the Earth. The former, which is proper to the Planets, is called by a name proper enough, the *First Inequality*; but the other is secondary, as it were to the Planet, and therefore justly called by Astronomers the *Second Inequality*: And he is said to get

it away, who shews that Place of a Planet, which an Observer would see from the Sun. Let S (in *Fig. 22.*) represent the Sun, T the Earth, P the Planet in its Orbit, NB the Ecliptic among the Fix'd Stars, NA the intersection of the Orbit of the Planet with the Sphere of the Fix'd Stars; N the Node: And seeing the Sun is in the Plane of the Orbit of each Planet, the right line SN will be the Line of the Nodes. Let the Earth T be seen from the Sun among the Fix'd Stars at B , and the Planet P at A . And because the Planet is either in Opposition to the Sun, (as in *Fig. 1st*) or in Conjunction with it (as in the 2^d;) the arc of the great Circle connecting the points A and B , is the arc of the Circle of Latitude, and consequently perpendicular to the Ecliptic. And in the Spherical right-angled Triangle ABN there are given the angle ANB the measure of the Inclination of the Plane of the Orbit of the Planet to the Ecliptic, (found by the preced *Prop.*) and the side BN , namely the inclination of the right line TS given (by the Theory of the Earth) by position to SN the Line of the Nodes found by position (by *Prop. 19.*) Therefore there will be known the Side AB , the Centric Latitude of the Planet, and AN the Distance of the Planet in its Orbit, seen from the Sun, from the Node N , which is to get away the Second Inequality of the Planet. Moreover in the rectilineal Triangle PST there are given ST from the Theory of the Earth, and the angle PTS the Latitude of the Planet by observation, or its complement to a Semicircle, and the angle PST the Centric Latitude of a Planet found above; wherefore PS and PT the Distances of the Planet from the Sun and Earth will be known.



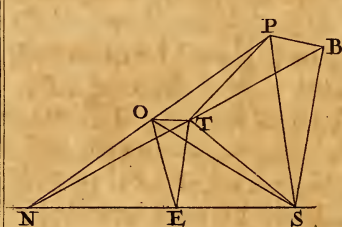


Fig. 23.
p. 427.

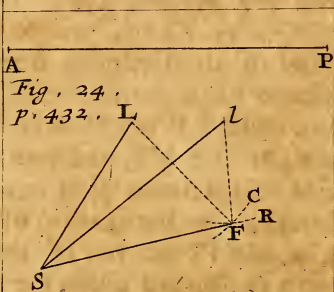
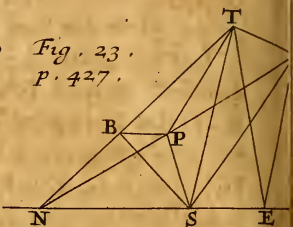


Fig. 24.
p. 432.

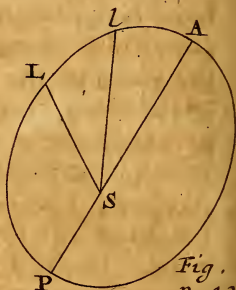


Fig. 25
p. 433.

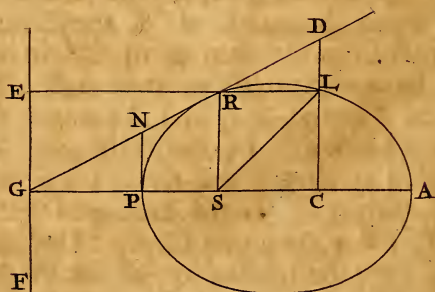


Fig. 26. p. 434.

PROPOSITION XXII.

B*Y any single observation of a Planet whatever, to clear it of its Second Inequality, and find its Distance from the Sun.*

Let the Planet P (in Fig. 23.) be observed, the Line of whose Nodes is NS , from the Earth T , with the apparent Latitude PTB . Let the Plane of the apparent Latitude be produced, till it cut the Plane of the Orbit of the Planet in PN , and the Plane of the Ecliptic in the right line BTN . From P let fall PB perpendicular to the right line NB , and from T erect TO perpendicular to the same NB , which (by Prop. 38. El. II.) will be perpendicular to the Plane of the Ecliptic, because the Plane of the Latitude PNB is perpendicular to the Plane of the Ecliptic. From T let fall TE perpendicular to the right line NS , and join OE , which is perpendicular to the same NS ; and the Angle OET will be equal to the Inclination of the Planes of the Orbit of the Planet and of the Ecliptic. In the Triangle NST , are given the Side ST , and angle TSN , by the Theory of the Earth and the known place of the Node, and by observation the angle NTS , the Elongation of the Planet from the Sun computed in the Ecliptic, or its complement to two right; therefore the Sides TN and NS , and the angle TNS will become known. In the Triangle TEN right angled at E , NT and TNE are given; and consequently TE . In the Triangle OTE right angled at T , there are given TE the Side found just now, and TEO the Inclination of the Planes of the Orbit of the Planet and of the Ecliptic, known by Prop. 20. of this Book; therefore OT will become known. In the Triangle OTN right angled at T , there are given OT and TN found before; and therefore the angle ONT also. In the first angle PNT ,

PNT , there are given NT , and the angle TNP , also PTN the apparent Latitude of the Planet known by observation, or its complement to two right; therefore NP will be known. In the Triangle NPB rightangled at B , the side NP and angle PNB lately found being given, PB and NB will be found. In the Triangle BNS , NB and NS and the angle BNS being given, NSB the Centric Longitude of the Planet computed from its Node, and the side SB will be known. Then in the Triangle PBS rightangled at B , PB , BS being known, PS the Distance of the Planet from the Sun will be known, and the angle PSB its Centric Latitude. Lastly, in the Triangle PNS all the sides being given, the angle NSP is known, the Centric Distance of the Planet in its Orbit, computed from the Line of the Nodes given by position (by *Prop. 19.*) in its Orbit. The Second Inequality of the Planet therefore is got rid of, and its Place seen from the Sun is determined: And besides the Distance of the Planet from the Sun is found in the parts of the mean Distance of the Earth from the Sun, which is taken for the measure.

PROPOSITION XXIII.

TO define the Periodic time of any Planet about the Sun.

Let the Planet proposed be observed in the Node (as was done in *Prop. 19.*) and again, when it returns next to the same Node: The Time between these observations is very nearly the Periodic Time of the Planet sought. For since the Lines of the Nodes are (by *Prop. 43. B. 1.*) almost at rest, when a Planet has described its whole Orbit, it returns to the same Node, & *vice versa*.

The Period of a Planet is determined not only by the observation of it twice in the same Node, but its Periodic time will become evident, if it be observed twice in any other point of its Orbit, cleared of the Second Inequality. But if the Centric Place of the Planet be computed from the Equinoxes, regard must be had to the Precession of the Equinoctial points made in the mean while. In defining the Periodic times of a Planet, it will be proper to make use of that advice given, in determinining exactly the Quantity of a Tropical Year; namely to take the most distant Observations that can be had, that the error may be insensible, the division being made by a great number of revolutions. And that an error of an intire revolution may not be committed, the knowledge of a Period near the truth, and expressed by a round number will suffice, which may be had, if in one revolution of a Planet two or more Centric Places be sought in Opposition or Conjunction with the Sun, (namely when it has no Second Inequality) and the Periodic time be taken in that ratio to the Time elapsed between any two Observations, as the distance between the centric Places is to an intire Circle. If the Planet should not happen to be observed in the Node itself, or in the Opposition with the Sun itself, (which generally happens;) then that moment is determined by observations made before and afterwards. If the period of a Planet be taken for that space of Time, wherein the Centric place of a Planet becomes the same again in the Mundane space or between the Fix'd Stars as it was before; tho' accurately speaking, there is no such Period at all, supposing the Nodes move; that is, that the planes of the Orbits themselves change; notwithstanding, if this Period be different from the Period defined above,

it

it may be defined (considered in regard of Longitude) by the help of the observations made by the preced. *Prop.* when the Centric Place of the Planet is near the same Fix'd Stars.

PROPOSITION XXIV.

TO find the length of the greater Axe of the Orbit of any Primary Planet.

We have hitherto defined the situation of the planes of the Orbits of the Planets to one another: We now come to describe the Orbits themselves in the planes thus defined. Which we shall do several ways, that this or that may be used according as the observations are, and the one may correct the other; and that the Orbits determined by all or several of these ways tending to one and the same thing, may be esteemed more accurate. And first we shall shew how their greater Axes may be determined. The greater Axe of the Orbit of any primary Planet, is to be taken to the greater Axe of the Orbit of the Earth in the sub-sesquuplicate ratio of the Periodic time of the proposed Planet, (found by the preced. *Prop.*) to the Periodic time of the Earth about the Sun, determined by *Prop.* 13. For then the Cubes of the greater Axes of the Orbit of the Planet and of the Earth, will be as the Squares of the Periodic times of those Bodies about the Sun, which has been demonstrated (in *Prop.* 40. B. 1.) to hold in Bodies revolving about the same Center, and acted upon by a Centripetal Force, which is reciprocally proportional to the square of the distance from that Center: And that this is the condition of the Primary Planets revolving about the Sun, has been shewn in *Prop.* 42. B. 1.

PROPOSITION XXV.

Three Centric Places of a Planet and its Periodic Time being given, to describe its Orbit, and find the moment of Time when the Planet was in the Aphelion.

After the same manner as in *Prop. 14.* three Places of the Earth seen from the Sun (that is, Centric places of the Earth) being given, together with the Periodic time, the Species of the Orbit of the Earth, and the Position of the Line of the Apfides is determined; the same may be done by the help of the same like *Data* in the Planet proposed, and the Orbit thus described, may be corrected by the method pointed at in *Prop. 15.* And as the Orbit of the Earth is described in the plane of the Ecliptic, determined before (in *Prop. 19. B. 2.*) so the plane of the Orbit of the Planet proposed, wherein its Orbit is to be described, is determined (by *Prop. 19* and *20.*) Moreover, the ratio of the greater Axes of the same Orbits is found from the Periodic times of the Earth and Planet (by *preced. Prop.*) The magnitude of the Orbit of the Planet will be had from the assumption of the greater Axe of the Earth's Orbit (or its half, the mean Distance of the Earth from the Sun:) The Orbit of the Planet therefore is wholly determined. And the Time when the Planet was in the Aphelion, is determin'd after the same manner, as the like was done *Prop. 14.* in the Earth.

Because the Species of the Elliptic Orbit of the Planet is only determined from these *Data*, (as is evident from *Prop. 14.*) and the magnitude of the Orbit in respect of the Orbit of the Earth is concluded by the *preced.* from the comparison of the Periodic times of these Planets; the same may be inferred other ways; namely, by making an
Orbit

Orbit of a certain Species (determined from three Centric Places) described about the Sun in a Focus, to pass thro' the very Place of the Planet in the line of the Nodes, found by the 19th Prop. of this: For this point given in the Orbit of the Planet, serves as well in the determining its Magnitude in respect of the Magnitude of the Orbit of the Earth, as its greater Axis defined above: For the distance of that point from the Sun is determined in the parts of the lines of the Earth's Orbit; and any one of these ways of describing the Orbit, may be examined and corrected by the other.

And from hence may the motion of the Ap-sides of any Primary Planet be defined; namely the same way as the motion of the Nodes was before.

PROPOSITION XXVI. LEMMA.

THE Focus and length of the greater Axe being given, to describe an Ellipse passing thro' two given Points.

Let *S* [in Fig. 24.] be the Focus of the Figure to be described, *AP* the length of the greater Axe, *L* and *l* the two points thro' which the Ellipse ought to pass. On the center *L*, at the distance equal to the excess of the right line *AP* above the right line *LS*, let a Circle *CF* be described; and after the like manner upon the Center *l*, with a distance equal to the excess of the right line *AP* above *lS*, describe a Circle *RF* cutting the former in *F*; and *F* will be one Focus of the Ellipse which is to be described: Consequently an Ellipse described upon the Foci *S* and *F*, having its greater Axe equal to the right line *AP*, will be the Ellipse sought. For since *LF* is equal to *AP*—*LS*, *LS* + *LF* will be equal to *AP*. After the same manner *AP* is equal to *lS* + *lF*: Consequently (by Prop. 52. B. 3. of Conics.)

Conics,) an Ellipse described upon the Foci S and F , having a greater Axe equal to the right line AP , will pass thro' the points L and l .

'Tis easy from such a Geometrical construction as this to invent a method of calculating suitable to Astronomy. For three points S , L and l being given, that is, the three sides of the Triangle SLl being known; the angles become known. Again in the Triangle LlF , all the sides are given; namely Ll as before, LF the difference of the given right lines AP and LS , and lF , the difference of AP , lS ; therefore the angle FLl becomes known: But SLL was known before, therefore the angle SLF will be given. Therefore in the Triangle SLF ; LS , LF , with the angle SLF being given, the side SF the distance of the Foci, and the angle LSF , between LS given by position and the greater axe, will be known; this axe therefore is given by position; and it is already given by magnitude: From whence all that belongs to the Ellipse may be drawn.

PROPOSITION XXVII.

Given two Centric Places of a Planet and the Distances from the Sun, together with its Periodic Time, to determine the Orbit of the Planet.

Let S be the Sun [Fig. 25.] let the two Centric places of the Planet given be in the right lines SL , Sl given by position, and let the Distances of the Planet from the Sun be the right lines SL , Sl given in magnitude. Wherefore L and l are the points thro' which the Ellipse is to pass. Moreover, the periodic Time being known (by Prop. 24.) PA the greater axe of the Elliptic Orbit is determined. Describe an Ellipse therefore (by the preced.) whose Focus is S , having its greater axe of a given length, that may pass thro' the given points L and l . The Centric places of the

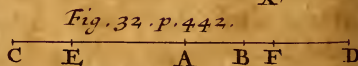
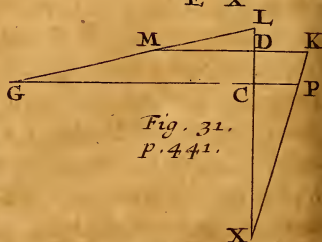
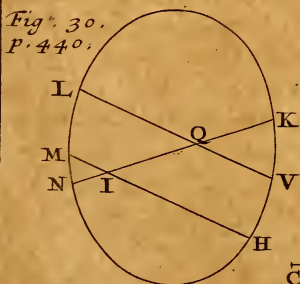
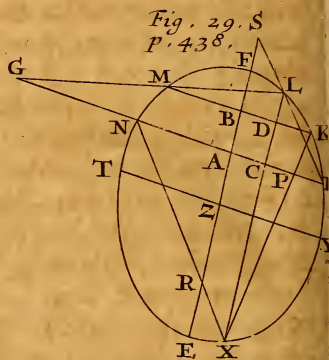
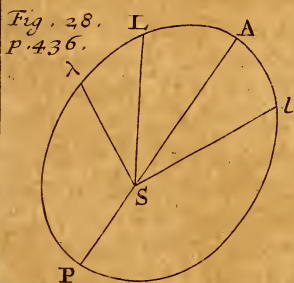
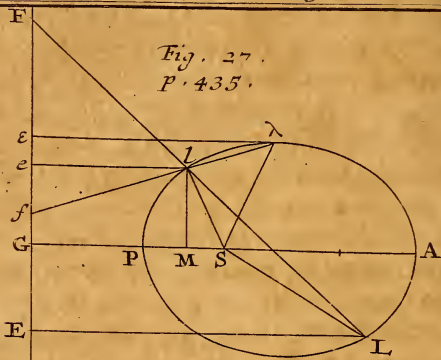
Planet and its Distances from the Sun, either in Opposition or Conjunction with the Sun are determined as in *Prop. 21*; or from an Observation made any where, as in *Prop. 22*. Nevertheless that may be assum'd, which is determined *Prop. 19*. when the situation of the Planet's Orbit in regard of the Ecliptic was defined. But if you have more than two Centric places with their corresponding Distances from the Sun, then their Orbit being defined by any two, you must see whether they agree to it thus defined, and that is to be retained (if they disagree) which is confirm'd by most Observations, as is usually done in most Astronomical Calculations.

PROPOSITION XXVIII. LEMMA.

TO the greater Axe *AP* of the Ellipse *ALP* [*Fig. 26.*] from the Focus *S* let a perpendicular *SR* be erected meeting the Ellipse in *R*, thro' which let a right line *DR* be drawn touching the Ellipse, and meeting with the Axe *AP* produced in *G*, and from *G* let *GF* be drawn perpendicular to the Axe. These things being suppos'd, if from any point *L* of the Ellipse two right lines *LS*, *LE* be drawn, one as *LS* to the Focus assum'd *S*, the other *LE* perpendicular to *FG*; I say, that *LE* is to *LS* in the given ratio of *GP* to *GS*.

Thro' the point *L* to the Axe *AP* draw the perpendicular *LC* meeting the Axe in *C* and the Tangent *DR* in *D*, and from *P* draw another line *PN* parallel to it: And *CD* will be equal to the right line *SL* (by *Prop. 139. B. 4. of Gregory St. Vincent*) and *PN* to *PS*. But *GC* is to *CD* in a given ratio, namely of *GP* to *PN*; and therefore *EL* is to *LS* in the given ratio of *GP* to *PN* or *GP* to *PS*. From hence it follows that *GA* is to *AS* in the same ratio of *GP* to *PS*; which is a particular Case of this Proposition, namely when the point *L* taken at pleasure coincides with

Plate 7. Book 3.



with A . And this is evident from hence, that (by Prop. 34. B. I. of Conics.) RG being a Tangent, AS is to SP as AG to GP ; from whence, by permutation and inversion, GA is to AS as GP to PS . This Proposition holds also in the other Sections of the Cone.

PROPOSITION XXIX. LEMMA.

TO describe an Ellipse about a given Focus, that shall pass thro' three given Points.

Let the Points given be L, l, λ , (in Fig. 27.) and the Focus S . Join the right line Ll , and produce it to F , so as that LF may be to lF , as SL to Sl . So let λl joined, be produced to f , so as that λf may be to lf , as $S\lambda$ to Sl . Join the right line Ff , and upon it, from L and S let fall the perpendiculars LE, SG , divide SG in P so as GP may be to PS as LE to LS , and produce it to A , so as that GA may be to AS in the same ratio: An Ellipse described having A and P for its Vertices and S for its Focus; that is, having its greater Axe PA , and its *Latus rectum* quadruple a right line which is to SP as SA to AP (by Prop. 54. B. I. of Conics.) will pass thro' the Points L, l , and λ .

From l and λ , upon FG let fall the perpendiculars $le, \lambda e$. Since by construction LF is to lF as SL to Sl ; LE will also be to le (by reason of the Parallels LE, le) as SL to Sl , or LE to SL as le to Sl . But by construction also GP is to PS , and GA to AS , as LE to SL ; and therefore as GP is to PS so is le to Sl . After the same manner λf is by Construction to lf (that is, λe to le) as $S\lambda$ to Sl ; and therefore λe to $S\lambda$ as (le to Sl ; that is, as was just now demonstrated, as) LE to SL . Since therefore SL is to LE , also Sl to le , and $S\lambda$ to λe , as GP to PS , or GA to AS , and this is a property of an Ellipse demonstrated in the preced Lem. 'tis evident that the Points L, l, λ ,

are in the Ellipse, whose Focus is S , and extremes of the greater Axe or Vertices, A and P . And from hence it is easy to be known, when the Problem is impossible, and when instead of an Ellipse, a Parabola or Hyperbola comes forth. If a Calculation familiar to Astronomers be requir'd, it may easily be drawn from the preceding Construction; for instance, after the following manner. Since $S\lambda$ is to Sl , as λf is to lf ; then dividedly $S\lambda - Sl$ to Sl as λl to lf : But the three first terms of the Proportion are given, from the Points S , l , λ , being given; therefore the fourth lf will be known. And after the like manner, because $SL : Sl :: LF : lf$. By dividing $SL - Sl : Sl :: Ll : lf$, and the three first terms being given the fourth term will be known; namely the right line lf . Moreover, the three Sides of the Triangle $Sl\lambda$ being given, the angle $Sl\lambda$ is given, and the three Sides of the Triangle SLl being known, SlL becomes known; therefore there is given both the Angle $Ll\lambda$ the difference or sum of them, and the Vertical to it Flf . Therefore in the Triangle Flf , the Sides lf , lf , just found, being given, together with the angle Flf contained; the angle Ffl will be found, and consequently its complement to a right fle . But the angle $Sl\lambda$ being known before, its complement to two right Slf will be known; therefore the sum or difference els of the known Angles Slf , elf , will be given, and consequently lSG its complement (because of the Parallels el , GS) to two right; that is, the inclination of the greater axe to the right line lS , given by position. Besides, in the Triangle fel rightangled at e , all the angles are given, together with the side fl ; therefore el , or GM is known, supposing lM perpendicular to GS . In like manner in the right angled Triangle SMl , the Angles and Side Sl being

being given, SM is given; and GS is the sum or difference of GM , MS . Again, because by the preced. Lem. $el : lS :: GP : PS$, by compounding $el + lS : lS :: GS : SP$, and the three first terms of this Proportion being given, the fourth SP will be known: In like manner, because $el : lS :: GA : AS$, by dividing $el - lS : lS :: GS : AS$; from whence AS becomes known: The sum of PS , SA is the greater Axe. The other Focus of the Ellipse is as far distant from A , as S is from P . The Foci and Vertices A and P being given, the rest will not be unknown.

PROPOSITION XXX.

Three Centric Places of a Planet, and the Distances from the Sun being given, to describe its Elliptic Orbit.

Let the Centric Places of the Planet be given, and its corresponding Distances from the Sun; namely the right lines SL , Sl , $S\lambda$, in position and magnitude [Fig. 28.] supposing S to be the Sun. By the prec. Prop. let an Ellipse, whose Focus is S , be described or determined by Calculation, passing thro' the Points L, l, λ . This will be the Orbit sought. The relation this has to the Orbit of the Earth is therefore known, because the magnitude of the Lines SL , Sl , $S\lambda$ is given, in the parts of the mean Distance from the Sun, and the situation of the Plane of the Orbit in respect to the Plane of the Ecliptic, is supposed to be determined by Prop. 19. and 20.

Upon this Proposition depends the Method that Dr. Halley lays down, in the *Philosop. Transact. An. 1676. N° 128.* for determining the Orbits of the Planets. Namely, if any Planet be observed thrice from the Earth in the same Point of its Orbit (which, when it may happen, is well enough known from its periodic Time nearly known)

there will be had three Places of the Earth, seen from the Sun, and three corresponding Distances of the Earth from the Sun, expressed in the parts of the distance of the Planet from the Sun; and therefore the Orbit of the Earth itself is determined by the preceding : Which being known, if any Planet be observ'd twice from the Earth in the same Point of its Orbit, and consequently (by *Prop. 19.*) its Centric Place and Distance from the Sun be determined, and this be repeated in two other Places of the same Planet; three Points of the Planet's Orbit will be had, by which together with one of the Foci (namely the Sun's Place) being given, the Orbit it self (by the prec.) is determined. If the Periodic Time be pretty exactly known, the same may be done by two Centric Places and Distances from the Sun being known, which may be gather'd by *Prop. 27.* from twice two Observations of a Planet observed in the same two points whatever they be: And by the same pains the Place of the Nodes and the Inclination of the Orbit to the Ecliptic are determined by the 19th and 20th Propositions.

PROPOSITION XXXI. LEMMA.

TO find the conjugate Diameters of an Ellipse, which passes thro' five given Points. [Fig. 29.]

Let the five given Points be *N, M, L, K, H*, that the Perimeter of the Ellipse passes thro'; a Diameter as *FE* of that Ellipse is to be found, together with its Conjugate *TY*, in magnitude and position. Let right lines be supposed to be drawn connecting the given Points, some two of which are either parallel, or there are no two such. This is easily seen; for since the Points *N, M, L, K, H* are given, the right lines that connect them all manner of ways are given, and the angles contained between those right line

If

If therefore it be found that two angles placed alternately, (for instance KMH , MHN) are equal, it may be concluded that the two right lines MK , NH , in which the said angles are alternately placed, are parallel.

CASE I. Suppose first, that there are two of them, as MK , NH parallel to one another. Let them be bisected in B and A , and let BA connecting them, be produced, till it meet the Ellipse in F and E ; and it will be (*by the Elements of Conics*) a Diameter of the Ellipse, and (by 26. *Dat.*) given by position; because passing thro' the given points B and A . Draw thro' L , the Line LX , parallel to EF , meeting the Ellipse in X , the right line MK in D , and NH in C . And let the right lines XK , LM connected, meet the right line HN (if need be produced) in the points P and G . By Prop. 17. B. 3. *Elem. Conic of Apollonius*, the rectangle $XD.L$, is to the rectangle MDK as ZFq to ZRq . And $XCL:NCH::ZFq:ZRq$. An therefore $XD.L:MDK::XCL:NCH$. But (as shall be shown hereafter,) $XD.L.MDK::XCL.GCP$. And therefore (by Prop. 9. *Elem.* 5.) the rectangle NCH is equal to the rectangle GCP . But the rectangle NCH is given, because its sides NC , CH are given; for the whole line NH is given at first as well in magnitude as position, also the right line LC by position (by 28. *Dat.*) wherefore (by 25. *Dat.*) the point C will be given: So likewise will the right line GC , since its extremes G and C are given: Therefore (by 57. *Dat.*) CP is given in magnitude: But it is also by position; therefore the point P itself is given. And the point K is given; therefore the right line KP connecting those points is also given by position, and consequently (by 25. *Dat.*) the point X in the Ellipse, where the lines LC , KD given by position meet)

is also given. Join the right lines NX , LH meeting the diameter EF (if need be produced) in R and S : And then (by the above cited *Prop. 17. B. 3. Elem. of Conic.*) both NCH will be to $XC L$, and NAH to EAF , as ZYq to ZFq ; from whence $NCH. XC L :: NAH. EAF$. And by what is below demonstrated, $NCH. XC L :: NAH. RAS$; therefore (by *Prop. 9. Elem. 5.*) the rectangle RAS is equal to the rectangle EAF . But RAS is given (since the three points A , R , and S are given, A as was shewn above, R and S by the 25 of the *Data*, namely the concurrence of the right line EF given by position with the right lines XN , HL given by position;) therefore the rectangle EAF will be given. After the like manner the rectangle EBF may be found; and the points A and B are given, therefore so is EF in magnitude; as we shall show hereafter. But its position was found before; wherefore the diameter EF is given in Position and Magnitude. Moreover TYq is to EFq as NCH to $XC L$, as was shewn above; that is, (by 1 *Dat.*) in a given ratio, because the rectangles NCH , $XC L$ are given; wherefore the subduplicate ratio of the former, namely the ratio of TY to EF (by 24 *Dat.*) is also given: But EF is given, as was shewn above; therefore by 2 *Dat.* TY will be given in magnitude. But its position is given by 28 *Dat.* because passing thro' the point Z , which is in the middle of the right line EF given in position and magnitude, parallel to the right line NH given in position. Therefore the right lines EF , TY are given in Position and Magnitude, namely the conjugate diameters of the Ellipse, passing thro' the five given points N , M , L , K , H .

CASE II. But now let us suppose that the right lines MH , NK , [*Ag. 30.*] are not parallel,

lel, nor any other two: Therefore these lines MH , NK (at least produced) will intersect one another; for instance in the point L , which will be given by 25 *Dat.* draw thro' L the right line LV parallel to MH , meeting the right line NK in Q , which (by 28 and 25 *Dat.*) will also be given. And the ratio of the rectangle NQK to the rectangle LQV will be given; namely the same (by *Prop.* 17. *B.* 3. of the *Elem. of Con.*) with that of the given rectangle NIK to the given MIH . But the rectangle NQK is given, since its sides NQ and KQ are given; and therefore the rectangle LQV is also given: From whence, since LQ is given by 57 *Dat.* the right line QV is given in magnitude; but also in position, because a parallel thro' the given point Q to the right line MH given by position; therefore the point V itself by 27 *Dat.* since the magnitude and position of the right line QV and the other extremity Q are given. Thus therefore we come back to the former Case; for there are given the five points N, M, L, V, H , or M, L, K, V, H , thro' which the Ellipse may pass, and yet such as the two right lines LV, MH may be parallel. By what has been said therefore the conjugate diameters of that Ellipse may be found. $Q. E. F.$

Let there be two right lines GP, MK , parallel to one another, on which let the right line $XC DL$ [Fig. 31.] fall, and from any points of it X and L draw any how the right lines LMG, XPK meeting the parallels in M, G, P and K : I say, the rectangle $XD L$, is to $MD K$ as $XC L$ to the rectangle GCP .

The ratio of the rectangle $XD L$ to the rectangle $MD K$ (by *Prop.* 23. *Elem.* 6.) is compounded of the ratio of XD to DK , and of the ratio of LD to DM ; and the ratio of the rectangle $XC L$ to the rectangle GCP of the ratios of XC to CP and LC to CG . But because of the parallels

parallels MDK , GCP , XD is to $DK :: XC. CP$; and $LD. DM :: LC. CG$. Since therefore the compounding ratios are equal, the compounded ratios will also be equal; that is, $XD L$ to MDK , as $XC L$ to GCP .

If both the rectangles EAF , EBF are given, and the points A and B are given; the points E and F are also given, supposing $EABF$ a rightline.

For suppose the rectangle BAC to be equal to the rectangle EAF , and the rectangle ABD equal to the rectangle EBF : And therefore, since (by *Hyp.*) EAF and BA are given, AC (by 57 *Dat.*) is also given. In like manner, since EBF and AB are given, AD will be given. By reason of the former equality (by 16. *Elem.* 6.) $FA. AB :: AC. AE$, and by conversion of ratios, $FA. BF :: AC. CE$. Again, by reason of the latter equality $EB. BD :: AB. BF$, [Fig. 32.] and by compounding $ED: BD :: FA. BF$. From whence (by *Prop.* 11. *El.* 5.) $AC. CE :: ED. BD$: And therefore the rectangle contained under AC and BD is equal to the rectangle under CE , ED . But the rectangle under AC and BD is given, (for both right lines are given) therefore the rectangle under CE , ED is also given. And because this given rectangle is applied to the given rightline CD , (composed of the parts CA and AD which are given) and wants of a square Figure, CE (by 58 *Dat.*) will be given; and consequently AE (by 4 *Dat.*) and because the rectangle EAF was given at first, the right line AF will also be given; that is, the points E and F will be given.

PROPOSITION XXXII. LEMMA.

TO find the Foci and Vertices of an Ellipse that is to pass thro' five given Points, and consequently to describe the Ellipse passing thro' the five given Points.

Pappus in Book 8. *Math. Coll.* proposes and constructs

fructs this Prob. and the prec. for the use of Architects : Namely, having a portion of the superficies of a right Cylinder given, that has no part preserved intire in the circumferences of the Bases, to find the thickness of the Cylinder. These the Astronomers afterwards converted to their use. By the preced. *Lemma* find any two conjugate Diameters of the Ellipse, that is to pass thro' the five given Points, and let them be AB , CD bisecting one another in the Center E . Produce BA to H [Fig. 33.] so as that the rectangle EAH may be equal to the square of DE ; by which means (by 58 *Dat.*) EH is given. Let a perpendicular KL , from K the middle of EH , meet with MAG ; a right line parallel to DC , thro' the point A in the point L , which consequently is given on the account of the right lines MG , KL given by position. On the Center L thro' E or H , draw a Circle intersecting the right line MAG in M and G , whose diameter MG consequently is given as well in position (because 'tis parallel thro' the given point A to the right line CD given by position,) as in magnitude, (since it is double to the right line connecting the given points E and L) the right lines GE , ME connecting them, are also given as well in position as magnitude. But their position is the same with the position of the Axes sought : For since the rectangle EAH (by 35. *Elem.* 3.) is equal to the rectangle MAG , and by construction EAH equal to the square of DE ; ED^2 will be equal to MAG , or (by 17 *Prop. Elem.* 6.) ED will be a mean proportional between MA and AG ; and consequently (by *Prop.* 88. *B.* 3. of *Greg. St. Vincent.*) the Axes of the Ellipse described upon the conjugate Diameters AB , CD are in the right lines EG , EM . And the magnitude of the Axes is thus expeditiously determined. Thro' A , let fall perpendiculars AQ , AN to EG , EM , which therefore are given.

ven. Take in the right line EG , EO a mean proportional between EG , EQ ; in like manner, in EM , ER a mean proportional between EM , EN ; the right lines EO , ER (given by 24 *Dat.*) will be the Semi-axes of the Ellipse (by *Prop. 37. B. I. Conic.*) If in GE produced you take EP equal to EO , and in ME produced, ET equal to ER , the Axes OP , RT of the Ellipse will be given in position and magnitude, together with the points Q , R , P , T , which are the principal Vertices of the same. Moreover, if the greater Axe PO be so divided in S and F , that the rectangle under PS and SO , as also the rectangle under PF and FO be equal to the Square made upon RE , S and F will be the Foci of the Ellipse (by *Prop. 52. B. 3. Elem. Conic.*) But these Points thus found (by 58 *Dat.*) are given: and if the Axes and Foci of the Ellipse are given, the Ellipse may be described by *Prop. 54. B. I. Conics of Apollonius.*

There is no need of annexing here the form of a Trigonometrical Calculation suited to this Problem: For it is well known to Geometers, that whatever is found out by the help of *Euclid's Data*, may also easily be found by a Calculation, proceeding exactly after the same manner, and generally in a Trigonometrical manner, as it happens here: Indeed the investigation of any Problem by the help of *Euclid's Data*, is an universal method of determining it, and very usual with the Ancients.

PROPOSITION XXXIII.

Five Centric places of a Planet and the Distances from the Sun being given, to find its Orbit.

In the preceding Propositions, while we were determining the Orbits of the Planets, we supposed nothing, but what was agreeable to Observations exactly made as well as to the Physical Causes of their



Plate 8. Book 3.

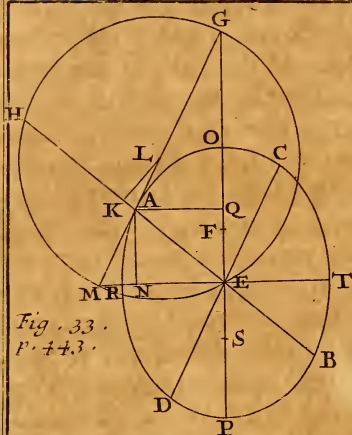


Fig. 33.
P. 443.

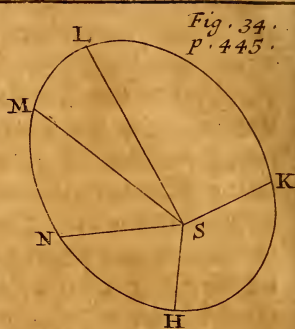


Fig. 34.
P. 445.

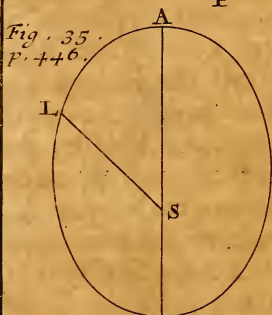


Fig. 35.
P. 446.

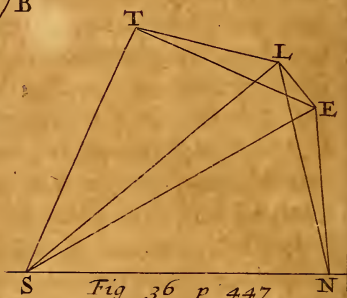


Fig. 36 p. 447

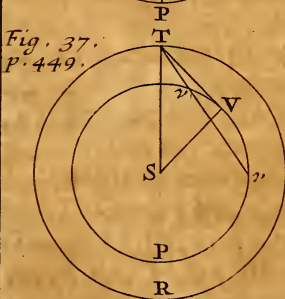


Fig. 37.
P. 449.

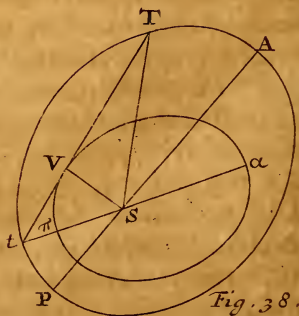


Fig. 38.
P. 450.

their Motions demonstrated in *Book 1*: Namely, that the Orbit of any Planet is an Ellipse, one of whose Foci is the Sun at rest, and that a Planet describes it after such a manner, as that its Area's or Sectors contained between right lines drawn to the Sun, and the Arcs of the Ellipse are proportional to the Times wherein the said Arcs are described by the Planet. For tho' we have supposed in *Prop. 14.* with *Ward*, for a little while, that a Planet describes its Orbit so, as that its angular motion at the Focus of the Ellipse in which the Sun is not; yet this was only done, because in the Orbit of the Earth, which is not very excentric, an approximation is made to the equal description of Areas by a radius drawn to the Sun, as is not to be disregarded, as was shown in *Prop. 7.* And we corrected the Orbit of the Earth defined according to that Hypothesis, as much as we pleased in the following 15th Proposition. But in this Proposition we shall not so much as suppose that the Sun is placed in the Focus of each Orbit; but shall describe the Orbit by the help of five given Centric Places, and as many Distances from the Sun, independently upon it: And by this means the truth of the preceding *Prop.* will appear clearer, since the Sun will be found to be placed in the common Focus of the Orbits thus described.

The Centric places of the Planet being given, the right lines, wherein it is seen from the Sun, are also given by position: Let them be SN, SM, SL, SK and SH ; [*Fig. 34.*] and the corresponding Distances from the Sun being given, the points N, M, L, K and H (by 27 *Dat.*) are given. Thro' these points (by the help of the preceding Lemma) describe an Ellipse, and it will be the Orbit of the Planet sought. And by the same Lemma the Vertices will be found, as also the Foci, Position
of

of the greater axe and the other things that are required in the Orbit of the Planet. $\mathcal{Q}$. *E. F.*

PROPOSITION XXXIV.

TO determine the Centric Place and Distance from the Sun, of any Planet proposed, at any given Time.

The plane of the Planets Orbit, the position of the Line of the Apfides and the Species of the Planet's Orbit being determined, (by the preced. Prop.) that is, the Orbit it self being described in its proper Situation about the Sun, suppose ALP [Fig. 25.] and the Moment being known (by Prop. 25.) when the Planet was some time or other in the Apfis also the Periodic Time of the Planet (by Prop. 23.) the Time since it was last in the Apfis, (*v. g.* A) will be known, or the ratio of the said Time to the whole Periodic Time will be known; that is, by the Law of Motion of the Planets, demonstrated in *B. I.* supposing the Place of the Planet L , the ratio of the area ALS to the whole Ellipse $ALPA$ is known. Find therefore the Angle ASL by Prop. 3, 4, 6, or 7. Therefore the right line SL is given by Position, since it is inclined in a given angle to PA the Line of the Apfides given in position: As also in magnitude, by the same Propositions, (since this is given in the parts of the Aphelion distance of the Sun SA , which is defined above in the parts of the Mean distances of the Earth from the Sun, which is looked upon as the common measure of these lines;) that is, the Place of the Planet proposed seen from the Sun, and its Distance from the same.

And *vice versa* the Time may be found when a Planet shall be in a given Heliocentric Place; by finding the mean Anomaly (by the preced. Prop.) from the coequated Anomaly given, to the former of which the Time is proportional.

PRO-

PROPOSITION XXXV.

A *T* a given Time to find the Geocentric Place of a Planet, as to Longitude and Latitude, and the Distance of the Planet from the Earth.

Find (by Prop. 18.) the Heliocentric Place of the Earth in its proper Orbit, at the Time given, and (by the preceding) the Heliocentric Place also of the Planet proposed in its Orbit, with the respective Distances from the Sun.

Let *S* (in Fig. 36.) represent the Sun, *T* the place of the Earth in its Orbit, *L* the place of the Planet in its Orbit. Let the right line *SN* be the common intersection of the Planes of these Orbits, which is extended thro' the Sun, (because the Sun is in both Planes) and whose position is given (by Prop. 19.) Besides, the place of the Earth *T* being given, the right line *ST* and the angle *TSN* in the Plane of the Ecliptic, are given; and the Place *L* of the Planet being given, the right line *SL* and the angle *LSN*, in the plane of the Orbit of the Planet are given. Draw thro' *L* and *S*, and through *L* and *T*, the Planes of the Circles of Latitude perpendicular to the plane of the Ecliptic, whose common intersection the right line *LE* (by Prop. 19. Elem. 11.) is perpendicular to the plane of the Ecliptic. Draw in the plane of the Planet's Orbit the right line *LN*, perpendicular to *SN*, and the line *EN* (by the converse of Prop. 11. Elem. 11.) will also be perpendicular to the same *SN*.

In the Triangle *SLN* rightangled at *N*, *SL* and *NSL* are given; from whence the Side *LN* will be given. In the Triangle *LEN* rightangled at *E*, there are given the Side *LN* lately found, and the angle *LNE*, which (by Def. 6. Elem. 11) is equal to the Inclination of the Orbit of the Planet to the Ecliptic, found by
Prop.

Prop. 20. LE and EN are therefore given. Besides in the Triangle LES , rightangled at E , the Sides LE and LS are given; therefore the Side SE , the *Curtate Distance of the Planet from the Sun* is given. In the Triangle ESN , besides the rightangle at N , there are given ES , and EN found before; therefore the angle ESN will be given, the Heliocentric Longitude of the Planet computed in the Ecliptic from the Node N : But the Place of the Node in the Ecliptic is given; wherefore the Heliocentric Place of the Planet in the Ecliptic will also be known. Again, in the Triangle TSE , TS is given at first, and SE was found just now; the angle TSE is also given (equal to the sum or difference of the given angles TSN , ESN , because the three right lines ST , SE , SN , are in the Plane of the Ecliptic, which is also called the *Angle of Commutation*: And therefore all the rest is given, namely, the angle TES , call'd, *The Parallax of the Orbit*, and STE the *Elongation of the Planet from the Sun*; that is, the position of the right line TE , or the Geocentric Place of the Planet in the Ecliptic, and TE the Magnitude of the same. Lastly, in the Triangle TEL , right-angled at E , the Sides TE and EL are given; therefore TL the Distance of the Planet from the Earth, and the angle LTE the Geocentric Latitude of the Planet will be known.

SECTION IV.

Of the greatest Elongation of the Planets from the Sun, their Direction, Station, and Retrogradation.

PROPOSITION XXXVI.

TO determine the greatest Elongation from the Sun of a given Planet, seen from a superior one also given, and its Time.

Let S be the Sun, TR the Orbit of a more remote Planet, [Fig. 37.] VP of a nearer. Let the radius SV of the nearer Planet be drawn any how, and perpendicular to it VT , meeting the more distant Orbit in T , which consequently (by 16. El. 3.) will touch the Circular Orbit VP in V . The inferior Planet will be seen by the superior in T , to be at its greatest Elongation from the Sun in V : For a right line drawn from any other point (as v) of the Orbit PV to T , will contain a lesser Angle with TS . Therefore in the Triangle TVS , rightangled at V , the ratio of the Sides ST , SV (namely the Distances of the given Planets from the Sun) is given; therefore the other angles TSV , STV are known, the latter of which is the greatest Elongation of the Planet from the Sun, the other TSV is the angle of Commutation or Heliocentric Distance of the Planets. Whenever therefore the angle of Commutation, found just now, happens, it at the same time happens that the inferior Planet, seen from the superior, is at its greatest Elongation from the Sun.

We have hitherto supposed the Orbit of both Planets to be Circular; but if due regard be had

to their Elliptic Figure, the Problem is solvable much after the same manner. For the angle SVT is given, that the right line VT [Fig. 38.] touching the Orbit, makes with the radius SV , inclined in an assumed Angle to the Line of the Apfides; besides SV is given in magnitude, since its Inclination to the Distance of the Aphelion $S\alpha$ already given in magnitude is known; and because the angles $AS\alpha$, αSV are known, also the angle VST , (very nearly equal to that, which would be were both Orbits circular;) the angle AST will also be known, and consequently ST will be known in magnitude; from whence what remains is done as before. In this case the greatest Elongation changes according to the different Distance of the point V (where it is celebrated) from the Aphelion of its proper Orbit: For it is greater (*cæteris paribus*) in α the Aphelion of the inferior, less in π the Perihelion, a mean in the mean Longitude; it is also various, the place V of the inferior remaining the same, according as the superior is situated in T or t .

PROPOSITION XXXVII.

ANY Primary Planet seen from any other Primary, appears in some part of its Orbit, Stationary, in another Direct, and lastly in another Retrograde.

Let S be the Sun, [Fig. 39.] about which two Planets revolve from D thro' C towards B , the Orbit of the nearer PT ; of the more distant RZ ; the arcs described by them in equal spaces of time AR , PB . Draw SR any how meeting the nearer Orbit in P . Between SR and SP let SM be a Geometric Mean. Then (by Prop. 27. B. 1.) the Velocity of the more distant Planet is to the Velocity of the nearer, as SP to SM : But the Spaces described by Bodies in Motion, in equal

equal Times are as their Velocities; and therefore AR is to PB , as SP to SM ; wherefore AR is less than PB . And it is plain that it is possible for two parallel right lines $HADb$, $GRCg$ to be drawn thro' A and R , so as that the arc of the nearer Orbit intercepted by them, is equal to the arc PB . For parallel right lines may be so drawn, that as much as CD (equal to PB) is greater than AR , so much it may be more oblique, (and the arc of the interior Orbit is given, which is as oblique to the parallels thro' A and R as you please,) that the arc CD compensating the greater length by its greater obliquity, may touch both of the parallel right lines in their extremities. Which being supposed, if the Planets be found in such a situation in regard of one another as A and D are here; that is, if the angle of Commutation at the Sun be equal to the angle ASD , (which, it is evident, it will be twice necessarily between the two next Conjunctions of the Planets proposed seen from the Sun,) the one seen from the other all that space of time, wou'd be refer'd to the same Fix'd Stars; that is, it would appear Stationary. For the superior at A is seen from the inferior at D , in the right line DAH , and while the superior is going to R , the inferior is going to C , (since by supposition DC is to AR as the Velocity of the Planet, which is nearer to the Sun, is to the Velocity of the more distant:) That therefore seen from this, in the right line CR will be referred to the same place among the Fix'd Stars, to which it was before referred, since the distance between the Parallels DA , CR vanishes in comparison with the distance of the Fix'd Stars. And, for the same reason, the inferior Planet describing the arc DC viewed from the superior, while it describes the arc AR in the same

time, will also appear Stationary. In like manner, if the Planet next the Sun be on the other side of the point P , (in which being viewed from the Sun, it seems in conjunction with the more distant Planet ; that is, in which the Sun and superior Planet viewed from the inferior, are seen in opposition, and where the inferior seen from the superior appears in conjunction with the Sun,) and as far distant from it, namely where the visual rays of any sensible time are again parallel to one another, for instance, at cd , the one seen from the other will again appear Stationary.

I say further, that before the Planet, which is nearer to the Sun, arrives at the beforemention'd situation in respect of the superior, that is, the superior being in R , before the inferior arrive at D ; the one seen from the other will appear to move *in consequentia*. For let the arc DF be taken equal to PB or DC , and join the right lines FA , DR , and produce them both ways in KAF and IRD ; they will not be parallel, but diverge towards the parts beyond RA (in respect of the Sun, but on this side they will converge and concur, for instance, in the point L , from whence they will again diverge. For since (by supposition) DA , CR are parallel, and DF equal to CD , but more oblique to RD than CD is to RC ; 'tis evident that DR , FA diverge on the other side of RA , and converge on this side of RA ; viz. either behind DF , if it be in that part of the nearer Orbit that is turned towards R (as in the case of this Figure) or between RA and DF , if in that part which is turned from it. If now the inferior being in F , the superior be in A , the latter viewed from the former will be seen in K : But the inferior being arrived at D , the superior is got to R (by supposition) and is seen from it at I . But

But to come from the situation of K to the situation of I is to move *in consequentia*, or to be Direct; for any of the Planets seen from the Sun, moves *in consequentia* from A to R , or from D to C . Nay, even the inferior seen from the superior at the same time seems Direct: For the inferior in F will be seen from the superior A at k ; and afterwards the inferior at D will be seen from the superior in R at i : But to change place from k to i , is also to tend *in consequentia*; for the Sun seen from any Planet seems to move *in consequentia*; namely from S towards $\mathcal{Q}$. In like manner, after the Planet, which is nearer to the Sun, has passed the second Station at d , it becomes direct again, and continues so till the inferior arrive again to a situation in respect of the superior, (as is D in regard of A ;) which it will do after it has compleated an intire revolution from the former place D , and so much of another besides, as is requisite for it to recover the said situation again, in regard of the more distant Planet, that had in the mean time gone forwards.

I say thirdly, that the one will appear Retrograde to the other, after that the Planet that is nearer to the Sun has got beyond the above described boundary DC of the first Station, and has not arrived at the second cd . Let CE be taken equal to PB , or DC ; draw the right lines CA , ER , and let them be produced both ways into $OCAr$, and $NErYn$, which are not parallel, but diverge towards the parts CE , and converge towards AR , and being produced meet, for instance in r , because by supposition DA , CR are parallel, and CE is equal to DC , but less oblique towards RE than CD is towards RC . If now while the superior is in A , the inferior be in C , this will be seen from that in O , and that *vice versa* will be

seen from this in o : But when the superior is got to R , and the inferior is found at E , this will be seen from that at N , but that will be seen from this at n . But 'tis evident, that to seem to move from O to N , or from o to n , is to move contrary to the order of the Signs; since from S to $\mathcal{Q}$, or from A to R is according to the order of the Signs; namely that wherein the Primary seen from the Sun, or *vice versa*, the Sun seen from the Primary seems to tend. And this apparent Retrogradation of the Superior seen from the Inferior, or of the Inferior from the Superior, continues from the celebration of the first station at DC , to the second at cd : But by reason of the motion of the more distant Planet, this second station is removed a little *in consequentia*, or beyond the place cd here expressed. 'Tis therefore evident, that any Primary seen from another Primary in some part of its Orbit is Direct, and in some Retrograde; and lastly, in some Stationary, *Q.E.D.*

COROLLARY 1.

'Tis evident from the demonstration of this Proposition, that a superior Planet seen from an inferior appears at the same time Direct, Stationary, and Retrograde, that the inferior seen from that superior does.

COROLLARY 2.

The more unequal the ratio between the Orbits of two Planets is, the nearer its place in the Orbit of the inferior, in which the inferior is found while the one seen from the other appears Stationary, is to that point of it, from which a right line drawn to the more distant Planet, touches the Orbit of the inferior; that is, where the inferior seen from the superior (by *Prop. 26.*) appears in its greatest Elongation from the Sun; and the angle greater wherein the Sun and superior

rior

rior Planet, seen from the inferior, seem to be distant from one another. For the greater the ratio of SR is to SP , the greater will the subduplicate of the latter, namely the ratio of RS be to MS ; that is, (by construction) the ratio of CD to RA . And therefore the line CD must be more oblique to RC or AD , that DA , RC may become parallel; but it is evident that CD is so much the more oblique as it is nearer to the point of the inferior Orbit, from whence a right line drawn to R touches it.

From hence it is, that to make Saturn Stationary there is need of so small a recession of the Earth from the point of its Orbit, where a right line drawn to Saturn touches it, towards the point where the Earth comes between the Sun and Saturn; to make Jupiter Stationary, a greater recession from the point of contact of the Orbit of the Earth, with a right line touching it drawn to Jupiter; to make Mars the greatest. Therefore Saturn is Stationary, when seen from the Earth it is distant a little more than a Quadrant from the Sun; that is, between the Quartile and Trine Aspect; Jupiter will appear Stationary almost in the Trine of the Sun; Mars beyond the Trine.

Hence it is also that the arc of the Orbit of Mercury intercepted between the point where Mercury is seen from the Earth at its greatest Elongation from the Sun, and the point where its next Station is seen, subtends a less angle at the Sun than the arc of the Orbit of Venus contains between its greatest Elongation from the Sun and next Station: And therefore since it subtends both a less angle at the Sun, and belongs to a less Circle, and besides is presented more Obliquely to the Earth, observing it, and is more distant from it; 'tis evident that the Stations of Mercury

are much nearer to its greatest Elongations from the Sun than those of Venus.

COROLLARY 3.

And from hence again it follows, that in respect of the same inferior, in two superior Planets compar'd together; the farther off the Sun the superior Planet is, the longer its Retrogradation continues; for (by the preced. *Corol.*) it begins sooner and ends later. But the angle of Retrogradation is less, because the Orbit of the given inferior Planet seen from the more distant superior Planet appears under a less angle, than if it were seen from a nearer: From this *Prop.* 37. it is evident, that the angle of Retrogradation (contain'd between two right lines from the superior Planet opposite to the Sun, to the places of the inferior Orbit, where the one appears Stationary to the other) is almost equal to the angle wherein the Orbit of the inferior is seen from the superior, since the above mention'd points of the Stations are not far from the points of contact.

Hence it is, that the Retrogradation of Saturn seen from the Earth is longer, but made along a less arc, than that of Jupiter; and that of Jupiter in like manner to that of Mars.

COROLLARY 4.

But in respect of the same superior, in two inferior Planets compared together; the farther off the superior Planet is from the Sun, the longer is its Retrogradation and along a greater arc. The former is evident from the preced. *Corol.* since the Retrogradation of two Planets is mutual by *Corol.* 1. The latter is evident from hence, that a Planet that is more distant from the Sun, while it appears Retrograde, describes a longer arc slower, and therefore spends more time.

Hence it is, that Venus seen from the Earth, is both longer Retrograde, and along a greater arc than Mercury.

COROL.

Plate 10. Book 3.

Fig. 40.
p. 457.

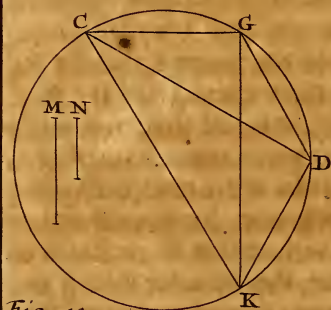
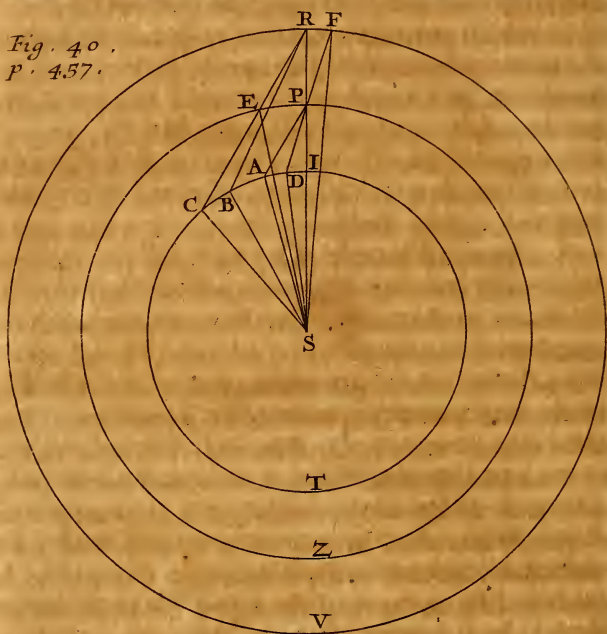


Fig. 41.
p. 458

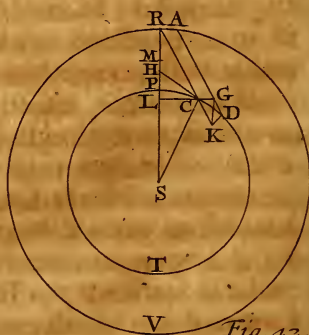


Fig. 43.
p. 460.

COROLLARY. 5.

Hence it also follows, that when two superior Planets are seen from an inferior in Conjunction, if that which is farther off from the Sun be Direct, that which is nearer the Sun will also be Direct; if that which is nearer the Sun be Retrograde, that which is farther off the Sun will also be Retrograde. Let S represent the Sun in [Fig. 40.] IT the Orbit of an inferior Planet; PZ, RV the Orbits of two superior Planets. Draw ST any how intersecting the Orbits of the superior in S and R . Let B be the Place in the Orbit of the inferior, where the inferior Planet is when the more distant one in R is seen from thence to be Stationary; and let A be the Place in the same, where the inferior is found when the next in P seen from the inferior appears to be Stationary. Join the right line AP, AS, BR, BS . From *Corol.* 2. it is evident that the Place A is nearer to the point I than B , and that the angle PAS is greater than RBS . When the Planet in R appears Direct from the inferior, the Place of the inferior ought to be farther distant from I than the Place B , for instance in C . Suppose now that the nearer of the superior Planets seen from the inferior is in conjunction with the more distant one, and it will be therefore in the point E , where the right line CR intersects the Orbit PZ . By *Prop.* 21. *Elem.* 1. the angle ECS is less than the angle RBS ; therefore it is much less than PAS . Therefore a Planet seen in E from C is Direct; because to make it Retrograde, 'tis requisite that ECS be greater than PAS , which is requisite to make it Stationary. But the inferior being between B and A , the more distant at R will appear Retrograde, and that which is nearer to it in conjunction, still Direct; because the Planet that takes the view is not as yet arriv'd to the limit A . But

But if the nearer of the superior Planets appears Retrograde at P from the inferior, the inferior will be between A and I , for instance in D . If now the more distant Planet appear in conjunction with it, that will be in the right line DS produced to the Orbit RV , namely in F : But the angle FDS is greater (by *Prop. 21 El. 1.*) than PAS , which again is greater (by what has been shown) than RBS . And therefore the more distant Planet seen in F from D , appears Retrograde, since to make it appear Direct, a lesser angle at the inferior than RBS is requisite.

And hence it is that when Saturn is seen from the Earth to be Direct, Jupiter and Mars can't be in conjunction with him unless they also be Direct; or Mars can't be in conjunction with Jupiter Direct, unless when he himself is Direct: But either Jupiter or Mars Direct may be in conjunction with Saturn Retrograde, and Mars Direct may be in conjunction with Jupiter Retrograde. But Jupiter and Saturn can't be in conjunction with Mars Retrograde, unless they themselves be Retrograde; or Saturn with Jupiter Retrograde, unless Retrograde.

LEMMA.

GIVEN in the quadrilateral figure $CGDK$ [Fig. 41.] inscribed in a Circle, (one of whose sides CK is a diameter of the Circle,) the side CG and the diagonal CD , together with the ratio between the opposite side KD and the other diagonal GK , (the same that is between the given right lines N and M ;) to find the angle GCD contained between the given side and the given diagonal.

Because the sum of the Squares of the right lines CG and KG is equal to the sum of the Squares of the right lines CD and KD , (for each of the sums are equal to the Square of CK ;) the difference of the Squares

Squares of the right lines GK and KD will be equal to the difference of the Squares of the right lines CD and CG , and consequently given. Moreover, because GK is to DK as M is to N , the Square of GK will be to the difference of the Squares of GK and DK , as the Square upon the right line M is to the difference of the Squares upon M and N ; being made out of those in the same ratio with them that are similarly made out of these, which are proportional to those: Therefore the Square upon GK (by *Dat. 2.*) will be given; the Square also of CK , made up of the given Square of GK and the Square of CG , is also given: Consequently the right lines themselves CK and GK are given, the right line DK is also given, because it has a given ratio to GK . But it is a known Theorem among Geometers, demonstrated by *Ptolemy, B. 1. of the great Construction, Chap. 9.* that in a quadrilateral $CGDK$ inscribed in a Circle, the rectangle contained under CD and GK is equal to the rectangle under CG and DK , together with the rectangle under CK and GD . And therefore the rectangle under CD and GK , less'n'd by the rectangle under CG and KD is equal to the rectangle under CK and GD . But the rectangled Parallelograms that are contained under the given right lines are given, and the difference of the given rectangles is given; and consequently the rectangle under CK and DG is given, and CK is given; therefore GD is given. And the three sides in the Triangle GCD being given, the angle GCD will be found.

Q. E. F.

PROPOSITION XXXVIII.

TO find the Angle of Commutation, wherein two given Planets seen from the Sun are distant, and the angle wherein the Sun and either of the Planets seen from

from the other are distant, when one of the given Planets seen from the other appears Stationary.

Let S be the Sun [Fig. 42.] RV , PT the Orbits of the given Planets ; and let CD , RA be the Arcs described in the same time by the Planets, while they are seen from the other Stationary, which are given by what has been said at Prop. 37. Join SR , SC , and RC , AD , which (by the preced.) will be parallel. Thro' C draw the lines CG parallel to RA , also thro' G the right line GK parallel to RS , and thro' D the right line DK parallel to CS , which may meet in K , join the right line CK . The Triangle GKD will be equiangular to the Triangle RSC , (since each of the sides of that, is parallel to each of this,) and consequently similar. Because the Time of the Station of the Planet is very small, if compared with the periodic Time even of the swiftest, the line CD does not sensibly differ from a right line, as also RA or CG , much less. The former therefore does not differ from HC a right line produced touching the Orbit PT in C , and the latter produced will be the right line CL perpendicular to SR , as AR is to the same. Moreover, a Circle made upon the diameter CK passes thro' D and G , because the angles CGK , CDK are right ; being by construction equal to the right ones SLC , SCH . $CGDK$ therefore is a Quadrilateral inscribed in a Circle, one of whose sides CK is the diameter of the Circle, and in which are given the side CG and the diagonal CD ; as also the ratio between the opposite side DK and the other diagonal GK , namely, the same that there is between the Semi-diameters CS , RS of the given Orbits. And therefore (by the Lemma premised) the angle GCD will be found, to which the vertical angle HCL is equal ; and to this again the angle RSC is equal, which is the Angle of Commutation, where-

whereby the two given Planets seen from the Sun are distant, when the one seen from the other appears Stationary. Moreover, in the Triangle RSC the two sides SR , SC , and the angle RSC contained by them being given, the angles SRC , SCR will be found; the former of which is that angle whereby the inferior is distant from the Sun, while it appears Stationary from the superior, the latter is the angle whereby the superior is distant from the Sun, while it appears Stationary from the inferior.

This Method of defining the Points of the Stations fit for Calculation and more natural, and as it were depending upon the motion of the Spectator about the Sun, I have demonstrated; passing over the Geometrical Construction of this Problem laid down by *Ptolemy in B. 12. of his Almagest.* accommodated to the Hypothesis of the Earth's being at rest, which *Apollonius Pergæus* formerly gave, and which *Copernicus* notwithstanding says, that it no less agrees with his Principles of the Motion of the Earth, in *B. 5. Ch. 35. Revolut.* and which you may see in the abovemention'd Authors.

If the plane of two Orbits be not the same, the one may be reduc'd to the other, as letting fall from all the points of the Orbit, which is to be reduced, perpendiculars to that other, by which means the circular Orbit becomes Elliptic, the the Elliptic is chang'd into another Ellipse, or perhaps into a Circle. But a new Orbit is more easily drawn (by *Prop. 33.*) thro' five Points, two of which are the same with two of the Orbit, to whose Plane the other is reduc'd, namely, the common Sections of that with this; the three others are found, by seeking the Curtated Distances from the Sun, as was done in *Prop. 35.* And if the other Orbit, or both after the reduction become Elliptic, the Problem is constructed in them
after

after the like manner, and the preceding Calculation applied to this case, after the same manner as was done in *Prop. 36.* in a case not much different. But to find the ratio between *RA* and *CD*, *Prop. 41. B. 1.* is to be used instead of *Prop. 27.* of the same Book; which was used in the case where both the Orbits were circular.

SECTION V.

Concerning Tables of the Primary Planets, and the Use of them.

FOR reasons much like those mention'd in *Sect. 9. B. 2.* concerning making Tables of the First Motion, Tables are also made for the more expeditious finding the situation of the Planets, both in regard of one another, and in respect to the Fix'd Stars at a given Time. The Art and Method of making them may be learnt from what goes before, but the Order from what follows.

PROPOSITION XXXIX.

TO describe the Tables, by the help of which the Heliocentric and Geocentric Place of any Primary Planet at a given Time is expeditiously determined.

In the first place, we have a Table of the Equation of Time, consisting of two Parts, of which we have treated in *Prop. 17.* And because in a complicated Calculation it would be a tedious piece of work to take out the two component parts with their Signs, add or subtract them as occasion requires, they form one Table made out of them mixt together, fitted to the Age the Calculator lives, in since in that Age the situation of the Points
of

of the Ecliptic, in regard of the Earth's Aphelion, is not sensibly changed.

Tho' this Table of the Equation of Time may be reckoned among the general Tables, since Time must be used to determine the Place of any moveable Body, and that marked out by something moved equably; and all Time is commonly (and most aptly) reckoned by the Sun: The motion of this Body which is in its own nature unequal, ought to be reduced to an equality; that is, such a Table as this ought to be used: Notwithstanding, since the Equation of Time marked by the Sun depends, on a double account, upon the situation of the Earth in regard of the Sun, this Table tho' general in its use, and necessary in determining the motion of the Fix'd Stars from the preced. Book, as of the Secondary Planets from the following, ought to be reckon'd from its own nature, among the Tables of the Primary Planets.

The Tables are to be made separately for each of the six Primaries, namely Saturn, Jupiter, Mars, the Earth, Venus and Mercury. Tho' this, or the inverse of this, be the true order of the Planets, and the Tables of them (each separately) ought to be disposed after this order naturally; yet with us, who are the Inhabitants of the Earth, 'tis necessary that the Tables of the Earth have the first place, both because by the help of these, the motion of the Earth is found, on which the apparent motion of the Sun (which leads all the other Planets) depends; which is necessary in the first place to determine the motion of any of the rest, by reason of the Equation of Time depending upon it.

In the Tables of each of the Planets first of all the Radix of the Motion of the said Planet is to be settled; that is, at any celebrated point of mean

mean Time pitched upon according to the reckoning of the place, for which the Tables are made, you have marked by the help of the preceding Propositions, the Heliocentric Longitude of the said Planet, of the Aphelion and ascending Node of the same; the first whereof, namely the Distance of the Planet from the Equinoctial, consists of two heterogeneous parts; namely the distance of the Aphelion of the Planet from the Equinox, expressed by the angle, and the distance of the Planet from the Aphelion, expressed by the area of its Orbit.

In the second place, the Mean Motions from the Equinox of the Planet it self, of the Aphelion and Node, for single Years, Tens, Hundreds, &c. also for Months, Days, Hours, and parts of Hours, are disposed into Tables.

In the third place, a Table is subjoined, exhibiting the true Anomaly of the Planet to the several degrees of the mean Anomaly of the same. This Table may be made either by *Prop. 3.* or by Approximation by the 4th, 6th, or 7th Propositions, according to the nature of the Approximation which a Person has a mind to use.

In the fourth place, you have a Table exhibiting the Distance of the Planet from the Sun, (found by the same Propositions, together with *Prop. 22.*) in parts, whereof the mean Distance from the Earth contains 100000, for the several degrees of the mean Anomaly. These two Tables are advantageously reduced into one, by setting next to each degree of the mean Anomaly, the agreeable true Anomaly and corresponding Distance. To these Distances, for ease of Calculation, are annex'd the corresponding Logarithms; or even the Logarithms of the Distances alone are disposed into Tables, since they are sufficient for Calculation.

In the fifth place, is put a Table of the Centric Latitude, or Inclination of a Planet to the Plane of the Ecliptic for the several degrees of the Distance from the nearest Node; which Distance from the Node is commonly called by Astronomers, *The Argument of the Latitude*.

In the sixth place, you have a Table of *Reduction*; whereby the Place of a Planet in its proper Orbit is reduced to a Place in the Ecliptic, shewing how much more or less forward it is in the latter than in the former.

In the seventh place, you have a Table shewing how much the interval of a Planet from the Sun in its proper Orbit is to be curtailed, to reduce it to the plane of the Ecliptic, by letting fall from the Planet a perpendicular upon it. And since these three (namely the Inclination, Reduction and Curtation) are fitted to the several degrees of the Argument of Latitude, the three preceding Tables will commodiously come into one; as was said before concerning the third and fourth Tables. These three things are found by common Trigonometry, having the Inclination of the Orbit of the Planet to the plane of the Ecliptic, which is done by *Prop. 20*.

In the eighth place, is advantageously placed the Angle of Commutation (found by *Prop. 38*.) making in that Planet an apparent Station from the Earth. And because neither the Orbit of the Planet nor of the Earth is circular, neither can the same Angle of Commutation serve in every case, nor the same in the two next Stations; the Angles of Commutation producing both the first and second Station of the Planet, are disposed into a Table, for four or eight of the principal Anomalies.

In every one of the Inferiors may commodiously be added a Table, (made by *Prop. 36*.) co-

lexing with the former, of the Angles of Commutation in the greatest Elongation of the Planet from the Sun, as well towards the East as towards the West, fitted to the same Anomalies. To these, the Depression of the Sun below the Horizon in the Moments of Heliacal Rising or Setting is also added. And these are the common Tables usually made by Astronomers for the ready finding out the Places of the Planets.

PROPOSITION XL.

TO calculate the Heliocentric and Geocentric Place of any Planet, in regard of Longitude and Latitude at any time proposed, by the help of the above described Tables, and to define its Passions.

First of all, let the Time proposed be equated, as was shewn in Prop. 17; let the Place of the Earth seen from the Sun, and its Distance from the Sun be found for that mean Time, as in the 18th. And since by the help of the first Table at the assum'd Epocha there are given the distances of the Planet proposed, and of its Aphelion and ascending Node from the Equinox; and by the second, their Mean Motions made between the said Epocha and the Time proposed, being equated, as was said; the mean Distance of the Planet from the Equinox, and the Distances of the Aphelion and the Nodes from the same will be given. The difference of the two former is the Planets mean Anomaly; to which, the correspondent true one or angular Heliocentric Distance from its Aphelion may be taken, out of the third Table; and out of the fourth, the Distance of the same from the Sun. The Distance of a Planet from the Node, or the Argument of Latitude will likewise be given; to which, the corresponding Inclination, or Centric Latitude may be

taken



Fig. 43.
p. 467.

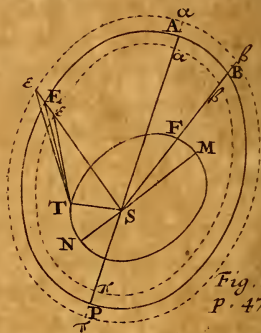
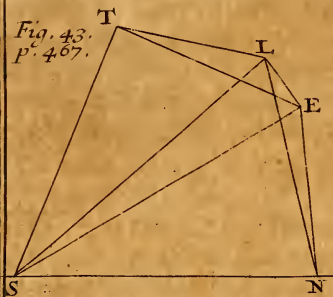


Fig. 44
p. 470.

Fig. 45.
p. 475.

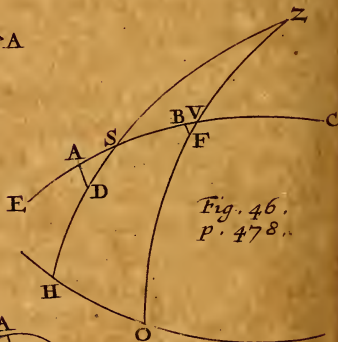
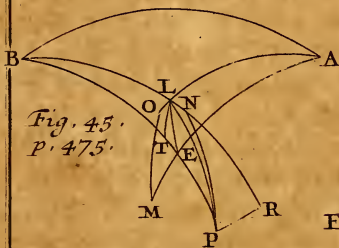


Fig. 46.
p. 478.

Fig. 47. p. 481.

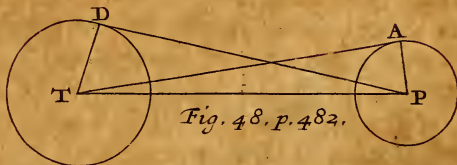
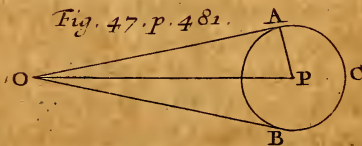


Fig. 48. p. 482.

taken, out of the fifth Table; and out of the sixth, the Reduction, whereby it is reduced to the Ecliptic; and out of the seventh, the Curtation, which being subtracted from the Distance of the Planet from the Sun in its own Orbit, first taken out of the fourth Table, leaves the Curtate Distance of the Planet from the Sun.

After this manner may be had the following things in the Diagram of *Prop. 35*; viz. the right line *ST* [*Fig. 43.*] in magnitude, namely the Distance of the Earth from the Sun; the same in position, namely the Heliocentric Place of the Earth; the right line *SE* in magnitude, viz. the Curtate Distance of the Planet from the Sun; and the same in position, namely the Place of the Planet reduced to the Ecliptic: And consequently the angle *TSE* is given, contained between right lines given by position, which is also the angle of Commutation; therefore the angle *STE* the Elongation of the Planet from the Sun seen from the Earth will be found. But the Place of the Sun seen from the Earth is given; and therefore the Geocentric place of the Planet in the Ecliptic, and *TE* the Distance of the Planet from the Earth are given.

Again, because *LE* is perpendicular to the plane of the Ecliptic *TSNE*, the Triangles *SEL*, *TEL* are rightangled at *E*; and therefore the Tangents of the angles *ETL*, *ESL* are in the same ratio with *ES*, *ET*: But *ESL* is the Heliocentric Latitude of the Planet first taken out of its proper Table, and the ratio of *ES* to *ET* is given; for the right lines themselves are given, (but the ratio of these is the same with the ratio of the Sines of the opposite angles *STE* of Elongation, and *TSE* of Commutation;) wherefore *ETL* the Geocentric Latitude of the Planet is known.

Moreover, by the angle of Commutation and Anomaly of the Planet found above, in the eighth Table it may be found whether the said Planet be in the first or second Station. But if the angle of Commutation be less than that which in this Planet makes the Station, the Planet will be Retrograde; if greater Direct; as was shewn above. And after the same manner, by the same Table it may be found whether Venus or Mercury be in their greatest Elongation from the Sun. And by the Place of the Planet found above, and the known Depression of the Sun requisite to it, annexed to this Table, it may be determined (by *Prop. 38. B. 2.*) whether the Planet is rising or setting Heliacally, or lies hid under the rays of the Sun. And lastly, by the same method, with that whereby the Phasis of the Moon is delineated, *Prop. 17. B. 1.*, the Phases of Mars, Venus or Mercury, at a given time may be delineated. But Jupiter and Saturn always appear full; as was said in *Prop. 9. B. 1.*

In this Proposition we do not attend to the variety of all cases, *viz.* whether this Arc be to be subtracted from that, or *vice versa* that from this; nor to the different cases of Inclination and Latitude Northern and Southern, Elongation Eastern and Western, and the like of the rest; nor how out of two numbers, the intermediate Tabular number sought is to be found by proportion: For we do not here give rules of calculation to be observed by a Calculator literally, but the Geometrical principles of a Calculation.

SCHOLIUM.

Tho' the Tables above described are natural and commonly made use of, yet to find the Places of the Planets more readily, Tables are made after this or the like manner advantageously enough, by such as look upon the Aphelia and
Nodes

Nodes of the Orbits of the Planets to be at rest ; as *Street* of all the Planets, *Flamsteed* at least of the Apfides of the Earth.

For every Planet's Radixes are to be settled, not of the Longitude of a Planet, and of its Aphelion and Node, as before, but of the mean Anomaly of the Planet : And the Motion of the Anomaly for Years and fums of Years, Months, Days, (or rather for the particular days of the feveral Months current,) Hours, &c.

To these are added the following things, which always continue the same, determining the situation of the Orbit of the Planet, and exhibiting the intire Theory expreffed by numbers ; namely 1. The Distance computed in the Ecliptic of the Aphelion from some one of the Fix'd Stars (for instance, the first of Aries, or the difference of Longitude of the said Fix'd Star and Aphelion of the Planet, which is called *the Longitude of the Aphelion from a Fix'd Star*. 2. The Longitude of the ascending Node of a Planet from the same Fix'd Star. 3. The Inclination of the Orbit of a Planet to the plane of the Ecliptic. 4. The Magnitudes of the greater Axe of the Orbit of a Planet and of its Excentricity, expreffed in parts, whereof the mean distance from the Earth contains 100000. For from these, by the respective Propositions, the Longitude of the Planet from the first Star of Aries, the Inclination or Heliocentric Latitude Northern or Southern, and the Curtate distance of a Planet from the Sun are found, for any mean Anomaly of the Planet. Out of these artificially annexed to the mean Anomaly, to which they belong, are made Tables ; though instead of the Curtate Distance itself, the Logarithm alone will suffice for Calculation. For since (by *Prop. 43. B. 1.*) the mutual situation of a Fix'd Star and the Orbits of the Planets remains the same, or the Lines of the Nodes

and Apfides are at reft, (at leaft they are taken to be fo by thofe Authors,) to the fame point of the fame Orbit, that is, to the fame mean Anomaly, thefe three continue the fame and unvaried in the fame Planet. Thefe being taken out of the Tables, the reft is done, as above in the Proposition. But the Longitude of the firft Star of Aries at a given Time is found by the Tables of the Fix'd Stars, of which fee *Prop. 68. B. 2.* What remains about the Paſſions of the Planets is to be done after the fame manner as above.

Such Tables as are more ready for ufe than thofe mention'd above, are called *Ephemerides of the Celeſtial Motions*, being calculated, from the perpetual Tables deſcribed above, for a certain number of Years, and they ſhew the Places of the ſeveral Planets as to Longitude and Latitude, their Paſſions, and mutual Aspects, as well when they are viewed from the center of the Sun, as from the center of the Earth, for every day; to which alfo are added the Places of the Moon computed for every day, its Phaſes, the Eclipses of the Sun and Moon, and other things found in the following Book.

PROPOSITION XLI.

IN the Primary Planets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Elliptic Orbits, which they deſcribe about the Sun.

Let *S* [Fig. 44.] represent the Sun; *MFTN* the Orbit of the Earth, whoſe Apfides are *M* and *N*, *AEPB* the Orbit of any other Planet, whoſe Apfides are *A* and *P*, which are ſuppoſed to be ſo delineated in the Scheme, as that the Inclination of the lines *MN*, *AP* may be the ſame with that found by what has been ſaid about the Lines of the Apfides of the ſaid Planets in the Heavens;
and

and the ratio of the Cubes of them, the same with that of the Squares of the Periodic Times of the same Planets. I say, that the Orbit of the Earth remaining, another Planet can't move in any other Orbit besides that just now described $AEPB$. For if it can, let the different Orbit of the said Planet be $\alpha\epsilon\pi\beta$; that is, greater or less than the Orbit $AEPB$. Let F denote the point in the Orbit of the Earth, wherein the Earth is when it is in conjunction with any other Planet Heliocentrally. The line SF drawn (and if need be, produced) will shew in the Orbit $AEPB$, the point B ; wherein the other Planet would then be found if it moved in this Orbit; and in the Orbit $\alpha\epsilon\pi\beta$, which it is supposed to describe, the point β , in which it is really found. The Earth removing from F and the other Planet from β , after a certain time, let the former be found in T , the latter in ϵ : $S\epsilon$ joined will meet the Orbit $AEPB$ in the point E , in which the Planet would have been found, if it had described the Orbit $AEPB$, since the Periodic Time is sufficiently known and certain. The right lines ET , ϵT being drawn, the angle $ST\epsilon$ will be the Elongation of the Planet from the Sun at the said time: But this Elongation is not agreeable to observation, for 'tis the Elongation STE (drawn from the supposition, that the Planet describes the Orbit $AEPB$) that does. And therefore supposing $MFTN$ to be the Orbit of the Earth, that other Planet does not describe the Orbit $\alpha\epsilon\pi\beta$ different from the Orbit $AEPB$. But the Cubes of the greater Axes of the Orbits $AEPB$, $MFTN$ are as the Squares of the Periodic Times of the Planets moved in those Orbits. And the same is collected of the other Planets compared with the Earth, for the like reasons: And therefore *ex æquo*, in any two Primary Pla-

nets compared together, the Squares of the Periodic Times are as the Cubes of the greater Axes of the Orbits, wherein they are carried.

And generally the agreement of the Places by calculation with those observed, argues the true Situation and true Species of the Orbit of any Planet, as well as the true Magnitude. For the Situations of the Earth and the other Planets are so variously intermingled, that any notable errors committed in the settling the Orbit and Theory of any Planet would sometimes manifestly discover themselves, and become very sensible: But on the contrary, the Places of the Planets drawn from Tables depending upon the said Theories, strangely agree with their Places by observation. The very small difference from observation in the mean while is owing to other causes; namely the action of the Planets upon one another, the effects whereof in disturbing the motions of the Planets are afterwards to be considered, and their Orbits rectified from thence.

PROPOSITION XLII.

THE Geocentric Place of a Planet at a given Time being given, to find by the Tables the Place of the same seen from a given Place or Habitation upon the surface of the Earth at that time.

If the Habitation given be not in the same Terrestrial Meridian with the Place to which the Tables are fitted, let the given Time be reduced to the reckoning of the Place of the Tables, by the Tables described *Sect. 9. B. 2.* To that Time find (by *Prop. 36. B. 2.*) its Altitude, seen from the center of the Earth, above the horizon of the Habitation given, and the Azimuth. Let the Parallax of the Sun, that corresponds with the Altitude thus found, be taken out of the Table made by *Prop. 47. B. 2;* from the Parallax of one Distance and

of one Altitude, or known by some Prop. of the 2^d Book, or by one of the following : And (by *Prop.* 48. B. 2. the Parallax of the Planet in the same Altitude above the Horizon, will be to the Parallax of the Sun taken above, as the Distance of the Sun from the Earth, to the Distance of the Planet from the same : But these Distances in the Place of the Planet to be computed by the preced. were known ; wherefore the Parallax of the Planet corresponding to that Altitude is found. This therefore is to be subtracted from the Geocentric Altitude found above, that it may become the Altitude once corrected in the given Habitation. To the Altitude thus corrected add the corresponding Refraction, from the Table of Refraction made by *Prop.* 66. B. 2. of the use of which we have spoken in *Prop.* 68. of the same ; and you will have the apparent Altitude of the Planet in the given Habitation, the Azimuth first found remaining unchanged. From this apparent Altitude and Azimuth, you may find by *Prop.* 26, and 27. B. 2. the Planet's place, which will be the apparent one in the given Habitation. The Time proposed reckoned in the Meridian to which the Tables are fitted, and afterwards reduced to the Meridian of the given Habitation, since it is a mean, (such as it ought to be when Tables are used) 'tis reduced to apparent Time, by *Prop.* 17. Therefore the apparent Place of the Planet, at the given Time now made the apparent, is found.

SECTION VI.

Of the Magnitude of the Orbits of the Planets.

IN what precedes, we have considered the Planets themselves, as so many points moving in their Orbits; that is, we have abstracted from their Magnitude. And in this case it is the same, as to the Phænomena, what Magnitude the Orbit of any of them is supposed to be of, provided all the rest have that ratio and position to this, as they have in nature. Therefore we have every where supposed the common measure of all the Orbits and their parts to be the greater Axe of the Orbit, that the Earth (the Habitation of the Spectator) describes about the Sun, or half of it, the mean Distance of the Earth from the Sun, without being concerned about the Magnitude of this apparent Distance. But now when the Magnitude of the other Planets comes to be compared with the Magnitude of the Earth, the aforesaid Distances must be compared with the Diameter of the Earth; which may be done by the help of the Parallax being known, as was shewn in *Book 2*. And tho' the Parallax of the Planet might be found by one of the methods of finding the Parallax delivered in *Book 2*, if you could observe it as accurately as you please; yet notwithstanding, because there are several things that are too obvious which are an hindrance to it, I shall propose two methods by which this observation, so slippery as it is, may be made without any danger of a notable error. In the first it shall be done by two observations made together in two Places of the Earth; in the second, by two made in the same Place.

PROPOSITION XLIII.

B*T the Conjunction of two Planets, observed in two given places of the Earth, to find the Parallax of both.*

Let the true Places of the two Planets observed in Corporal Conjunction (that is, where the one seems to touch or cover the other) be L and E . Imagine the Circle AB [Fig. 45.] to be drawn thro' the vertices A and B of the Places on the Earth. Draw the vertical Circles AEM , ALO , BEP , BLN . Let the Planets be observed from one of the given Places (namely that whose vertex is A) in O and M . And since the Places are given where the observations are made, (by Prop. 32. B 2.) the difference of the Meridians will be given; and so the names of the Hours, whereby any the same instant of time is expressed in the said two Places: And therefore the same moment of Time may be pitched upon to make the observation in the given Places. Therefore at the same instant that the Planets in O and M are observed from the Place whose vertex is A , let them be observed from the Place whose vertex is B , for instance, in N and P ; Draw the great Circles LE , OM , NP . Observe also (either by a Micrometer, or what is instead of it) Threads stretched various ways in the Focus of the Telescope, or by the help of an Icoscope throwing the Image upon a Plane, or by any other method to be left to the industry of the Artist) the magnitudes of the Arcs NP , OM , (which in a corporal Conjunction, by reason of their smallness, will be right lines) and of the Angles PNB , NPB , AOM , and AMO . Now, by the Theory of these Planets, there is given the ratio of their Distances from the Center of the Earth, from whence, and their Altitudes (observed

ved exactly enough for this purpose) there are also given (by *Prop. 49. B. 2.*) the ratios between the Sines of the Arcs LN , EP , LO , EM , and consequently (by 2. *Dat.*) one being given all are known. From the point P let the Arc PR be let fall perpendicular upon the Azimuth BN produced: And in the rightangled spherical Triangle PNR , the side PN and angle PNR being given, you may find the sides NR , RP , and the angle RPN ; therefore the angle RPE is known. Suppose (after the manner of Analysts) the Sine of the Parallax LN in the computation to be the Root: And by the given Sines of the Arcs LN , NR , you may find (by what *Ptolemy* delivers in *Cap. 9. B. 1.* of the *Almagest*) the Sine of the Arc LR , their sum. Then, in the spherical Triangle LPR rightangled at R , by the given Sines of the sides LR , RP , you may find the Sines of the side LP and Angle LPR . Therefore, by the given Sines of the angles LPR , and $EP R$ (by what is demonstrated by *Ptolemy* in the place quoted above) you may find the Sine of their difference; namely of the angle LPE . From the point L let fall the Arc LT perpendicular to the Azimuth BP . Therefore in the spherical Triangle PLT rightangled at T , by the given Sines of the side LP and angle LPT , you may find the Sines of the sides LT , TP : Therefore the Sine of the arc ET , namely the difference of the Arcs EP , TP is given. And lastly, in the spherical Triangle ETL rightangled at T , by the given Sines of the sides LT , ET , you may find the Sine of the side EL . Just after the same manner you may find the Sine of the side EL , from the Sines of the Arcs LO , EM given, expressed by the before supposed Root. Therefore you have given an Equation between the Sine of the Arc EL found first, and, the same Sine found

secondly,

secondly, the resolution whereof will give the value of the Root, or sine of the Arc LN ; from whence all the rest that is sought will become known. But if such Places upon the Earth be chosen, as have the Planets passing the common Azimuth of the Places, and that instant of Time wherein one of them is in the said common Azimuth; this construction will become that particular one in *Prop. 87. Opt. Prom.* of James Gregory.

LEMMA.

THE difference of two Quantities and the ratio between them being given, to find the Quantities themselves.

Let a quantity be taken, that is to the known difference, as the the greater term of the given ratio, is to the difference of the terms; this quantity will be the greater of the quantities sought: For in four proportional quantities, the products of the two former are in the same ratio with the similar products of the two latter. If the greater be lessen'd by the given difference, what remains will be the less.

PROPOSITION XLIV.

BY two Observations of the same Corporal Conjunction of two Planets, made in the same Place, to determine the Parallax of both Planets.

Let the apparent motion of the two Planets reduced to the Ecliptic, made mutually by one another (or to one another) in a given time, be observed near the Corporal Conjunction: And by the Theory of both Planets, known by the preceding, let their motion from one another mutually be determined, computed in the Ecliptic, viewed from the Center of the Earth, made in the same time; which may be done accurately enough, if the time be not too great. From these arcs

arcs therefore given, namely, the true and apparent motion of the Planets from one another, made in the same time, the Parallax of both is to be determined.

Let HO [Fig. 46.] represent the Horizon; Z the Zenith, and EC the Ecliptic: And let SV be the arc of the Ecliptic, wherein the Planets (viewed from the Center of the Earth) move mutually from one another in a given time, known from the Theory; and AB the arc of the Ecliptic, whereby they seem to move from one another in the same time, known by Observation; therefore their difference is given. Thro' A and B imagine Circles of Latitude AD , BF drawn, meeting the Verticals ZH , ZO , drawn thro' S and V , in D and F ; and SA is the Parallax of the Longitude of the nearer Planet from the more remote, or the excess of the greater Parallax of Longitude above the less in this situation; and VB the excess of the greater Parallax above the less in the other. The ratio of the arc AS to the arc BV is compounded of the ratios of AS to SD , SD to VF , and VF to BV ; but the ratio of AS to SD is known, because in the rectilineal Triangle ASD , rightangled at A , the Triangle ASD is given, found by *Prop. 36. B. 2*; the ratio of SD to VF is found, by *Prop. 47. B. 2*; and the ratio of VF to BV is found as before, in the ratio of AS to SD ; wherefore the ratio of AS to BV compounded of them is known: But the difference of the arcs AS and BV is also known, namely the same with the given difference of the arcs AB and SV ; and therefore (by Lem. preced.) the arcs themselves AS and BV will be given. And therefore (by *Dat. 2.*) SD and VF will be given; that is, the excess of the greater Parallax above the less in a given Altitude: For the Altitude HD or OF may be observed exact enough

enough for that purpose. And from thence the excess of the greater Horizontal Parallax above the less (by *Prop.* 47. and 63. *B.* 2.) will also be given: But (by *Prop.* 35.) the ratio between the Distances of the given Planets from the Earth at the time of Observation is given, (for the Distance of both from the Earth is expressed in the parts of the mean Distance of the Earth from the Sun;) and therefore the ratio of the Horizontal Parallaxes of the same Planets is also given, because (by *Prop.* 48. *B.* 2.) the reciprocal of the former: And therefore (by *Lem.* preced.) the Horizontal Parallaxes of the Planets themselves are also given.

And from hence the Parallaxes of all the Planets are determined. For the Parallax of one of the Planets being given, the Parallax of all of them (by *Prop.* 49. *B.* 2.) are given; because (by the preceding Proposition) at a given time the ratio between their Distances from the center of the Earth is given.

SCHOLIUM.

Tho' any two Planets in any Conjunction (after the manner shewn in the preced. Proposition) may be used to determine the Parallax of both, nay any Planet (Mars, for instance, in his Achronical situation, and nearest to the Earth) in conjunction with a Fix'd Star, whose Parallax is nothing, may suffice to determine its Parallax, (at that time the greatest,) one of the inferior Planets passing over the disc of the Sun, is fittest for that purpose; especially Venus, whose Parallax is very sensible, because she is then nearer; and the Theory, so far as it is necessary for this purpose, is accurately enough known.

PROPOSITION XLV.

TO determine the greater Axes of the Orbits of the Planets, Excentricities, &c. in known measures.

For any time (by one of the two preceding, or any other method shewn in *B. 2.*) find the Parallax of any Planet, corresponding with any known apparent Altitude; and from thence the ratio of the Distance of the Planet from the center of the Earth to the Semidiameter of the Earth will be given; namely the same (by *Prop. 46. B. 2.*) with that which is between the sine of the apparent Distance from the vertex and the sine of the Parallax. But (by *Prop. 35.*) the ratio between the Distance of the Planet from the center of the Earth and the Distance of the same from the Sun at that known time; and (by the preced. *Prop.*) for the same Time the ratio of the distance of the Planet from the Sun to the greater Axe of the Orbit, and its Excentricity, &c. is given: And therefore, *ex æquo*, and by (*Dat. 2.*) the ratio of those right lines to the Diameter of the Earth will be given; that is, all these Diameters of the Orbits will be given, expressed by Diameters of the Earth. And since, by the *Schol.* of *Prop. 17. B. 2.* the Diameter of the Earth is found in known and familiar measures, the aforesaid Diameters of the Orbits will be given in the same.

By this means the Distances of the Sun and Primary Planets from the Sun at any time proposed, expressed by Semidiameters of the Earth or any other known measures, become known. For those Distances at that time expressed in the parts of the mean Distance of the Earth from the Sun are known, (by *Prop. 34.* and *35.*) and (by this) the said mean Distance of the Earth from the Sun is known in known measures.

SECTION VII.

Of the Magnitude and Density of the Sun and Primary Planets.

PROPOSITION XLVI.

TO define the Magnitude of the Sun and Primary Planets.

'Tis very proper in this place, having laid down the Theory of the Planets, to treat also of their Magnitude and Density, upon which conjunctly the Quantity of Matter in each depends; especially since some things in their Motion and the dimension of their Orbits come to be rectified by this means.

Let some one of them be ABC , [Fig. 47.] whose Center is P , and let O be the Eye of the Observer. Join the right line OP , along which imagine a plane drawn cutting the spherical Body of the Sun or Planet in its greatest Circle BAC . From O draw right lines OA , OB touching it, and join the right line PA . The angle AOB is the same with the apparent Diameter of the said Body at O , and is bisected by the right line OP . Let this be observed by the methods well known to Astronomers, by which means, in the Triangle OP , all the angles will be known: For (by *Prop. 16. El. 3.*) A is a right angle, and OP is half the observ'd apparent Diameter; hence the ratio of the sides OP and AP is known: But (by *Prop. 45.*) the ratio between the Semidiameter of the Earth, and OP the Distance of the Planet is given; therefore *ex æquo*, the ratio between the Semidiameters and the Earth and Planet is given; therefore the triplicate of this is also given, namely the ratio between the Earth and the Planet. But the Magnitude of the Earth is given by *Schol. Prop. 17. B. 2*; and therefore the Magnitude of the Planet (by 2 *Dat.*) is given. Q. E. F.

If thro' T and P the centers of the Earth and Planet [*Fig. 48.*] a Plane be drawn cutting them both, the common sections of this plane with those Bodies will be great Circles in them. But if you draw the Tangents TA , PD , and join TP , TD , AP ; these two last do almost coincide with arcs described upon the Centers T and P and at the distance TP ; and consequently they are in the same ratio with the angles TPD , PTA that they subtend, the latter whereof is the apparent Semidiameter of the Planet, the former its Horizontal Parallax: And therefore reciprocally the Diameter of the Planet is to the Diameter of the Earth, as the apparent Semidiameter of the Planet to its Horizontal Parallax. But the three latter terms of this Proportion are given; and therefore the first, namely the Diameter of the Planet.

SCHOLIUM.

By the aforesaid method, the Magnitudes of the Sun and all the Planets, excepting the Earth, may be accurately compar'd together. For their Diameters are in a ratio compounded of the ratio of the Distances, and of the ratio of the Sines of the semi-angles under which they appear, that is, almost of the apparent Diameters: But the ratio of the Distances found by *Prop. 35.* is the same with the true by *Prop. 41*; and the ratio of the apparent Diameters is certain by observation. But the comparison of the Magnitudes of those Bodies with the Magnitude of the Earth depends upon the Parallax of some Planet. If this be not accurately given, the said comparison can't be made accurately. We have already given the best methods of finding it in their proper places: But the last, which you have in *Prop. 44.* seems most fit for use, if it be applied to Venus seen in the disc of the Sun; which has not been yet done, nor can be

Plate 12. Book 3.

Fig. 49. p. 483.

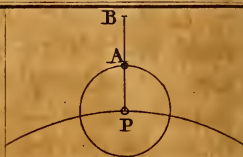
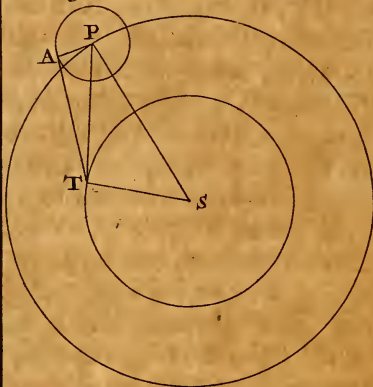


Fig. 50. p. 484.

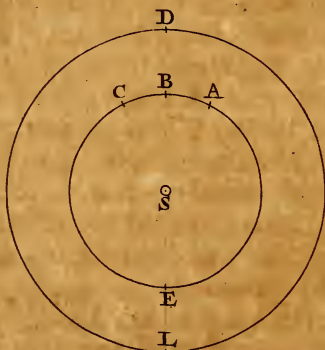


Fig. 51. p. 488.

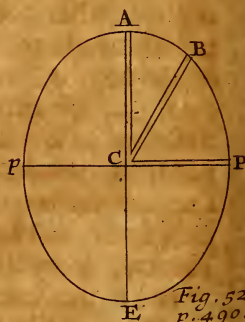


Fig. 52.
p. 490.

be done till the year 1761, (which will be the second time of Venus being to be seen by us in the Sun.) If in the mean while the Diameter of the Earth seen from the Sun be made a mean betwixt the Diameters of the five others seen from the same, (which 'tis very probable is not far from the truth, and besides agrees very nearly with the Parallax of the Sun determined by an observation of Mercury seen in the Sun, and other observations made other ways,) the Magnitudes of all the Planets and Sun will come out near enough to their true Magnitudes, and such as are to be taken for the true, 'till something more certain be settled about the Parallax of the Sun by observations.

PROPOSITION XLVII. LEMMA.

TO find the ratio of the Distance of a given Secondary from its Primary, to the Distance of the given Primary from the Sun.

Let S represent the Sun; [Fig. 49.] T the Earth; P any Planet, whose Satellite is A . At T observe its greatest Elongation from P , namely the angle ATP . Therefore in the Triangle PAT right-angled at A , the angle T being given, the ratio of the side AP to PT will be given: And by Prop. 35. the ratio of PT to TS is given; and the ratio of TS to the Distance of any other Planet from the Sun is also known; and therefore the ratio between the Distance of the given Satellites from its Primary and the Distance of any Primary from the Sun will not be unknown.

But if the Satellite proposed be our Moon, the Problem can be solved no otherwise than by finding the Distance of the Sun from the Earth, (by one of the methods above, or some like it) and the Distance of the Moon from the same (by the same, or some other already delivered or hereafter to be delivered in Book 4th.)

PROPOSITION XLVIII.

TO find the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in a given Primary, about which a Satellite revolves.

Let S be the Sun ; [Fig. 50.] P the Primary Planet given, about which its attendant A revolves; and let V be any other of the Primaries taken at pleasure : Make PB equal to SV , and imagine any Body placed at B ; and (by Prop. 49. B. I.) the Quantity of Matter in S to the Quantity of Matter in P , will be as the accelerative gravity of the Body V towards S to the accelerative gravity of the Body B towards P . And the ratio of the accelerative gravity of the Body V towards S to the accelerative gravity of the Body B towards P , is compounded of the ratio of the accelerative gravity of the Body V towards S , to the accelerative gravity of the Body A towards P , and of the ratio of the accelerative gravity of the Body A towards P , to the accelerative gravity of the Body B towards the same P . But (by Prop. 26. B. I.) the accelerative gravity of the Body V towards S , is to the accelerative gravity of the Body A towards P in a ratio compounded of the ratio of the right line SV to the right line PA , and of the duplicate ratio of the periodic Time of the Satellites A about P , to the periodic Time of the Primary V about the Sun ; and the accelerative gravity of the Body A towards the Planet P is to the accelerative gravity of the Body B towards the same, in the duplicate ratio of the right line PB (or its equal SV) to the right line PA . And therefore the accelerative gravity of the Body V towards S is to the accelerative gravity of the Body B towards P ; that is (as was shewn above) the Quantity of Matter in S is to the Quantity of Matter in P , in a ratio compounded of the three follow-

following; namely, the ratio of the right line SV to the right line PA , the duplicate ratio of the periodic Time of the Satellite A about the Planet P , to the periodic Time of the Primary V about the Sun, and the duplicate ratio of the right line SV to the right line PA . But the first and last compounding ratio make the triplicate ratio of the right line SV to PA : From whence the Quantity of Matter in S is to the Quantity of Matter in P in a ratio compounded of the triplicate ratio of SV to PA , and the duplicate ratio of the periodic Time of the Satellite A about P to the periodic Time of the Primary V about the Sun. But both the compounding ratios are given; namely the ratio of SV to PA , by the *Lemma* premised, and the periodic Times themselves by observation: Therefore the ratio compounded of them will not be unknown; namely the ratio of the Quantity of Matter in the Sun to the Quantity of Matter in the Primary P .

Hence the ratio of the Quantity of Matter in the three Planets that have Satellites may be found; *viz.* by finding the ratio of the Quantity of Matter in every one of them to the Quantity of Matter in the Sun.

SCHOLIUM.

After the same manner you may find the ratio of the accelerative Gravity in the superficies of the Sun to the accelerative Gravity in the superficies of any Planet that has a Satellite. For the ratio of Gravity in the surface of the Sun, to the Gravity in V . *viz.* the duplicate of the ratio of the Distance SV , to the Semidiameter of the Sun, known from what goes before, and the ratio of Gravity in V towards the Sun to the Gravity in A towards the Planet, found as above, are had; as also the ratio of Gravity in A to the Gravity in the superficies of the Planet P , the

duplicate of the known ratio that the Semidiameter of the Planet has to the Distance of the Satellite from the Primary. Now it is evident, that the ratio compounded of these three known ratios, is the same with the ratio of Gravity in the superficies of the Sun to the Gravity in the surface of the Planet *P*. And from hence again, *ex æquo*, the ratio of Gravity in the superficies of one Planet to the Gravity in the superficies of the other Planet, if both have Satellites, will be determined.

PROPOSITION XLIX.

TO find the ratio of the Density of the Sun to the Density of any Primary Planet, about which a Satellite revolves.

If there be two Bodies *A* and *B* of the same magnitude, 'tis evident the Quantity of Matter in *A* is to the Quantity of Matter in *B* as the density of *A* is to the density of *B*: And if they be of the same density, the quantity of matter in *A* is to the quantity of matter in *B*, as the magnitude of the Body *A* to the magnitude of the Body *B*: And therefore, tho' neither the magnitudes nor densities are equal, the ratio of the quantity of matter in *A* to the quantity of matter in *B*, is compounded of the direct ratio of the densities and the direct ratio of the magnitudes of them. Wherefore the ratio of the density of the Body *A* to the density of the Body *B*, is compounded of the ratio of the quantity of matter in *A* to the quantity of matter in *B*, and the ratio of the magnitude of the Body *B* to the magnitude of the Body *A*; that is, (in the present case) the Density of the Sun is to the Density of a Planet in a ratio compounded of the ratio of the Quantity of matter in the Sun to the Quantity of matter in the said Planet, and the ratio of the Magnitude of a Planet

Planet to the Magnitude of the Sun. But (by *prec. Prop.*) the former of the compounding ratios is given, if the Planet have a Satellite, and (by *Prop. 46.*) the latter is given; therefore the ratio compounded of them is given, namely of the Density of the Sun to the Density of the Planet.

COROLLARY

From hence, *ex æquo*, the ratio of the Densities of three of the Primary Planets is had, namely Saturn, Jupiter and the Earth. The Densities of the three others may be analogically derived from hence. No doubt the Divine Being, the Wise Creator of the World, has placed the Planets at different distances from the Sun, that each of them may enjoy a greater or less degree of the Sun's heat, according to the degree of its Density: For denser Matter requires a greater heat to perform its natural operations withal. But without doubt, some respect is had to the Magnitude of the Planet, since of like Bodies the lesser, *cæteris paribus*, are heated stronger and more intimately; because in respect of their Bulk, they have a greater superficies, and consequently receive more rays.

PROPOSITION L.

TO correct the greater Axes of the Orbits of the Planets found before.

In Proposition 24, the greater Axes of the Orbits are taken in the subsepticimate ratio of the periodic Time; that is, the Masses of the Planets are abstracted from, as ought to be done before they are determined, and the Planets are looked upon as points revolving in Ellipses about the center of the Sun at rest and placed in the Focus. But because the actions of the Sun and Planet are mutual, and a Planet is made by them to describe an Ellipse (by *Prop. 58. B I.*) whose

Focus is the common center of Gravity of its self and the Sun, similar to that which it would describe (by *Prop. 51. B. 1.*) attracted by the same Forces, about the Sun at rest ; and (by *Prop. 54. B. 1.*) the greater Axe of the Ellipse that any Planet describes about the Sun, revolving at the same time about the common center of Gravity, is shown to be to the greater Axe of the Ellipse, that the same Planet would describe about the Sun at rest in the same periodic Time, in the subtriplicate ratio of the sum of the Masses of the Sun and Planet to the Mass of the Sun ; therefore the greater Axe of the Orbit of each Planet (found by *Prop. 24.*) is to be increased in the said subtriplicate ratio of the sums of the Masses of the Sun and Planet to the Mass of the Sun, that it may be correct. But (by *Prop. 48.*) the ratio between the Masses of the Sun and Planets are given, and therefore the ratio is given, wherein the greater Axes (found by *Prop. 24.*) of the Orbits are to be increased, that they may be correct.

PROPOSITION LI.

TO estimate the Errors of the motion of the Planets about the Sun, arising from their Action upon one another.

If *S* represent the Sun, [*Fig. 51.*] *ABCE* and *DL* the Orbits of two of the Planets ; 'tis evident that while the nearer to the Sun is in *A*, and the more remote in *D*, (namely when the Planets are tending towards an Heliocentric Conjunction,) the nearer to the Sun will be a little accelerated, and the more distant retarded by the Attraction upon one another ; and when they are arrived after the Conjunction to such a situation as *C* and *D* is, that the more distant one is accelerated, and the nearer is retarded ; and that near the Heliocentric Conjunction the Distance of the more remote

Planet from the Sun is lessened, of the nearer is increased. And since the Masses of the Sun and both the Planets (by *Prop. 48.*) are given, and the mutual Distances from the superiors are also given, and that (by the preced. *Prop.*) correctly; the ratio of the Attraction of each towards the Sun to the Attraction towards the other will be given, and consequently the ratio of the Error arising from the mutual Attraction to the effect of that Force, whereby each is retained in its proper Orbit about the Sun.

But if all the Error is to be thrown upon the Planet that is more remote from the Sun, the Orbit of the nearer remaining, this will be done almost (by *Prop. 58. B. 1.*) by placing the Focus of the exterior Orbit in the common Center of Gravity of the Sun and of the interior.

These Errors are most sensible in Jupiter and Saturn, because of the vast Masses of those Planets.

SCHOLIUM.

There are also some inequalities arising from the mutual Action of the Primary and their Satellites: For (by *Prop. 63. B. 1.*) 'tis not the Center of the Primary, but the Center of Gravity of the Primary and the Satellites, that describes the Elliptic Orbit about the Sun. But because these things depend upon the Quantities of Matter in these Secondaries, the method of determining which properly belongs to the following Book, we shall at present say nothing of them.

SECTION VIII.

Of the Figure of the Sun and Planets.

PROPOSITION LII.

TO determine the Figure of the Earth; that is, the ratio that its Axe has to the Diameters that are perpendicular to them. In

In the first Book, at *Prop. 31*, we shewed in general, from the proper causes, that the Sun and Planets are of a Figure depressed towards the Poles, but elevated towards the Circle, that lies exactly between the Poles, approaching that of an oblate Spheroid. Now we come to determine this Figure by observations. And first the Figure of the Earth is to be defined.

Let the Figure *APEp*, [*Fig. 52.*] whose Center is *C*, represent the Earth; *Pp* its Axis, *AE* any Diameter of the Equator, perpendicular to the Axis. Let the lengths of Pendulums oscillating in a Cycloid, in a given space of time, in the Places *A* and *P*; and *AC*, *PC* will be reciprocally as the said lengths of the Pendulums, and therefore in a known ratio. Let the same be done in any intermediate Place *B*, whose Latitude (*viz.* the angle *ACB*) is known, (by *Prop. 18. B. 2*;) and *AC*, *BC* will be reciprocally as the lengths of the Pendulums oscillating in an equal space of Time in those Places *A* and *B*, and therefore in a given ratio.

For imagine a Canal filled with a Fluid and bent from *A* to *C*, and from thence to *P*. From the rest of the Fluid 'tis evident that the Fluid in the Leg of the Canal *AC* is in equilibrio with the Fluid in the Leg *PC*, the centrifugal Force arising from the motion about the Earth's Axis, and the greater distance from the Earth's Center raising and retaining the Fluid in the Leg *CA*, at a greater height than in the Leg *CP*. Wherefore any portion of the Fluid in *CA*, is in equilibrio (or equally heavy) with the like similarly posited portion of the Fluid in the Leg *CP*, (which consequently is also true of any homogeneous Bodies, tho' they be not Fluid;) and the upper points *A* and *P* are similarly posited in the Legs: And therefore homogeneous Bodies placed in *A* and *P*
which

which are as AC , PC are equiponderant towards the Center of the Earth. But the Gravity of a Body placed in A , which is as PC , is to the Gravity of another homogeneous Body standing there, that is, as AC , as PC to AC , (namely the weights of homogeneous Bodies placed near one another, are as the Bodies themselves;) and therefore the Gravities of homogeneous equal Bodies placed in A and P are as PC and AC ; that is, reciprocally as the distances from the Center. Just after the same manner the Gravity of the Body in B may be shewn to be to the Gravity of an equal and homogeneous Body in P , as CP to CB ; for the Fluid will continue at rest in the Canal BCP , as in the former ACP : Wherefore *ex equo*, the absolute Gravities of equal and homogeneous Bodies placed any where on the Surface of the Earth are reciprocally as the Distances from the Center. But the accelerative Gravity of a Body is as its absolute Gravity applied to its Mass. And therefore the accelerative Gravities of Bodies placed on the Surface of the Earth are as the distances of the Places from the Center reciprocally. Moreover, because the oscillation of a pendulous Body placed at A , and moved in a Cycloid, is supposed (by observation) to be performed in the same space of time with the oscillation of another at P ; and (by *Prop. 25. Part 2. of Hugenius's Horologium Oscillatorium*) the time of a free descent along a given axe of a Cycloid has a given ratio to the time of Oscillation in that Cycloid; and (by *Prop. 6. and 7. Part 3. of the said Book*) the axis of the Cycloid, which a heavy Pendulum describes, has a given ratio to the length of the String suspending the weight; and consequently the time of a free descent along the given axis of the Cycloid, has also a given ratio to the time of a free descent along the length

length of the string, namely, the subduplicate of the former: The time of a free descent along the length of a Pendulum placed at A , will be equal to the time of a free descent along the length of a Pendulum placed at P . But the spaces at A and P , run in equal times by a falling Body, are as the accelerative Gravities in the said Places A and P , *viz.* the effects are as their causes; and the said accelerative Gravities have already been shewn to be reciprocally as the distances from the Center; and therefore the length of a Pendulum oscillating at A , is to the length of a Pendulum oscillating at P in an equal space of time as CP to CA . And after the same manner may it be shewn, that the length of a Pendulum at B is to the length of a Pendulum at P oscillating in an equal space of time as CP to CB : Wherefore, *ex æquo*, universally the lengths of Pendulums oscillating in equal Times are reciprocally as the distances of the Places from the Center of the Earth. Since therefore the ratio of AC to any BC and the inclination of the said BC to AC , and (among the rest) the ratio of AC to PC , are found; the nature of the Figure $APEp$ is defined, namely, of the Section of the Earth with the plane of the Meridian, by whose rotation about the Axis Pp , the Figure of the Earth is generated. *Q. E. F.*

If the Figure of the Earth were to be determined after the manner above described, there would be no need of measuring the length of a Pendulum oscillating at the Pole P , in a given space of time: For by observing several such lengths in different Latitudes, the length of a Pendulum oscillating at the Pole in the said space of time, may safely be concluded upon.

Again, in solving this Problem, there will be no need of fitting a Pendulum after such



Plate 13. Book 3.

Fig. 53.
P. 493.

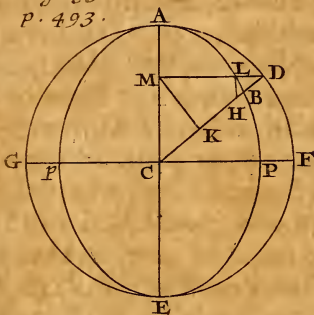


Fig. 54.
P. 498.

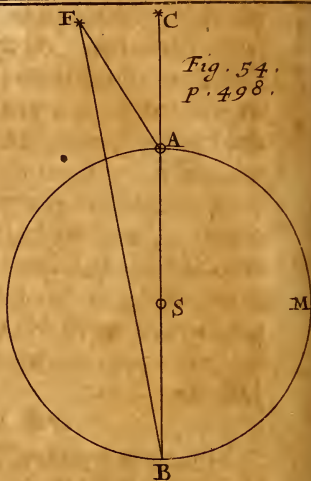


Fig. 55.
P. 499.

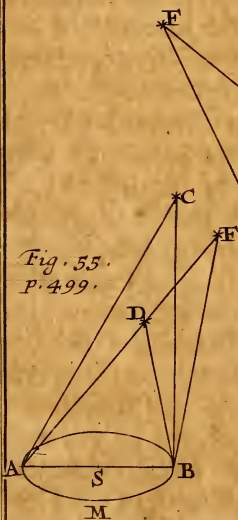


Fig. 56.
P. 501.



a manner in different places, as to make it perform its oscillations in the same space of time: It will be sufficient to carry into the other places a Pendulum fitted to one place, so as that its several single oscillations may answer a given space of time, and mark how much a Clock, that has the said Pendulum for its Ballance, is swifter or slower during one apparent revolution of a Fix'd Star to the same place, according as the places, into which it is carried, are nearer the Equator or Pole, than that place is to which the Movement is adjusted, and from that acceleration or retardation it may be found by calculation, what length is necessary to make the single oscillations to be performed in the same space of time, as they are in the place to which the Pendulum was first adjusted.

C O R O L L A R Y.

Hence it follows, (the same things remaining) if a Circle be described upon the interval CA or CE [Fig. 53.] and the line CB drawn any how be produced to D , that DB is as the increment of the accelerative Gravity in the recess from the Equator towards either Pole, or as the increment above the length of the Pendulum oscillating at the Equator in the given time, agreeable to the place B . For it has been shewn above, that if AC represent the gravity or length of a Pendulum at the Pole, CP will represent the increment of the gravity or length of the said Pendulum oscillating in an equal time at the Equator; and therefore (producing Pp to F and G) PF will represent the increment of the gravity or of the length of the said Pendulum at the Pole P . In like manner, if CB represent the gravity or length of the Pendulum at A , AC (that is, DC) will express the gravity or length of the Pendulum at B ; and consequently DB the excess or increment of the gravity

vity or length of the Pendulum at B , above the gravity or length of the Pendulum oscillating in the same time at A .

If $APEp$, the section of the Earth with the plane of the Meridian be an Ellipse, the increments of the gravity or of the length of the Pendulum in the different places of the Surface of the Earth, above the gravities or length of the Pendulum at the Equator, will be very nearly in the duplicate ratio of the right Sines of the Latitudes of those Places. Thro' D draw DM parallel to the Axe Pp , meeting the Ellipse in L , and the right line AE in the point M , from whence let fall the perpendicular MK upon CD ; take DH equal to PF , wherefore CH is equal to CP , and join LH : Because APE is an Ellipse, (by *Prop. 31. B. 1. El. Conic.*) CP^2 is to ML^2 as the rectangle under AC and CE is to the rectangle under AM and ME : And, by the same, in the Circle AFE , CF^2 is to MD^2 as the rectangle under AC and CE to the rectangle under AM and ME ; and therefore CF^2 will be to MD^2 as CP^2 to ML^2 . Wherefore CF is to MD as CP to ML ; and therefore (by *Prop. 19. El. 5.*) PF is to LD as CP to LM ; that is, HD is to LD as CH to ML ; and therefore (by *Prop. 2. El. 6.*) LH is parallel to CM ; and consequently the Triangle LDH is similar to the Triangle MDC , and similarly posited. But from the right angle M of the Triangle CMD the right line MK is drawn perpendicular to the Hypotenuse DC , which also is very nearly done in the Triangle $HL D$, the particle LB of the curve of the Ellipse being almost a right line and perpendicular to CD , because the Ellipse $APEp$ does not differ much from the Circle $AFE G$: Wherefore CD is similarly divided in K , as HD is divided in B ; that is, HD is to DB as CD to DK , or PF to BD as CD to DK .
Again,

Again, because of the rightangled Triangle CMD divided by the right line MK perpendicular to CD , (by *Prop. 8. Elem. 6.*) CD, DM, DK will be proportional, or the ratio of CD to DK the duplicate of the ratio of CD to DM ; that is, of the ratio of CF to MD . But it has already been shewn, that PF is to BD as CD to DK ; and therefore PF is to BD in the duplicate ratio of CF to MD : But CF is the whole Sine, and MD the right Sine of the angle ACB , namely of the Latitude of the place B ; and PF, BD , are the increments of the gravity or length of the Pendulum oscillating in the given time, agreeable to the Places P and B , above the gravity or length of the Pendulum at the Equator: And therefore supposing $APEp$ an Ellipse, the increment of the gravity or length of the Pendulum at the Pole, above the gravity or length of the Pendulum at the Equator, is to the like increment in any other Place B of the surface of the Earth, very nearly in the duplicate ratio of the Radius to the right Sine of the Latitude of the said place B . And the same may be shewn in any other place on the surface of the Earth: And therefore, *ex æquo*, if $APEp$ be an Ellipse, the increments of the gravity or of the length of the Pendulum in different places of the surface of the Earth, above the gravity or length of a Pendulum oscillating the like Time at the Equator, in the duplicate ratio of the right Sines of the Latitude of those places very nearly.

SCHOLIUM.

All these Premises are true, supposing the Earth to consist of uniform and equally dense matter, For we supposed the Fluid, wherewith the Canals ACP, BCP are filled, to be uniform and homogeneous. But if the matter whereof the Earth consists, be denser at the Center than towards the surface, the above mentioned increments of the

di-

distances from the Center going from the Poles towards the Equator, and the proportional increments of Gravity and of the length of the Pendulum oscillating in the given Time going from the Equator towards the Poles, will be greater than the Calculus upon the former supposition makes them; and the Axis of the Earth will be less in respect of the Diameters perpendicular to the same. For if this matter towards the Center, by which the Earth is denser there, be imagined to be removed, or rather be looked upon separately, that the remaining matter may be every where equally dense; from what precedes 'tis evident, that the accelerative Gravity in different places of the surface of this equally dense Earth, is reciprocally proportional to the distance from the Center. But the said accelerative Gravity is still increased, by reason of that redundant matter about the Center; and that (as is shewn in *Book I.*) nearly in the duplicate reciprocal ratio of the Distance from the Center; Since therefore in the case of the uniformly dense Earth its superficies is elevated towards the Equator and depressed towards the Poles; and the Gravity at the Equator less than at the Pole in the ratio of the distance of the Pole from the Center to the semidiameter of the Equator; and further the gravity at the Equator to the aforesaid redundant matter about the Center should be less than at the Pole, in a duplicate ratio of the distance of the Pole from the Center to the semidiameter of the Equator, which is less than the former simple ratio, since that ratio is a ratio of lesser inequality; and therefore from both causes together, or in the case of the Earth's being denser toward the Center, the Gravity at the Center compounded of the two former, is less than the Gravity at the Pole in a ratio less than that is which the distance

distance of the Pole from the Center has to the semidiameter of the Equator. And therefore from this lesser Gravity at the Equator in respect of that which holds at the Poles, the Earth will be more elevated at the Equator, than what it would be from what goes before. And the same holds in the length of the Pendulum oscillating in a given space of time; because it is shewn above to be proportional to the accelerative Gravity.

PROPOSITION LIII.

TO determine the ratio that the Axis of the Sun, or of a given Planet, has to the Diameters perpendicular to it.

Find by *Prop. 26. B. 1.* the ratio of the centrifugal Force in the Circle that is in the middle between the Poles of the Sun or of the given Planet to the centrifugal Force in the Equator of the Earth: But the ratio between the centrifugal Force in the Equator of the Earth to the Gravity on the Surface of the Earth is given, *viz.* the same with the ratio found above, that the excess of the Diameter of the Earth along the Equator has to the Axis of the same; and by *Schol. Prop. 48.* the ratio of the Gravity on the surface of the Earth to the Gravity on the surface of the Sun or of the given Planet is given: And therefore the ratio compounded of these will be given, namely the ratio that the centrifugal Force in the middle Circle, between the Poles of the Sun or of the given Planet, has to the Gravity on the surface of the same. And this is the same with that which the excess of the Diameter of the Equator of the Sun or of the given Planet above the Axis of the same to the said Axis, since (by *preced. Prop.*) that excess arises only from that centrifugal Force, and consequently is proportional to it: There-

fore (by *Dat. 6.*) the ratio between the Axes of the Sun or of a given Planet and its Diameter perpendicular to the Axe will be given.

SECTION IX.

Of the Distances of the Fix'd Stars.

PROPOSITION LIV.

TO determine the Distance of a Fix'd Star by Observation.

Having determined the Distance and Magnitude of the Sun and Primary Planets, what remains is only to discourse a little of the Distance of the Fix'd Stars. And tho' the Distance of the Fix'd Stars be so immensely great in comparison of the Diameter of the Earth, that the latter intirely vanishes and becomes insensible; and therefore all the ways laid down in this and the preceding Book for finding the Parallax of any Phenomenon, and consequently the Distance, are useless in this matter: Yet the Diameter of the Orbit that the Earth describes in the space of a Year about the Sun, may seem big enough to be compared even with the Distance of the Fix'd Stars: and since we, who are carried on the Earth, possess sometimes this, sometimes that extremity of the Diameter of the said Orbit, there ought to arise a very sensible Parallax of the Fix'd Stars from so sensible a removal of the Observer. The Observation may be made very commodiously after the following or some such like method.

Let *S* [*Fig. 54.*] represent the Sun, *AMB* the *Orbis Magnus*. Chuse two Stars *C* and *F* near
one

one another, situated not far from the Ecliptic. Observe their Distance, when one, as C , is in opposition to the Sun, *viz.* the angle FAC . Observe their Distance again, when C is in conjunction with the Sun, namely the angle FBC . Therefore in the Triangle FAB all the angles being given, the ratio of the sides is given, namely of FA the Distance of the Fix'd Star from A to AB the Diameter of the *Orbis Magnus* known by *Prop.* 45; therefore AF is determined. After the like manner the Distance of a Fix'd Star may be found, if the second observation of the apparent Distance of the Fix'd Star be made, when C is not in conjunction with the Sun, (which perhaps would be difficult, by reason of the Sun's brightness,) but in any other known situation of the Earth, as when it is in M : For the right line AM , and its ratio to the known one AB , is known by the above cited *Prop.* 45.

Having once determined the Distance of a Fix'd Star, its Magnitude will be defined by the method laid down in *Prop.* 46, provided the apparent Diameter of the Fix'd Star be sensible.

S C H O L I U M.

Because the Observations described above, or others like them necessary to determine the Distance of the Fix'd Stars, are so slippery and require so much nicety, that Artists dare not promise they shall obtain it by those means; yet they think they have done something very considerable, if they can show by observation that there is a Parallax of the Fix'd Stars in respect to the *Orbis Magnus*: For by this means they could put the Motion of the Earth out of all doubt, which all acknowledge to be a matter worth their while to do. But this may be done pretty well by the following method.

The other things remaining, let the two Fix'd Stars F and C (*fig.* 55.) be supposed not to be near

the Ecliptic. Let a Plane drawn thro' them and the Sun cut the Orbit of the Earth in A and B , which Points will be given exactly enough for the purpose, by what has been said in the preced. Book. Let the angles CAF , CBF (or the apparent Distances of the Fix'd Stars F and C , seen from the Earth, when she was in A and B ,) be observed, which will be remarkably enough different from one another, if AB be sensible in regard of AC , and the Stars C and F very unequally distant from the Sun, which seems probable to hold in Stars of a different Magnitude. Perhaps if instead of the two Stars C and F , two, such as C and D , be observed, D will be seen from A to be on this side of C , from B on that side, (which may happen in so great a multitude of Fix'd Stars) or it may be, these two Stars will appear to coincide at one time of the Year, and to be sensibly distant from one another at another time of the Year: In which case the Annual motion of the Earth about the Sun will be evident by the bare use of a Telescope, without any Instrument necessary to take the distance of the Stars,.

The above mentioned method of observing, besides that it is capable of receiving all the improvement, that any Instruments have or will receive, is hardly any ways chargable, since there needs only a Telescope with a Micrometer, there is no need of erecting an Observatory nor of rectifying a Perpendicular, all which are very uncertain, since it is not absolutely certain that either Walls tho' the firmest, or Rocks or Mountains themselves always retain the same situation. To this head we may refer perhaps what that celebrated Astronomer Mr. *Cassini* has discovered about the Fix'd Stars; namely that the First of Aries sometimes appears split into two equal ones, and distant by the length of the Diameter of either from one another,

ther, which he has also observed of the preceding Head of Gemini ; and some others, as some of the Pleiades ; and the middle in Orion's Belt, appear to be three, nay, four sometimes.

PROPOSITION LV.

TO observe the Parallax of the *Orbis Magnus*, by the access and recess of a Fix'd Star to or from the Pole of the Equator made in different Seasons of the Year. [Fig. 56.]

Let *S* represent the Sun ; *HME* the Orbit of the Earth, whose points *E* and *H* are the places of the Earth in the Solstices. Further let *F* be a Fix'd Star near the Pole of the World, that is, let *AB* the Axis of the Earth *ATB* contain with a right line connecting the Earth and the Fix'd Star an angle *BHF* or *BEF* pretty small ; and let this Star be not far from a Solstitial Colure, and therefore in conjunction almost with the Sun in the other Solstice, or let the Plane *EFH* be perpendicular to *HME* the plane of the Ecliptic. 'Tis evident that the angle *BHF* the distance of the Fix'd Star from the Pole of the World, when the Star is in opposition to the Sun, is less than the angle *BEF* the distance of the same from the Pole, when the Fix'd Star is in conjunction with the Sun. And conversely, if the angle *BHF* be found less by observation than *BEF*, this inequality must be thrown upon the Parallax of the Annual Orbit. And the Distance of the Fix'd Star is gained after this manner: The difference of the angles *BEF*, *BHF* is equal to the angle *F* ; and in the Triangle *FEH* the other angle *FEH* is given, namely the Latitude of the Fix'd Star at the Time, when the Earth is at *E* : From whence the ratio between *HE* the Diameter of the *Orbis Magnus*, and *HF* the Distance of the Fix'd Star will be known.

SCHOLIUM.

This method of finding by Observation the Parallax of the *Orbis Magnus*, that most industrious Observer Mr. *John Flamsteed* makes use of, and so answers the objection of such, as with *Ricciolus*, deny the motion of the Earth about the Sun, because it is confirmed by the observation of a Parallax of the *Orbis Magnus*. For he found the Distance of the Pole Star from the North Pole to be greater about the Summer Solstice than about the Winter one, by about 40 or 45 Seconds, and that by a continued series of Observations for seven Years, allowing for the correction of the Instrument, and the change of the Star's place on the account of the recession of the Equinoctial points: as Mr. *Flamsteed* learnedly discourses in a Letter to Dr. *Wallis*, dated Decemb. 20, 1698 published in 3 Vols. of *Wallis's Mathematic Works*. But this method of determining the Parallax of a Fix'd Star, supposes the Axis of the Earth *AB* [*Fig. 57.*] to be exactly parallel to itself, when the Earth is in the points *E* and *H* of its Orbit, when the Observations are made. And tho' that small Nutation of the Axis which arises from the diminution of the Inclination of the Ecliptic and Equator at the Solstices, and the increase of it at the Equinoxes (by *Corol. 20. Prop. 66. B. I. of Newton's Math. Princip. Nat. Philos.*) does not affect or hinder this observation, as Mr. *Flamsteed* observes in the said Epistle; yet another Nutation arising from another cause may produce all this diversity in the distance of the Pole-Star from the Pole: Namely, if the density of the Southern Hemisphere of the Earth be a little greater than that of the Northern, (either because of the lesser Heat there and greater Cold here, or because of the inequalities of the Continents of the Earth placed about the Poles, or some other cause unknown to us,) for then since in the Winter Solstice the



Plate 14. Book 3.

Fig. 58.
P. 503.

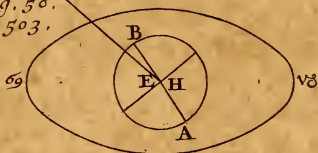


Fig. 57.
P. 502.

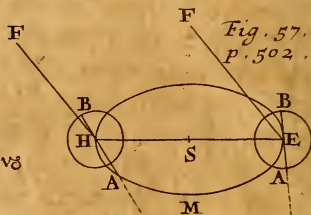


Fig. 59. P. 505.

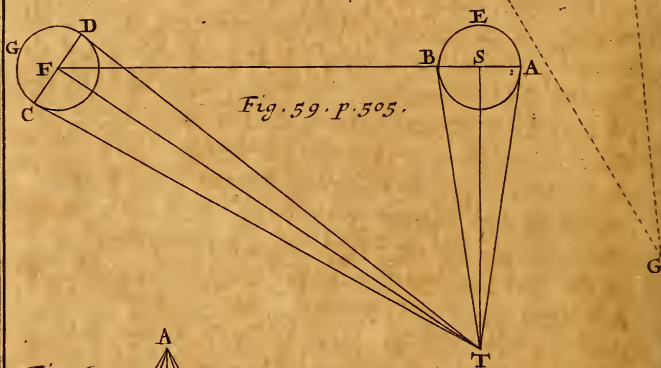


Fig. 60.
P. 506.

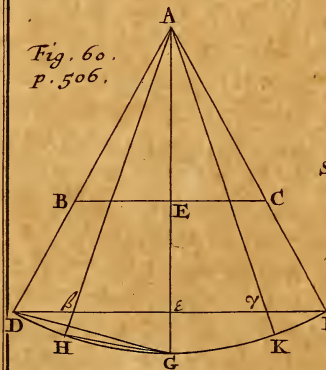
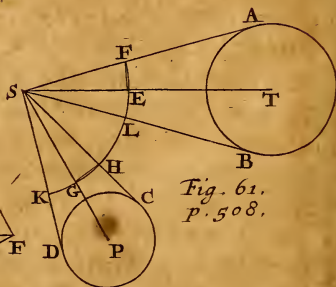


Fig. 61.
P. 508.



the South Pole A is inclined toward to the Sun, and at the same time is nearer to it than the North Pole B , when in the time of the Summer Solstice the latter is inclined towards the Sun, HA will be more inclined to the plane of the Ecliptic in the Winter time than EA in the Summer time; on which account BHF will be less than BEF , tho' the right lines HF , EF should be parallel. Since therefore all that can be made out of Mr. *Flamsteed's* observation is, that the angle BHF is less than the angle BEF ; and this may arise from a double cause, namely from the concurrence of the right lines HF , EF towards F , if the Axis of the Earth in one observation be parallel to the same in the other (which is supposed to be the case by Mr. *Flamsteed*, and expressed by the Fig. 56.) or from the concurrence of the lines right HA , EA towards G the quite contrary way to F , right lines HF , EF being supposed to be parallel, as in the Figure 57; from this observation the Parallax of the Fix'd Stars is not evinced, because the observation may stand intire though the right lines EF , HF remain parallel; that is, tho' the Parallax of the *Orbis Magnus* be nothing.

Nay, this observation does not immediately prove even the Annual Motion of the Earth: For tho' the Earth be in the middle (revolving about its Axe, as in the *Semi-Tychonic* System, [Fig. 58.] and so make the apparent Diurnal Revolution of the Celestial Bodies;) the Sun in the Southern Signs may then attract the Southern Hemisphere of the Earth turned to it, being nearer and perhaps denser, so as to make the angle BHF , the distance of the Fix'd Star from the Pole, less than BEF , when the Sun in the Northern Signs being more distant, attracts the Northern Hemisphere less, which also perhaps is less dense.

The same likewise may be said of the different

Distance of a Fix'd Star from the Zenith, which is the way Dr. *Hook* formerly sought after the Parallax of the Fix'd Stars. For if the direction of the Axis of the Earth be changed, the direction of any other right line in the Earth (whose position to the Axe is given) will likewise be changed: And the motion of the Earth about the Sun, can't be certainly concluded, from the various inclination of a right line connecting the Fix'd Star and the Earth to that right line in the Earth. It were therefore to be wished, that our exact Observers would for the future manage their observations for proving the Annual Motion of the Earth after this manner, according to the *prec. Prop.* or its *Schol.* For so at last the Parallax of the Fix'd Stars in regard of the *Orbis Magnus* will with any certainty be found (if it be sensible,) since the change of those angles can't arise from any other cause, but the change of the Earth's place or of the situation of the Fix'd Stars among themselves; but the identity of the situation of the Fix'd Stars in regard of one another is taken as a principle by all such as endeavor to find their Parallax. In the mean while the reasons laid down in the following Proposition incline me to believe with *Copernicus*, that the Diameter of the *Orbis Magnus* in regard of the Fix'd Stars (even the nearest) is insensible.

PROPOSITION LVI.

THE Distance of the Sun from the Earth in respect of the Distance of a Fix'd Star (even the nearest) from the Earth, vanishes almost and is insensible.

Since the Sun and Fix'd Stars are the only great Bodies of the Universe that have any native Light, they are justly esteemed by Philosophers to be of the same kind, and designed for the same uses; and 'tis the effect of a Man's temper, that sets a greater value upon his own things than

he

he ought, that makes him judge the Sun to be the biggest of them all. Let them therefore be supposed equal, since that the one exceeds the other has no ground or foundation to stand upon.

Let T be the Earth, [Fig. 59.] AEB the Sun whose Center is S ; CGD a Fix'd Star equal to the Sun, whose Center is F . Since the angle BTS , that the semidiameter of the Sun seen from the Earth appears under, exceeds 16 Minutes, then will BS or CF equal to it, be almost the two hundredth part of ST ; or 200 times CF will be equal to TS . In the Triangle FST rightangled at S , the Sine of the angle SFT is to the side ST as the radius to the side CT , equal to FT : From whence (by Prop. 11. Elem. 5.) the Sine of SFT is to ST as the Sine of the angle CTF to CF . And since, in these small angles, the angles themselves are as their Sines, the angle SFT will be to the angle CTF as the right line ST to CF ; and therefore 200 CTF is equal to SFT , or 100 CTD to TFS . But that most acute Observer *Christian Hugen* (in his *System of Saturn*) affirms, that he, assisted by a Telescope, (by the help of which the diameter of an object was magnified an hundred times) could not observe any magnitude in the Diameters of the Fix'd Stars tho' the brightest, but that they appeared like a point; that is, the angle DTC made an hundred times bigger, was still insensible; or an hundred times CTD (that is, SFT) could not be taken notice of by Sense. Therefore the Semidiameter of the *Orbis Magnus* ST seen from the nearest Fix'd Star (such as we are to suppose the brightest to be) would appear like the smallest Point; that is, TS in respect of SF or TF vanishes and is insensible.

This smallness of the apparent Diameters of the Fix'd Stars is the cause of their Twinkling: For the smallest opaque corpuscles, which are always moving

moving in the Atmosphere, intirely cover any Fix'd Star, and then immediately uncover it, that is, make it Twinkle.

SCHOLIUM.

Because ST compared with SF is insensible, 'tis no matter in what point of the *Orbis Magnus* the Fix'd Stars be observed; for the whole *Orbis Magnus* is but as a point in respect of the Distance of the Fix'd Stars: And since Saturn the outmost of the Planets, is not above ten times farther off from the Sun than the Earth, there will be immense Spaces between this outmost Planet and the nearest Fix'd Stars to the Sun, into which the very excentric Orbits of the Comets may run, of which we have treated in *Prop. 35. B. I.* And 'tis on the account of this immense Distance, that the Fix'd Stars have no sensible effects upon our Solar System, and don't disturb the Planets contain'd in it.

Since therefore the Distance of the Fix'd Stars can't be determined by observations made in different places of the Earth's Orbit, it remains that we show how to measure it by some Optical Methods: Two of which we shall propose, one by Mr. James Gregory publish'd in the year 1668, the other by Mr. Hugen in his *Cosmo-Theoria*.

PROPOSITION LVII. LEMMA.

THE illuminations of the same Sphere, in different distances from the lucid Body, are reciprocally as the Squares of the distances.

Tho' this Proposition be an easy consequence of *Prop. 48. B. I.* yet I shall take the pains to demonstrate it here. Let A [*Fig. 60.*] be any point of a lucid Body taken at pleasure; and let BC be the diameter of the Sphere that is illuminated, directly opposed to the Rays, at a smaller distance from the lucid Body; & by the same diameter at a greater distance:

distance; that is, let AE , a ray reaching to the Center E be perpendicular to BC , and A^e to $\beta\gamma$: Join AB , AC , and being produced let them meet $\beta\gamma$, also produced, in D and F . On the Center A , at the distance AD or AF , describe an arc of a Circle DF , which let AB , A^e and $A\gamma$ produced, meet in H , G and K ; and let this Figure be imagined to revolve about the axe AG . 'Tis evident that all the Rays falling upon the Sphere, whose diameter is BC , upon the removal of it, will be spread upon the Sphere whose diameter is DF : And therefore the illumination of the Sphere, whose diameter is BC , is to the illumination of the Sphere, whose diameter is $\beta\gamma$, as all the rays, contained within the Sector of the Sphere generated by the rotation of ADG , to all the rays contained within the Sector of a Sphere generated by the rotation of AHG ; that is, as the Surface of a Sphere generated by the rotation of the arc DG , to the Surface of a Sphere generated by the rotation of the arc GH ; that is, (by *Prop. 49. B. 1. of Archimed. of the Sph. and Cyliind.*) as the Circle whose radius is DG , to a Circle whose radius is HG , or as the Square of DG to the Square of HG . But in small angles, such as we suppose DAG and HAG to be, DG is to HG very nearly as DE to β^e : And therefore the illumination of a Sphere, whose Center is in E , is to the illumination of the same or an equal Sphere, whose Center is in e , in the duplicate ratio of D^e to β^e , or D^e to BE , or A^e to AE ; that is, reciprocally as the Square of the distances. And since the same, for the same reasons, is true of any other point of a lucid Body, as well as of A , what was proposed becomes evident.

And after the like manner, the illuminations of the same Sphere made by equal lucid Bodies at unequal distances, are reciprocally as the Squares of the distances from those lucid Bodies.

PROPOSITION LVIII. LEMMA.

ALL the Rays of the Sun falling upon one Planet are to all the Rays of the Sun falling upon another Planet, as the Squares of the Chords of the Horizontal Parallaxes of the Sun seen from those Planets. [Fig. 61.]

Let S represent any radiating point of the Sun; and the Spheres AB , CD , whose centers are T and P , any two Planets. Join ST , SP , let a plane drawn thro' them cut the Spheres in the Circles AB , CD ; and from S draw Tangents to the said Circles. Upon the Center S , at any distance, draw an arc of a Circle meeting the right lines SA , ST , SB , SC , SP and SD in F , E , L , H , G and K ; join the right lines EF , GH . Let the Figure $AFSET$ be imagined to revolve about its Axis ST , and $CHSGP$ about the Axis SP . All the Rays flowing from the point S that fall upon the Planet T (*viz.* all within the Cone ASB) will be to all the Rays falling upon the Planet P (namely all in the Cone CSD) as the Superficies of a Sphere described by the rotation of the arc EF , to the Superficies of a Sphere described by the rotation of the arc GH ; that is, (by *Prop. 49. B. I. of Archimedes of the Sphæ. and Cyl.*) as a Circle described upon the radius EF to a Circle described upon the radius GH , or as the Squares of the right lines EF , GH : But EF and GH are Chords of the angles AST , CSP to the radius SE , and the said angles are the horizontal Parallaxes of the Sun S , seen from the Planets T and P ; and therefore all the Rays flowing from the point S and falling upon the Planet T , are to all falling upon P , as the Square of the Chord of the Sun's Parallax seen from T , to the Square of the Chord of the Sun's Parallax seen from P . And since the same is in like manner true of Rays flowing from any other point of the Sun, the Proposition is evident.

Instead of the Chords the Arcs themselves, or the Angles proportional to them, and subtended by them, may be taken, as in the preced. Prop.

SCHOLIUM.

After the like manner, all the rays of a lucid and every way radiating Sphere, are to all its rays that fall upon a given Sphere and illuminate it, in the duplicate ratio of the Diameter (or Chord of the Semicircle) to the Chord of the horizontal Parallax of the lucid Body, seen from the illuminated Body; or as the Square of the Diameter to the Square of the Chord of the said horizontal Parallax. And consequently (by taking the halves of the antecedents,) the half of all the Rays of the lucid Sphere, or all the Rays flowing from a radiating Hemisphere (supposing the other Hemisphere opaque) are to all its Rays falling upon the Sphere to be illuminated, to which it appears full, as the half of the Square of the diameter (that is, the Square inscribed in a Circle) to the Square of the Chord of the Parallax of the lucid Body, seen from the Sphere to be illuminated.

PROPOSITION LIX. LEMMA.

THE ratio of the illumination of the Earth by the Sun to its illumination by a Planet shining at the full, is compounded of the duplicate ratio of the Chords of the Parallaxes of the Sun seen from the Earth and that Planet, and the ratio of a Square inscribed in a Circle to the Square of the Chord of the horizontal Parallax of the said Planet seen from the Earth.

The Rays, with which the Planet illuminates the Earth, are such as flowing originally from the Sun, are reflected back again upon the Earth. We suppose therefore that each Planet reflects every way all the Rays of the Sun that fall upon it; that is (for greater ease) we suppose a Planet to emit Rays every way from that Hemisphere of it
that

that is turned towards the Sun, *viz.* so many as fall upon it from the Sun. 'Tis evident that the illumination of the Earth by the Sun is to the illumination of the same by a Planet shining in the full, as all the Rays of the Sun falling upon the Earth to all the Rays flowing from the Planet and falling upon it: But the ratio of the Sun's Rays falling upon the Earth to the Planet's Rays falling upon the same, is compounded of the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, (that is, all the Rays flowing from the Planet's Hemisphere,) and the ratio of all the Rays flowing from the Planet's Hemisphere to all those of them that fall upon the Earth. And the former of these compounding ratios, or the ratio of the Sun's Rays falling upon the Earth to the Sun's Rays falling upon the Planet, is equal (by the *preced. Prop.*) to the duplicate ratio of the horizontal Parallax of the Sun seen from the Earth to the horizontal Parallax of the Sun seen from that Planet; but the latter, namely the ratio of all the Rays flowing from the Planet's Hemisphere illuminating the Earth to all of them that fall upon the Earth and inlighten it, is equal (by the *preced. Schol.*) to the ratio that a Square inscribed in a Circle has to the Square of the Chord of the horizontal Parallax of the said Planet seen from the Earth. And therefore the ratio of the illumination of the Earth by the Sun to the illumination of the same by a Planet shining in the full, is compounded of the duplicate ratio of the Chords of the horizontal Parallaxes of the Sun seen from the Earth and that Planet, and of the ratio of the Square inscribed in a Circle to the Square of the Chord of the horizontal Parallax of the said Planet seen from the Earth. *Q. E. D.*

PROPOSITION LX.

TO determine the Distance of a Fix'd Star.

Chuse a Fix'd Star of nearly the same brightness with some Planet, when it is in the Full; in which case the Earth is equally illuminated by these two Stars. But the mutual Distances of the Sun, Earth, and Planet at the time of the above-mentioned Observation, and the Magnitudes of these three Bodies (by *Prop. 46.*) being known, the horizontal Parallaxes of the Sun and Planet seen from the Earth (by *Prop. 48. B. 2.*) become known, together with the angle the said Planet appears under from the Sun, whose half is the horizontal Parallax of the Sun seen from that Planet. And consequently (by the preced. *Prop.*) the ratio of the illumination of the Earth by the Sun to the illumination of the same by the said Planet becomes known; that is, the ratio between the illumination of the Earth by the Sun and by the Fix'd Star: But (by *Prop. 57.*) the illumination of the Earth by the Sun is to the illumination of the same by a Fix'd Star (equal to the Sun and equally lucid) in the duplicate ratio of the Distance of the Fix'd Star from the Earth to the Distance of the Sun from the same; therefore the ratio between the Distances of the Fix'd Star and the Sun from the Earth is known; and consequently the Distance it self of a Fix'd Star from the Earth is known.

S C H O L I U M.

For the same purpose you may commodiously take Jupiter in its Achronical situation, and Sirius, which (if any other) may be looked upon as equal to the Sun. But since Jupiter in the said situation exceeds Sirius in brightness, and besides the Rays of the Sun falling upon Jupiter are not reflected towards

towards the Earth without being weakened, but are most of them stifled near the surface of Jupiter, the Distance of Sirius determin'd after this manner will be less than the true Distance.

PROPOSITION LXI.

TO measure the Distance of a Fix'd Star another way.

Tho' the diameter of the brightest Fix'd Star can't by any Optical Instrument yet invented, be made so big as to become sensible, and such as may be compared with the Sun's Diameter; however, the Sun's Diameter may be lessen'd so much, as that its Light thus lessen'd may (in appearance) be no greater than the light of Sirius twinkling in the Night; which will be done if its Rays be let into a Tube thro' a hole small enough; or if its image be lessened as much by means of a Lens, or partly by one partly by the other method: For the diameter of the image of the Sun, tho' insensible (if looked for inconsiderately) becomes known by calculation founded upon Optics and Dioptrics, having the Diameter of the hole, the Lens and situation of the Eye given. But the Diameter of an image of the Sun, as bright as Sirius to an Eye given by position, is equal to the Diameter of Sirius; therefore the apparent Diameter of Sirius, before insensible, becomes known by calculation: But the Sun and Sirius are supposed equal; therefore their Diameters are equal: And therefore the Distance of Sirius will be to the Distance of the Sun, as the Sine of the apparent Diameter of the Sun to the Sine of the apparent Diameter of Sirius, or (by reason of the smallness of these angles) as the apparent Diameter of the Sun to the apparent Diameter of Sirius; therefore the Distance of Sirius will be known.

Q. E. I.

The End of the First Volume.







